流体运动经典分析

杨本洛 著

科学出版社

（京）新登字 092 号

内 容 简 介

本书是分析流体运动的一本专著，包括涡量分析、不可压缩流分析等经典内容．本书还有一个关于流体运动全局性分析的专题．流体运动的全局性分析为“湍流”问题的解决提供了途径．

全书共分七章．在第一章和第二章中，采用了现代张量分析的方法，对流体力学基本定理和涡量场性质作了较系统的描述；对在连续介质力学中具有重要应用的 Gibbs-Duhem 方程的适用性条件，以及对 Reynolds 输运方程的解释，提出了不同的看法．

第三章到第六章集中于涡量场分析的讨论．在这几章中，除了对涡量场分析的现状作了一般性介绍外，还就双旋度 Poisson 方程解的性态和相应积分方程的构造问题，并对用散度和涡量表述速度场的唯一性分析、及其积分表述等一系列问题进行了讨论；论证了作为涡动力学中基本定理及基本公式的“经典唯一性定理”和“广义 Biot-Savart”公式的错误，指出将速度场划分为单独决定于散度、涡量及边界上流体速度分布等几个速度分量的思想是不恰当的．

第七章讨论了不可压缩流的流动问题，指出经典不可压缩流力学模型中，流体绝对不可压缩假设造成该模型有悖于物理学的基本定律，是不可压缩流分析出现困难的本质原因之一．针对这一问题，构造了基本数学表述形式，提出相应的经验公式，并对可能出现的误差进行了分析．

本书最后所列专题是针对“湍流”研究中的哲学和数学问题而进行的讨论，并给出刻画流体一般运动规律的理论方程．

本书可作为流体力学、应用数学、工程热物理、航空、造船、核能、水利等领域的高校教师、研究生、高年级本科生及有关研究人员的教学和参考用书．

流体运动经典分析

杨本洛 著

责任编辑 陈文芳

科学出版社出版

北京东黄城根北街 16 号

邮政编码 100717

中国科学院印刷厂印刷

新华书店北京发行所发行 各地新华书店经售

*

1996 年 5 月第 一 版	开本：850×1168 1/32
1996 年 5 月第一次印刷	印张：10 1/2
印数：1—1500	字数：245 000

ISBN 7-03-005071-1/O・839

定价：20.00 元

出版说明

上海交通大学杨本洛教授所著《流体运动经典分析》一书，对流体力学理论中一些基本问题提出了自己的观点，以达到百家争鸣、促进学科发展的目的。至于这些观点是否正确，还有待从事流体力学、应用数学、工程热物理等领域研究工作的专家、学者充分讨论，在实践中加以验证。

科学出版社

1996-06-03

Brief Introduction

This is a specialist book about fluid dynamics that includes vorticity analysis, incompressible fluid analysis, etc. It also contains a topic about global analysis of fluid motion which gaves a useful way for the resolution of turbulent flow.

The fluid dynamics basic principles and vorticity field characters are systematic ally described by modern tensor analysis in Chapter I and II. And some different view about the adaptability condition of Gibbs-Duhem equation is raised, which play a important role in continuous medium mechanics and explanation of Reynolds equation.

Chapter III through VI focus on the discussion of vorticity field analysis. Besides the general introduction of vorticity field analysis, it discussed a series of problems about the characters of the solution of bispinor Poisson equation, the building of relevant integral equation and the uniqueness of description of velocity field by using dilatation and vorticity. Proved that the "classical uniqueness theorem" and the "generalized Biot-Savart formula" as the basic principle and formula in vorticity-vortex dynamics is wrong, also pointed out that it's not property to say the velocity field is possible to be divided into the three parts that are separately determined by dilatation, vorticity and the distribution of fluid velocity at the boundary.

Chapter VII talks about the flow of incompressible fluid. Concluded that the hypothesis of fluid is absolutely incompressible, which is opposite to physical basic principals, is one of the reasons that makes the difficulty in analysis incompressible fluid motion. To solve this problem, this chapter created the basic mathematics model, gave out corresponding experience formula, and analyzed possible error.

The last topic in this book is about philosophy and mathematics problem in the research of turbulent flow, and gave out a set of equations to depict general law of the movement of fluid.

I trust it will be of help to the under-graduated and graduated students as well as to the researchers and specialist in the field such as fluid dynamics, applied mathematics, engineering thermal physical, aviation, ship building, nuclear power and hydraulic power. It could also be a teaching material or reference for them.

前　言

在本书即将脱稿之际，首先感谢科学出版社给予了我这样一次机会，得以对流体力学中的许多问题进行较为系统的深入探讨．本书的内容从最初仅仅着眼于涡量场分析以解决涡动力学理论上不封闭的问题而扩展到不可压缩流分析，以致最终完成了对“湍流”运动的哲学和数学思考，导出反映流体一般运动规律的流体力学理论方程．从而在涡量场、不可压缩流和“湍流”等几个当今流体力学最富挑战性的领域做出了一些属于自己的工作．这是我写书之初完全始料未及的．由于讨论内容的逐步扩展，以及构造了新的流体力学理论体系，书名从原先拟定的《涡量场分析》，几经修改，最终定为《流体运动经典分析》．所谓经典分析是指，本书除了最后一个专题以外的全部内容仍然属于流体力学经典框架下的讨论．

1988 年吴介之教授（美国 Tennessee 大学、宇航研究院）在江苏理工大学举办了涡动力学的讲座，我从讲座中得到许多教益．涡动力学的许多问题都属于一个理论框架下的逻辑演绎问题，和数学密切相关，我比较喜欢研究这一类问题．讲座以后，吴介之教授从美国寄来许多有关涡动力学方面的论文和资料，向我指出用散度和涡量表述速度场时，粘附性边界条件会使相应偏微分方程组的求解产生“超定”，从而使整个涡动力学在理论上不能封闭，提议我能否针对这一问题从数值计算方法上进行一些探讨．我欣然接受了这一提议．但是，在读了人们在这方面所做工作的有关文献以后，我有一种直觉，人们至今在这个问题研究中存在方法上的问题．涡量场力学中的所谓“超定”问题，决不可能仅仅是一个数值计算问题．解决“超定”问题的途径不能依靠构造新的数值计算方法来解决．同时我也断定，涡动力学中的基本公式：“广义 Biot-Savart 公式”是错误的．它的不能应用不能像某些著述所说的那

样，归于流场的物理原因，而是这个公式本身就是一个错误的数学公式. 我于 1989 年写就《论"广义 Biot-Savart 公式"谬误》一文. 对于我的这一疑问，吴介之教授特地寄来了 H. B. Phillips 的名著："Vector Analysis"一书中的相关内容. Phillips 的证明，以及诸如电磁理论，流体力学有关著述的论证似乎是"严密的". 但从这个公式的数学形式以及物理背景考虑，我仍然坚信我的判断. 只不过因为我觉得对前人推导错误的分析尚不足以说服自己，所以一直没有将此文投寄任何刊物. 对一般可压缩流中"广义 Biot-Savart 公式"推导中的错误，拿出可以说服自己的理由是经历了差不多六年的反复推敲，到今年年初，在书写本书的过程中才得以完成.

本书共分七章和一个专题.

第一章以整体张量为工具，讲述了经典流体力学的基本概念，推导了经典流体力学的基本方程组. 我以为流体力学像一些固体力学那样，采取整体张量的方法，绝不仅仅单纯为了书写简洁，更基本的原因在于，使用了整体张量的表述方法，力学量及相关力学规律的描述具有明确的物理意义，有利于人们在物理上的深刻理解和进行更为深入的探讨. 第一章的绝大部分都是流体力学的经典内容，只是对 Reynolds 输运定理给以和经典著述不同的物理解释，对流体力学中应用 Gibbs-Duhem 方程的合理性给予了论证. 虽然 Gibbs-Duhem 方程是经典连续介质力学中重要的基本方程，但是应该说，人们以往对这个方程应用条件的认识不统一，基本观点并不恰当.

第二章叙述了涡量场的一些数学物理性态. 这些叙述基本上仍然是经典流体力学中的内容. 然而，即使在经典流体力学的框架下，关于"物质涡"的定义及有关数学推导都是错误的，本章给予了纠正.

第三章至第六章是本书最初拟定讨论的主要内容. 这三章基本围绕如何用散度和涡量表述速度场的运动学问题进行了分析. 在这些分析中完全属于自己的工作主要有：

(1) 给出双旋度 Poissin 方程的两种积分表述形式，分析了双

旋度 Poissin 方程的主要数学性态. 其中有双旋度 Poissin 方程对于解函数,即矢势函数任意散度假设的自洽性分析;对习惯所作"正则假定"数学依据的分析;对双旋度 Poissin 方程解函数的"势"分析;以及双旋度 Poissin 方程定解问题的提法和数学构造分析. 论证了双旋度 Poissin 方程经典导数形式的积分表述,即一种"广义 Biot-Savart 公式"的错误. 另外也明确指出,人们通常在正则假定下将双旋度 Poissin 方程转化为向量 Poissin 方程进行讨论在数学和物理上是不恰当的. 双旋度 Poissin 方程和向量 Poissin 方程的定解问题在数理上是两个完全不同一的问题.

(2) 论证了"经典唯一性定理"的错误. 给出用散度和涡量唯一表述速度场的边界条件,和固壁处流体所应满足的粘附性边界条件之间的自洽性证明. 指出在求解一个向量场,使该向量场的散度和旋度分别等于一个任意给定的标量场和一个任意给定的无散向量场时,恰当的定解条件是给出边界上待求向量函数的切向分量,而不是习惯认为的那样,给出待求向量函数在边界上的法向分量. 仅仅对于二维问题,这两种边界条件才可以互为替代.

(3) 给出吻合于粘附性边界条件,用散度和涡量表述速度场的积分形式. 对积分表述的有关性态进行了分析. 并论证了所谓涡动力学基本公式,即另一种形式"广义 Biot-Savart 公式"的错误. 同时分析了将速度场变为无旋有散场,无散有旋场与无散无旋调和场之和的所谓速度场总体分解是不恰当的.

第七章,对不可压缩流问题进行了分析. 进一步明确,不可压缩流仅仅是指流场中流体密度的变化和流体密度相比是小量而已. 如果由不可压缩流的上述力学意义,推得流场中速度散度处处为零,强制流体绝对不可压缩不仅在数学上是不严谨的,而且由于破坏了任何一种物理现象所必须遵守的热力学基本定律,使得流场中的压力失去了它原有的物理意义. 这一问题可能是造成求解不可压缩流困难的原因之一. 在该章,根据经典不可压缩流体的物理意义,采用严格数学演绎方法建立了在经典流体力学框架下的理论模型. 这个模型和经典模型的基本差异在于,用有限量关系

表示的连续性微分方程取代被略去的小量关系表示的连续性微分方程，从而保留了压力的热力学意义，为人们运用各种经验模型计算不可压缩流流场的合理性提供了判断的基准. 该章还对包括流体绝对不可压缩等几种不同经验方法所可能出现的误差进行了大概的分析.

本书最后是题为《流体运动全局性分析——"湍流"研究中的哲学和数学思考》的专题. 该专题明确提出"湍流"研究是一个错误逻辑指导下的错误命题，提出不从流体运动的本原现象进行深入的考虑，分析经典流体力学体系和物理真实之间的差异，而仅仅从Navier-Stokes 方程出发，把"湍流"定义为不规则流，再研究它的运动规则完全陷入了循环逻辑之中，是永远不会成功的.

曾经考虑将这个专题作为前后相贯的一章单独写出，但是囿于以下想法，以为还是采用目前的形式为好. 首先，流体的全局性分析作为流体运动经典分析，即流体运动局部分析的一种简单继续，在逻辑结构上是不合适的. 流体的全局性分析既是流体局部性分析在认识层次上的一次深化，也是对流体局部分析的一种否定. 其次，作为著述的一个正式章节应该在所用数学工具的储备和相关数学公式的推导上，满足一个演绎系统应该满足的连贯和完备性. 而这一点目前对于我，无论在精力、时间以及一些相关材料的准备和深入思考上都是不够的.

一个人的能力毕竟十分有限. 面对"湍流"这样一个世界性的、经历了一个多世纪的重大难题，寄希望于一个人的工作是不现实的. 然而，发现经典模型和物理真实之间的差异，提出一个理性的逻辑演绎体系本身就是需要予以重视的事实. 我并不担心我的思想会有某些不周甚至错误之处. 我真诚地期待着人们对我可能的错误和不足进行批判，同时我也期待稍事休整以后，能够将流体力学全局性分析的研究工作继续深入下去.

科学发展的历史表明，人类对于未知的认识，往往是在对于错误的逐步认识过程中得以深化的. 如果一个问题长时间得不到解决，人们需要跳出关于这个问题长期所形成的思维框框，从哲学的

高度对这个问题自身的存在及许多相关问题重新进行思考.将自己和别人的思想强制地束缚于一个固定的模式,绝不是一个科学的研究方法.涡动力学理论长时间不能封闭和"湍流"研究长时间不能解决都说明了这同样一个事实.本书所做的工作同样依循这样的科学规律.只不过有关涡的工作,是属于流体力学经典框架下逻辑演绎体系的一种严格化,而有关"湍流"的论述则是对物质运动现象的重新认识,属于演绎指导下的认识归纳总结阶段.以往,理性力学没有对宏观物质的变形运动和流动加以区分,这是不恰当的.事实上,宏观物质的这两种宏观运动形式的物理内含和数学表述都不相同.注意到这一点,则"湍流"所表现的种种不规则现象就是流体运动的本原现象,人们需要对流体运动的这种本原现象的规律构造完整的逻辑体系.

目前"湍流"研究中的一些提法和观点,实际上反映了目前科学研究中的一种哲学思潮.随着人们认识物质世界的深化,人们对于物质世界认识的进一步深化越发地困难了.然而在人们面临复杂的物质世界时,一方面大谈复杂性,一方面又试图将复杂的物质世界纳入一两个固定的模式中,我认为这种说法本身就是对物质世界复杂性的否定,值得为人们重视.尽管我的看法可能难免偏颇,提出来供大家讨论和商榷.

和经典流体力学相比,流体运动的全局性分析没有增加任何新的公理假设.相反,流体运动全局性分析取消了经典分析中不合理的"不可切入公理"等假设,使得"湍流"表现的种种断裂、交混现象成为流动的基本现象,而Navier-Stokes方程仍然是这个新理论体系中的一个重要方程.因此,前面七章所讨论的绝大部分内容,像散度、涡量和速度之间的关系,以及不可压缩流各种经验模型的分析等等仍然是适用的.由于流体运动全局性分析保留了经典框架中连续速度场存在性的假定.对于流体运动中实际存在的速度场非连续现象则需要构造新的理论模型进行讨论.显然,在新的理论模型中,微分流型将是一个基本的数学工具.任何一个自洽的理论模型都是若干公理假设下的逻辑演绎体系.随着公理假设的减

少，理论体系所反映的运动规律和物质运动真实之间的差距就越小．事实上，流体力学中 Euler 的无粘理论，Navier-Stokes 的局部分析理论和今天的全局性分析理论，以及将要研究的无连续速度场假设的一般性理论都是在不断地减少公理假设的过程中，和流体运动的真实逐步加以吻合的．

数学是一切科学研究的必要工具．没有数学就不能以逻辑的形式表达物质运动的本质规律，也无法透过错综复杂的表面现象发现物质运动的本质规律．在科学的探索中，人们必须不断地学习、了解和掌握新的数学工具，并且需要对所应用的数学概念有较为深切和准确的理解，而不满足于一些新的数学工具的简单应用和模仿，同样也不能仅仅寄希望于依靠数学的逻辑推理．只有如此，人们才能运用数学这个强大的逻辑武器去宏观地审视一个复杂的物理现象，使人们在探索未知追求真理的过程中找寻到科学的大门．

在学术交往中，不少前辈和朋友告诫我，文章中尽量不要使用“错误”两字．对于一个认真严肃的科学工作者，使用“错误”两字的确应该十分谨慎．但是，如果在自己的认识水平上，对于一个自己确实认为是错误的结论而不明白地表明自己的观点，同样是不认真和不严肃的．科学需要极其认真和严肃的精神，更需要一种追求真理的真诚．事实上，书中一些观点的提出，往往开始都很粗糙，经过多次变更和修改以后，到最终才能说服自己．而且相隔一段时间以后重新阅读，仍然会觉得在一些地方有不能令人满意之处．在构造一个思想的过程中，前辈的指点，同道间的交谈，我的研究生和学生们的质疑和不同观点之间的争论都给了我许多启发和帮助．围绕本书内容所作的论文，差不多每一篇都经历了这样反反复复的修改过程．对一个问题的认识常常经历十次乃至数十次的反复方可形成一个较为稳定的看法．工作中，我面对流体力学如此众多的基本问题，深深感到仅仅凭借个人的力量是难臻完善的．因此在把个人所做的这些有限工作展示出来之际，我再次真诚地期望展开真正科学意义上的研讨，对书中不足和错误之处给以指正和批

判，使每一个有志于探索流体运动规律的人们在流体力学这些重大领域的研究工作得以深入下去.

借此机会向所有曾给予支持和帮助的前辈、同事和朋友们致以深切的谢意.

杨本洛

1995年冬于上海交通大学

Preface

First, I would like to express my thanks to Beijing Science Press to give me a chance to have a deep and systematic discussion of many interesting topics in fluid dynamics. It was focused on vorticity field analysis to solve the theoretical non-closed problem in vorticity-vortex dynamics at the very beginning, then expanded to the analysis of incompressible fluid motion and finished the research of turbulent flow in philosophy and mathematics at last. Little did I expect that some works are made by myself in several most challenging fields such as vorticity, incompressible fluid and turbulent flow. Since that, the name of this book is changed from "Analysis of Vorticity Field" to "Classical Analysis of Fluid Motion" at last. To be classical, it means except the last topic of this book, everything else is still under the principle of classical fluid dynamics.

In 1988, Prof. Jiezhi Wu (Tennessee University, Space Institute) made a lecture about vorticity-vortex dynamics in Jiang Su University of Science and Technology. I attend his presentation and learned a lot then. Much theory in vorticity-vortex dynamics is by logical deduction and depends on mathematics very much, that's the field I really prefer. And I received many theses and documents about vorticity-vortex dynamics from Prof. Wu after he went back to US. I learned that the boundary adherence condition makes the corresponding differential equation set "over specified" when use dilatation and vorticity as the main parameters to describe the velocity field, and result in the non-closed problem in vorticity-vortex dynamics. Prof. Wu suggests to me to solve this by numerical computing. However, once I read the corresponding documents in this field, I got an intuition that we still in the non perfect direction of this problem. The "over specified" problem in vorticity-vortex dynamics will not be just a numerical computing problem and will not be solved by some kines of new numerical computing models only. I also concluded that the "generalized Biot-Savart formula," as the basic formula in vorticity-vortex dynamics, is wrong. The reason for this formula's unsuitable is not because of the physical reason of fluid field but because the formula itself is wrong. That's the main point of my thesis named "Dissertation of the mistake of the 'generalized Biot-Savart formula'" in 1989. For my information, Prof. Wu send me an article in H. B. Phillips' book "Vector Analysis." The proof provided by Phillips seems quite strict. But I still stick on my judgment. For almost six years' research, I just finished the proof of the mistake in "generalized Biot-Savart formula" during the writing of this book.

This book contains seven chapters and one special topic.

Chapter I contain the main concept and the inference of the basic equation set of classical fluid dynamic by general tensor, the kernel representation. I think to use general tensor is not only clear in express on the paper but also more meaningful in the physical meaning of mechanics concepts and laws and help understanding. Most part of this chapter is about classical fluid dynamics, only the explanation of Reynolds transmission principle is different with the classical one, I also give out a proof for the reasonability of Gibbs-Duhem equation. Although Gibbs-Duhem equation is an important equation in classical continuum mechanics, people still don't have a consensus.

In chapter II, I described some mathematics and physical characteristic of vorticity fields and that's almost in classical fluid dynamic too. But the definition of material vorticity is wrong in classical fluid dynamic and is corrected in my book.

Chapter III through VI is the main content of this book at first. These three chapters focus on how to solve kinematics problem to express velocity field by using dilatation and vorticity. The following tasks are finished totally by myself.

(1) Give out two integral expressions of the bispinor Poisson equation and analyze the main mathematics characteristic of them, including the analysis of bispinor Poisson equation self consistency with the assumption that the divergence of solution function can be arbitrarily given, the analysis of the mathematics basis of the "normal hypothesis", the potential analysis of bispinor Poisson equation and the construction of the complete mathematical model of bispinor Poisson equation. It also proves the error of the classical integral expression of bispinor Poisson equation, a. k. a. "generalized Biot-Savart formula". And points out it's not property in mathematics and physics to transfer the bispinor Poisson equation to a general vector Poisson equation on the normal hypothesis. The definite problem of the bispinor Poisson equation and the vector Poisson equation is totally different in mathematics.

(2) Prove the error of the "classical uniqueness theorem", give out the perfect boundary conditions to determine velocity field with dilatation and vorticity, and give out the self consistency demonstration of the boundary adherence conditions which should be satisfied to the fluid on the wall. It points out that while finding the solution of a vector field to make its divergence and curl equal to an arbitrary scalar quantity field and solinoidal vector field, the proper definite condition is to give out the tangent part of the unsolved vector function on the boundary instead of giving out the normal part as usual. Only in two dimension problems, the two boundary conditions could be replaced by each other.

(3) Give out the integral representation of the velocity field with dilatation and vorticity that consists with boundary adherence conditions, analyze the relative characteristic of the integral formula, prove the error of the so called basic formula of vorticity-vortex dynamics, a. k. a. another type of "generalized Biot-Savart formula". And analyze improperty of the decomposition

of velocity field to the sum of the potential-diatational field, the rotational-solenoidal field and the harmonic, potential-solenoidal field, that called as a global decomposition of velocity field.

Chapter VII analyzes the incompressible fluid flow. Point out that the incompressible only means the change of fluid density is very small compared to the density of fluid itself. Assuming the velocity dilatation to be zero is not rigorous in mathematics and broken the thermal physical principle, which makes pressure lose its physical sense. This may be the one of the reasons for the difficulty to solve the incompressible fluid problem. In this chapter, a new theoritecal model is built by using some strict mathematical deductive method, which based on the classical definition of incompressible fluid. This model differs fundamentally from the classical one in the fact that the continuum equation is expressed in terms of finite quantity, instead of the omitted tiny form. The expression makes the pressure more sensible and supplies a criterion to value various empirical formulae. This chapter analyzes the possible error in them as well.

The last topic of this book named"Global analysis of fluid motion - - philosophic and mathematical thinking on"turbulence" research". This topic signalizes that the "turbulence" research is a wrong proposition under incorrect logic. It emphasizes there is no sense in researching turbulent flow based only on Navier-Stokes equation, instead of examining into the essential phenomena of fluid flow and considering the differences between the classical theory and the physical facts. While definiting turbulence as an irregular flow, the research of turbulence rule falls into a logical circle and will never be successful.

I have thought to write this topic as a sequential chapter but changed to the current way because of the following reason: First, it's not exact in logic to say the global analysis of fluid flow is a simple expansion of local analysis of fluid flow. The global analysis of fluid motion is a deeply development in thoughtfulness and a revolution of the local analysis. Second, as a formal chapter, I have to provide necessary and whole mathematics tools in to satisfy the completeness and coherency in deductive reasoning. And it isn't competent for me either in vigour, time and deep thinking, or in preparation for the related material.

The ability of a person is rather finite. Facing such a great difficult problem, which is a world wide problem and has been investigated over for one century, it is absolutely not possible to solve turbulence problem by a single person. But it is important enough to find out the difference of the classical model and the real physics and construct a deductive logic system. I am not afraid of existing some insufficiencies and rather mistakes in my research. And I am sincerely looking forward to have any critique on them, and expect to continue my investigation after a short rest.

The history of science development shows that the knowledge of the unknown world is always by correcting error step by step. If one problem keeps unsolved for a long time, we need to jump out of the traditional model of thinking and take the some essential problem, such as if the problem exists itself, into deliberation by philosophy. To constrain people's thinking to a fixed frame must not be a science method. The non-closed problem of the vorticity-vortex dynamics and turbulence problems haven't been solved for such a long time explains the same fact. All the works in this book are following the way of science research. For the study of vorticity-vortex dynamics, it's only belong a process to make the deduction more rigors and exact under the classical fluid dynamics. And for the discussion of turbulence, it's a reconsideration of fluid flow phenomenon, which is belong the logic induction stage of our knowledge. It's not suitable that the solid deform motion and the fluid flow motions haven't been distinguished by each other in rational mechanics. In fact, both motions are not the same in physical content or in mathematical form. If the fact is noticed, the turbulence is consequently considered as the nature phenomenon of fluid flow and a logical deduction system describing the laws of the nature phenomenon of fluid flow must be constructed by people.

At present, some propositions and theses about turbulence reflect a kind of philosophy trend of thought. The more we learned the world the more difficulty to keep on studying. But when facing such a complete world, some people talk animatedly the world complexity on one hand, and just try to find one or two fixed model to reflect the whole world on the other hand. I think that's just a deny of the complexity of the world although that's only my personal opinion and could be wrong.

Comparing to the classical fluid dynamics, the global analysis of fluid flow did not add any more assumptions. On the contrary, it takes the reasonless "non-mixed hypotheses" away and makes the turbulence as a basic flowing phenomenon. The Navier-Stokes equation is still an important equation in it. So, the most contains in the first seven chapters are still suitable. Because the continuum velocity field hypothesis is kept in it, we need to build a new model when the actual non-continuum phenomenon must be regarded. In this new model, differential manifold would be a basic mathematics tool. Every self-consistent theory model is a logical deductive system on the some axiomatizational assumption. The fewer assumptions we have, the less difference between the model and the truth. In fact, the Euler's non-viscous theory, Navier-Stokes's local analysis theory, the global analysis theory and generalization theory without the assumption of the continuos in velocity field are becoming more correspond to the truth of the fluid flow, while deducing the number of assumptions step by step.

Mathematics is the basic tool for science research. We cannot describe the characteristic of the physical world and discover the laws of material motion by the appearances without mathematics. In the research of science, we need to keep on learning