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by Shi Xiuhong

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动态波动模型的检验

史秀红／著

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内容提要

波动模型是分析金融数据、估算金融风险的主要工具之一，受到了学界和业界的广泛关注。虽然金融波动模型的参数估计和实证方法备受重视，但对模型的检验方法的研究却被人们严重地忽视了。众所周知，模型的前提条件关系到模型的应用以及结论的正确与否。遗憾的是，由于仅有的几篇关于模型检验的文献都是基于沃尔德检验或尤度比检验的思想，面临着不可定义参数检验的难题，无法保证其检验结果的正确性。

本书借鉴量子力学[redacted]思想，率先建立拉格朗日乘数检验统计量，以[redacted]的检验问题。各章的主要内容分别如下：

第一章，介绍金融波[redacted]系；

第二章，在随机波[redacted]议检验 EGARCH 模型的拉格朗日乘数检验统[redacted]仿真和实证分析，验证该检验统计量的检验能力；

第三章，在 Jump-GARCH 模型的基础上，提议检验跳跃现象存在与否的拉格朗日乘数检验统计量，并应用计算机仿真，验证该检验统计量的正确性；

第四章，在 Jump-GARCH(t) 模型的基础上，提议检验跳跃现象的拉格朗日检验统计量，并用计算机仿真和实证分析加以验证；

第五章，分别在 Jump-EGARCH 模型和 Jump-EGARCH(t) 模型的基础上，提议检验跳跃现象的拉格朗日乘数检验统计量，并通过计算机仿真和实证分析加以验证；

第六章,在 Jump-SV 模型的基础上,提议检验跳跃现象的拉格朗日乘数检验统计量,并通过计算机仿真和实证分析验证该检验统计量的检验效率。

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1

Financial Volatility Models

1.1 Stylized Facts

Financial time series data has several "stylized facts". The first is volatility clustering. As Mandebrot (1963) wrote, "... large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes ... ". This volatility clustering is evident when returns are plotted through time. The second is fat tails of the distribution. Asset returns tend to be leptokurtic as documented by Mandelbort (1963), Fama (1965), and others. They modeled stock returns as identically and independently distributed series sampled from fat-tailed distributions. The third is leverage effects, which refers to the tendency for changes in the first moment to be negatively correlated with changes in the second moment. The forth is the presence of non-trading periods. Information that accumulates while financial markets are closed is reflected in prices after the market reopens. The fifth is the presence of forecast for events. Forecast releases of information are associated with high ex ante volatility. The sixth is the effect of macroeconomic variables and volatility.

Volatility is the central concept in understanding these features.

The study for volatility should be a main issue in financial time series analysis, since volatility is an important element in the asset pricing model of Sharpe (1964) and the option's pricing model of Black and Scholes (1973). However, until recently, the focus of most statistical analysis of financial time series has been centered on the conditional first moment, with any temporal dependencies in the higher order moments treated as a nuisances. The increased importance of risk and uncertainty considered in modern finance theory, however, has necessitated the development of new econometric time series techniques that allow for the modeling of time-varying variances and covariances, i. e. time-varying volatility.

There are two types of the time varying volatility models. The first one is the class of Autoregressive Conditional Heteroskedasticity model (hereafter ARCH-type model), which contains the ARCH model introduced by Engle (1982) and its variants. The second is stochastic volatility (SV) model. However, jump process is so popular in statistical analysis of financial time series recently that it should be considered as the third approach of the volatility model, even though it is not a branch in the view of specification of volatility model. The following three subsections would give detailed evaluation of theoretical developments on the three approaches respectively from the econometrics point of the view.

1.1.1 ARCH-type Model

The ARCH-type model is the most widely used model of the three approaches. It offers a key insight in the distinction between the conditional and unconditional second-order moments. The unconditional covariance matrix for the variables of interest may be time invariant, the conditional variances and covariance often depend non-trivially on the past states.

Understanding this temporal dependence exactly is crucially important not only for many issues in finance theory, such as option pricing, the term structure of interest rates, general dynamic assert pricing; but also for statistical analysis. In particular, a time series $\{x_t\}$ can be represented as

$$x_t = \sqrt{h_t} u_t \qquad (1-1)$$

where $u_t \sim N(0,1)$, and

$$h_t \equiv Var(x_t | \Omega_{t-1}) \qquad (1-2)$$

suggests that the conditional variance for variable x_t, as well as Ω_{t-1} denotes the information set at time $t-1$. The precise parameterization of this conditional variance function is an important issue of econometric specification, just as the specification of the mean.

Numerous specifications for the time varying conditional variance have been proposed in the literatures. In the ARCH (p) model proposed by Engle (1982), the conditional variance is defined as a linear function of the past p squared innovations,

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 \qquad (1-3)$$

where $\alpha_0 \geqslant 0, \alpha_i \geqslant 0, i = 1,2,\cdots p, and \sum_{i=1}^{p} \alpha_i < 1$.

For a ARCH(p) model, the value of p is large in practice, and then the ARCH(p) model was generalized to GARCH (p,q) model by Bollerslev (1986). In particular:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_{1i} x_{t-i}^2 + \sum_{j=1}^{q} \alpha_{2j} h_{t-j} \qquad (1-4)$$

where $\alpha_0 > 0; \alpha_{1i} \geqslant 0, \forall i \in [1,p]; \alpha_{2j} \geqslant 0, \forall j \in [1,q]; \sum_{i=1}^{p} \sum_{j=1}^{q} (\alpha_{1i} + \alpha_{2j}) < 1$.

Apparently, the GARCH model can be rewritten as an infinite order ARCH model and often provides a highly parsimonious lag shape. Empirically GARCH(1,1) has been very successful with in this vast class. Furthermore, these applications typically revealed that there is a long-term persistence in the effect of shocks at time t onto the conditional volatility.

In spite of the apparent success of these simple parameterizations, there are some features of the data in which these models are unable to capture. The most interesting of these is the "leverage effect", emphasized by Nelson (1990) based on an argument by Black (1976). Statistically, this effect says that negative surprises to asset markets increase predictable volatility more than positive surprises. Thus the conditional variance function ought not to be constrained to be symmetric in past information. Nelson (1990) proposed the exponential GARCH or EGARCH model:

$$\ln(h_t) = \alpha_0 + \sum_{i=1}^{p} \alpha_{1i} \times \frac{x_{t-i}}{\sqrt{h_{t-i}}} + \sum_{j=1}^{q} \alpha_{2j} \ln(h_{t-j}) + \sum_{i=1}^{p} \beta_i \left[\left| \frac{x_{t-i}}{\sqrt{h_{t-i}}} \right| - E\left(\left| \frac{x_{t-i}}{\sqrt{h_{t-i}}} \right| \right) \right] \qquad (1-5)$$

Applying logarithm, parameters α_0, α_{1i}, *and* α_{2j} are free from the non-negative constraints. Furthermore, β_i, $\forall i \in [1, p]$ would typically be negative reflecting that positive shocks generate less volatility with all else being equal. Because of asymmetry, EGARCH model can express this feature, whereas ARCH and GARCH models cannot express.

More generally, the GARCH models allow for h_t to be an arbitrary function of past conditional variances and past residuals; thus the number of variants is vast, though the number of formulations of the

ARCH-type model considered here is limited because we are interested in expressing the stylized facts regarding asset volatility given above.

Specifying the volatility of current returns as a non-stochastic function of past observations, the ARCH-type model can be estimated by the Maximum likelihood (ML) procedure. A recent survey of the ARCH family can be found in Bollerslev et al. (1994).

1.1.2 Stochastic Volatility (SV) Model

In a Stochastic Volatility (SV) model, the logarithm of volatility is specified as a linear stochastic process similar to an ARMA process. SV model containing nonlinear state-dependent volatility appeared in early financial researches, such as Tauchen and Pitts (1983), and Taylor (1982), and Hull, and White (1987), and so on. The canonical model in this class for evenly spaced data is:

$$x_t = \sqrt{h_t}\varepsilon_t \qquad (1-6)$$

$$\ln(h_t) = \beta \ln(h_{t-1}) + \eta_t \qquad (1-7)$$

$$\eta_t \sim N(0, \sigma_\eta^2) \qquad (1-8)$$

$$Cov(\eta_t, \varepsilon_i) = 0, for \ \forall \ t \ and \ i \qquad (1-9)$$

where $|\beta| < 1$ is set to ensure the stationary of h_t. In most cases, the volatility process begins with the initial condition

$$h_1 \sim N\left(0, \frac{\sigma_\eta^2}{1-\beta^2}\right) \qquad (1-10)$$

assuming stationary.

The evaluation of the likelihood of the SV model is much more difficult than that of the ARCH-type model, and hence they have had less empirical applications than the latter. Research on SV model mainly has been performed in the area of parameter estimation and several estimation methods alternative to the conventional maximum likelihood method are now available, Ghysels et al. (1996), and Shephard (1996) provide good reviews of the literature.

A simple approach to the estimation of SV models is based on the moments, which includes the methods of the simple moment matching (MM) (Taylor, 1986), the generalized method of moments (GMM) (Melino and Turnbull, 1990; Anderson and Sorensen, 1996), and the simulated method of moments (SMM) (Duffie and Singleton, 1993; Gourieroux et al., 1993).

Another simple approach is the quasi-maximum likelihood (QML) estimation developed by Nelson (1988), Harvey et al. (1993), Ruiz (1994), and Shephard (1994). They employ Kalman filtering to estimate the unobservable log-volatility, and use the Gaussian quasi-likelihood to perform parameter estimation. Unfortunately, the Gaussian QML approach of Harvey, Ruiz, and Shephard (1994), which initially seemed appealing because of its simplicity, fell by the wayside as it become apparent that stochastic volatility models are highly non-Gaussian. The problem is that standard volatility proxies such as log absolute or squared returns are contaminated by highly non-Gaussian measurement error (Anderson and Sorensen, 1997), which produces highly inefficient inference about latent volatility. This quasi-likelihood estimator is poorly approximately by the normal distribution. It means that the quasi-likelihood maximum estimator based on the normal approximation has poor finite sample properties, for not depending on the exact likelihood, even though the usual quasi-likelihood asymptotic theory is correct.

Better alternatives based on the exact likelihood are simulation-based maximum likelihood (SML) (Danielsson and Richard, 1993; Danielsson, 1994a), and Bayesian approaches, which relies on the

Markov chain Monte Carlo (MCMC) method, namely the Metropolis-Hastings and Gibbs sampling algorithms, to sample from the joint posterior distribution. These methods have found a number of applications in the recent statistical literature. Early work on these methods appears in Mertropolis, Rosenbluth, and Teller (1953), Hastings (1970), Ripley (1977) and Geman (1984) while the more recent developments spurred by Gelfand and Smith (1990) are summarized in Gilks, Richardson, and Spiegelhalter (1996), and Tanner (1996, Ch. 6), and Chen et al. (2000). The econometrics work on this topic is reviewed in Chib and Greenberg (1996). A tutorial introduction to the Mertropolis-Hastings algorithm including its derivation from the logic of reversibility, is given by Chib and Greenberg (1995).

1.1.3 Jump Process

The jump process proposed by Press (1967), and introduced into financial econometrics by Merton (1976) have been proven to be a useful tool in financial time series analysis, even when volatility models are taken into account. The jump processes are widely used in empirical analysis such as pricing models, event studies, and so on.

The general parametric jump process, as a mixture of both continuous diffusion path and discontinuous jump path, can be written as:

$$x_t = \sqrt{h_t}\varepsilon_t + z_t \qquad (1-11)$$

$$z_t = \sum_{i=1}^{J_t} y_i \qquad (1-12)$$

where x_t is return at time t, and h_t denotes the instantaneous volatility of the asset's return conditional on that the Poisson jump event not occur. According to the specification of h_t, Jump-ARCH, and Jump-

SV process can be modeled, as follows:

J_t: the arrival number of jumps at time t, assumed to follow a Poisson distribution with intensity λ;

λ: the intensity parameter of the Poisson distribution, defined as the expected number of events of per interval;

y_i: the size offered by the ith jump at time t, $Y_i \geqslant 0$,

$$y_i \sim i.\ i.\ d.\ N(\theta, \sigma^2), \tag{1-13}$$

$$Cov(y_i, y_j) = 0, j \in [1, J_t], Cov(\varepsilon_t, y_i) = 0 \tag{1-14}$$

Assuming of strong Markov process ensures that the equation mentioned above has unique solution in probability.

The jump models are designed to capture "surprise effects" e. g., large changes attributable to the arrival of unexpected information (Merton, 1990). But the estimation for the parameters representing the jump arrival intensity and the distribution of the jump size is particularly cumbersome when data is sampled at discrete time interval. It is empirically difficult to discriminate the variances caused by the continuous part, e. g. volatility model, from the variances caused by the discontinuous part, i. e. jumps.

Thanks to the advance of computation and econometric methods, the following methods can be used in the estimation of the model with jumps: the maximum likelihood estimation (QML see Fehr and Rosenfeld, 1979; Ball and Torous, 1983; Ball and Torous, 1985; Jorion, 1988; S ϕrensen, 1991; Eraker et al., 2001; Beine et al., 2003; Maheu et al., 2004).

Another is the method of moments (MM), including cumulants matching (CM, see Press, 1967; Beckers, 1981; Ball and Torous, 1983), the generalized method of moments (GMM), efficient method of moment (EMM, see Anderson et al., 2002;), simulated method of moment (SMM, see Duffie et al., 1998; Chernov et al., 1999; Craine et al., 2000; Anderson et al. 2002), and indirect inference method (IIM, see Gouri e' roux et al., 1993; Gallant and Tauchen, 1996; Jiang, 2000). Markov Chain Monte Carlo (MCMC, see Eraker et al., 2003).

Based on the assumption of Markov process, ML is a convenient method, because the calculation of the likelihood function from discretely sampled data is simplified. For large samples, ML is one of the best estimation methods, because estimators are consistent, asymptotically normal and asymptotically efficient. However, ML requires a complete specification of the transition density, which for nonlinear models may not have an explicit expression.

The estimators offered by MM are consistent, but inefficient. In particular, estimators offered by CM, may be functions of the sample moments, may be inexistent, or have the wrong sign. The estimators obtained by GMM, EMM, and SMM are based on arbitrarily chosen moments of a jump model, originate from the difficulty in distinguishing between whether movements are part of the jump path dynamics. These drawbacks have limited the usefulness of these methods in empirical work.

1.2 The Relationships of the Three Models

The ARCH-type models are easy to estimate because the exact likelihood function can be explicitly defined. Thus, there are a large number of variants of the ARCH models, since introduced by Engle (1982). It is suspected that the development of ARCH-type models is very near to the saturation point. The study on the SV and jump process are now mainly on parameter estimation, so that the number of