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工商管理精要系列 · 影印版

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MATHEMATICS FOR BUSINESS

H. A. 斯波那
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456758

图书在版编目（CIP）数据

管理数学：英文/斯波那（Spooner，H.A.）、威尔逊（Wilson，D.A.L.）著·影印版
北京：中国人民大学出版社，1997.10
（工商管理精要系列·影印版/巴克利主编）
书名原文：The Essence of Mathematics for Business

ISBN 7-300-02480-7/F·761
Ⅰ.管…
Ⅱ.①斯… ②威…
Ⅲ.管理学-数学-英文-影印版
Ⅳ.C931.1

中国版本图书馆 CIP 数据核字（97）第 21932 号

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出　版：中国人民大学出版社
（北京海淀路 175 号 邮码 100872）
Prentice Hall 出版公司
发　行：中国人民大学出版社
经　销：新华书店
印　刷：中国人民大学出版社印刷厂

开本：787×1092 毫米 16 开　印张：17.25 插页 1
1997 年 10 月第 1 版　1997 年 10 月第 1 次印刷
印数：1-5 000 册

定价：32.00 元
（图书出现印装问题，本社负责调换）

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出 版 说 明

《工商管理精要系列·影印版》是中国人民大学出版社和西蒙与舒斯特国际出版公司继《工商管理经典译丛》之后，共同合作出版的一套大型工商管理精品影印丛书。

本丛书由欧洲著名管理学院和管理咨询公司的教授和专家撰写，它将90年代以来国际上工商管理各专业的最新研究成果，分门别类加以精练浓缩，由享誉世界的最大教育图书出版商Prentice Hall出版公司出版。每一本书都给出了该专业学生应掌握的理论框架和知识信息，并对该专业的核心问题和关键理论作了全面而精当的阐述。本丛书虽然篇幅不长，但内容充实，信息量大，语言精练，易于操作且系统性强。因此，自90年代初陆续出版以来，受到欧洲、北美及世界各地管理教育界和工商企业界读者的普遍欢迎，累计发行量已达数百万册，是当今国际工商管理方面最优秀的精品图书之一。

这套影印版的出版发行，旨在推动我国工商管理教育和MBA事业的发展，为广大师生和工商企业界读者，提供一套原汁原味反映国外管理科学研究成果的浓缩精品图书。有助于读者尽快提高专业外语水平，扩大知识面，掌握工商管理各专业的核心理论和管理技巧。

本丛书可作为管理院校的专业外语教材和各类企业的培训教材，对于那些接受短期培训的企业管理者、MBA学生，以及想迅

速了解工商管理各专业核心领域的师生来说，本丛书更是极具价值的藏书和参考资料。

为了能及时反映国际上工商管理的研究成果，中国人民大学出版社今后将与 Prentice Hall 出版公司同步出版本丛书的其他最新内容并更新版本，使中国读者能借助本丛书，跟踪了解国际管理科学发展的最新动态。

1997 年 8 月

Preface

Mathematics provides some of the most important tools for modern management – but many managers find mathematics baffling, and so they never come to appreciate its relevance to their problems. Some of them will have closed their minds to the subject while they were still at school, convinced that they could never make any headway with it. Others will have progressed further, but allowed what they had learnt to become rusty through lack of use.

This book is a sympathetic response to the needs of all those who approach mathematics with dread, but realize that they have got to get to grips with it at last. Their objective is usually twofold:

1. to obtain the skills which are needed to achieve their MBA or other qualification;
2. to obtain the confidence to talk sensibly about problems that are formulated in mathematical terms.

It is usually a great morale booster to realize that the second objective can be achieved, but this can only be done by putting aside all preconceived ideas of the inherent difficulty of mathematics and realizing that the great attraction of the subject is that it is logical and consistent, and that quite powerful methods can be developed from simple basic rules. In this respect, mathematics is really no different from any other structured discipline, say music or law or language.

With this readership in mind, we have assumed very few pre-requisites – virtually nothing apart from some knowledge of simple arithmetic and the ownership of a good medium-priced scientific calculator. Starting with a firm foundation and slowly building a basic framework of knowledge, the reader will be able to go on to benefit from more advanced texts on the applications of mathematics in management subjects. This book is intended to be an introductory text; we hope that our readers will go on to consult other authors who may develop the ideas which we have introduced in different ways, using different notations, so we have purposely tried to convey the idea of *flexibility in notation but consistency in reasoning.*

The early chapters introduce the basic ideas of algebra, taking the reader through varied examples of algebraic reasoning and numerical work to be done on the calculator. Simple applications of these ideas to management problems are introduced from Chapter 1 and the range of applications expands as the book progresses. We feel that it is important for the reader to develop a feel for problem formulation as well as a facility for manipulating numbers and formulae. Numerous fully worked examples are included in the text and additional examples are given at the end of each chapter (with answers at the end of the book). Some applications run on from chapter to chapter, being developed further as more advanced techniques are introduced.

In the final chapter we discuss general matters concerning the application of mathematics in a business context, including some advice on how to formulate and solve real-life problems. We emphasize that a manager cannot avoid the need to understand something about mathematics by thinking that mathematical problems can simply be entrusted to an expert. Such experts need to be briefed and their findings need to be interpreted, so all parties must be able to communicate in the common language of mathematics.

This text is designed either to be used for individual study or as the basis of a course with lectures and tutorials. Our experience is that classes requiring this sort of course include people with widely different perceptions of the difficulties of each of the topics that we have included. Because of this, we do not recommend how long should be spent on each topic. We simply say: 'Take it at your own pace, and don't panic!'

Ann Spooner
David Wilson

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1

Elementary algebra

Introduction

The main aim in any application of mathematics is to formulate and solve problems. To do this it is essential to have the correct tools. Basic algebra is one such tool. A proficiency in manipulating algebraic expressions makes the understanding and solution of mathematically expressed problems much easier. This chapter starts from 'square one' and no prior knowledge is assumed. Readers already proficient in the use of algebra should skip to Chapter 3. The examples at the end of the chapter have been graded in order of difficulty. Working through them should provide a reasonable level of familiarity with algebraic notation and the ability to use an algebraic approach to the formulation and solution of problems.

1.1 Algebraic expressions and equations

The definitions of algebraic expressions and equations are best illustrated by means of examples. Simple examples have been used in this section; more complicated examples will appear in the later chapters.

Example 1.1
If a profit of £5 is made on every unit of a certain product that is sold, the profit (in £) on two units is 5×2, the profit on three units is 5×3,

and so on. If we wish to generalize this kind of statement, we introduce a letter to denote the number of units sold. Any letter may be used; in this case we shall call the number of units sold n.

The profit on n units sold will then be $5 \times n$. The n is called a *variable* and the constant 5 is its *coefficient*.

The expression $5 \times n$ could also have been written as $5 . n$. The dot between the 5 and n signifies multiplication. Where there is no ambiguity the multiplication sign is omitted altogether. In this case the expression would be written as $5n$. (In computer programming the asterisk symbol * is used to denote multiplication. Similarly the symbol / is used instead of ÷ to denote division.)

The gross profit on sales depends upon the number of units sold, which we called the variable n, and is said to be a *function* of the variable n. We may write this as

$$f(n) = 5n.$$

A function can be denoted by any letter; in this case the letter f has been chosen. The variable(s) upon which the value of the function depends are listed within brackets after the letter.

In Example 1.1 above, if no units are sold ($n = 0$) there is no loss to the company. However, in practice there are usually some fixed overhead costs such as rates and rent to be covered. Let us suppose that the overheads are £50, so that £50 has to be deducted from the value of any sales in order to calculate the profit. Working in pounds sterling, the new relationship between profit and number of units sold is

$$\text{net profit} = 5 \times (\text{no. of units sold}) - 50$$

or

$$F(n) = 5n - 50.$$

The notation $f(n)$ was used for the gross profit, so the slightly different notation, $F(n)$, is used for the net profit.

In order to break even (make zero net profit), we must have a value of n which satisfies the *equation*

$$0 = 5n - 50 \quad \text{or} \quad 50 = 5n.$$

The *solution* to this is $n = 50/5 = 10$. In other words, with a profit of £5 on each unit ten units must be sold in order to cover the cost of the overheads; more than ten units sold will result in an actual net profit.

Terms such as $5n - 50$ are referred to as *algebraic expressions*. When the expression is set equal to a particular value it is called an *algebraic equation* or simply an equation.

The use of symbols such as n, $f(n)$ and $F(n)$ to represent quantities that may take various values enables us to simplify and manipulate relationships such as those between profit and numbers of units sold in our examples above. This becomes especially useful when the relationships are more complicated.

In the same way as firms are distinguished by using distinct trading names, algebraic functions which differ even to the smallest extent must have different names because they represent different expressions. Thus gross profit and net profit have different meanings in practice which are reflected by their different algebraic 'names', $f(n)$ and $F(n)$.

1.2 The addition and subtraction of algebraic forms

Example 1.2
If a pencil costs x pence and two pencils are purchased they cost $2x$ pence. If a further three pencils are purchased they cost an additional $3x$ pence. In all, five pencils have been bought at a total cost of $5x$ pence. The total cost could have been obtained as the sum

$$2x + 3x = 5x \text{ pence.}$$

Example 1.3
If ten pencils are bought at a cost of x pence each and then four pencils are returned and their cost refunded, the total expenditure on pencils is the cost of six pencils which is $6x$ pence. This could be written as the difference between the cost of the initial purchase and the refund, that is

$$10x - 4x = 6x \text{ pence.}$$

Rule 1.1: Collecting up identical terms

Terms where the letters are *exactly* the same may be simplified or 'collected up' by adding and subtracting the coefficients as specified.

Example 1.4
Simplify the expression

$$12x + 6x - 3x + 4x - 2x.$$

Solution
First check that all of the terms in the expression contain only the variable x. Since this is true, Rule 1.1 applies.

The sum of the coefficients of x is $12 + 6 - 3 + 4 - 2 = 17$. Therefore the answer is

$$12x + 6x - 3x + 4x - 2x = 17x.$$

Example 1.5
If five pencils were bought at a cost of x pence each and three pens were bought at a cost of y pence each, the total purchase cost would be $5x + 3y$. This cannot be simplified further since the costs x and y are different variables relating to different items.

Rule 1.2: Collecting up different terms

Algebraic terms in an equation may only be added or subtracted if they have *exactly* the same configuration of variables.

Example 1.6
Simplify the expression

$$5y + 3z + 6y + 4z.$$

Solution
First collect up the terms involving y to give $5y + 6y = 11y$.
Next collect up the terms in z to give $3z + 4z = 7z$.
The answer is

$$5y + 3z + 6y + 4z = 11y + 7z.$$

Example 1.7
Simplify the expression

$$11a + 7b - 6a - 4b.$$

Solution
Collect up the terms in a and then the terms in b to give the answer

$$11a + 7b - 6a - 4b = 5a + 3b.$$

Rule 1.3: Multiplication of variables

The term xy means 'variable x times variable y'. Terms containing xy must be collected up separately from the 'x' terms and the 'y' terms.

Example 1.8
In a particular transaction, n items are sold. The profit on each item varies with currency fluctuations. The profit in pounds sterling per item might be called p. The total profit per transaction is then £np. The individual variables, n, the number of items sold per transaction, and p, the profit per item at that date, clearly have a different meaning from the term np, which expresses the total profit per transaction in pounds sterling.

Example 1.9
Simplify the expression

$$2d + 5w + 4wd + 4w + 2h - 6d + 7wd.$$

Solution
Collect up the terms in d; these are

$$2d - 6d = -4d.$$

Next collect up the terms in w; these are

$$5w + 4w = 9w.$$

Next collect up the terms in wd; these are

$$4wd + 7wd = 11wd.$$

The answer is then

$$-4d + 9w + 11wd + 2h.$$

Note the result

$$2d - 6d = -4d.$$

Rule 1.4: Positive and negative terms

The general rule for combining positive and negative terms is to subtract the smallest term from the largest term and give the result the sign of the largest term.

Thus $120 - 200 = -80$.

If your revenue was £120 and your expenses £200 you would be in debt to the tune of £80. In other words, your balance would be −£80.

Note
In mathematics a loss or negative quantity is indicated by the use of the minus sign and *never* by the use of brackets as in accounting.

1.3 Products of positive and negative real numbers

Let a and b denote any positive real numbers. Then:

$$(+a)\times(+b) = +ab; \qquad (+a)\times(-b) = -ab;$$
$$(-a)\times(-b) = +ab; \qquad (-a)\times(+b) = -ab.$$

It is usually easier to remember these rules in terms of words as follows:

Plus times plus = plus

Plus times minus = minus

Minus times plus = minus

Minus times minus = plus.

If there is no sign attached to a particular term it is understood to be positive.

In some cases it helps to clarify where the signs should go if brackets are used to indicate precisely the term to which the sign is attached.

Example 1.10

$$2\times 3 = (+2)\times(+3) = 6 \qquad (-2)\times(-3) = 6$$
$$2\times(-3) = -6 \qquad (-2)\times 3 = -6.$$

Example 1.11

$$2\times 3\times 4 = 24 \qquad (-2)\times 3\times 4 = -24$$
$$(-2)(-3)\times 4 = 24 \qquad (-2)(-3)(-4) = -24.$$

Note the use of the brackets to define the negative numbers. When several quantities are multiplied together mistakes with the signs can often be avoided by using brackets in this way. An odd number of minus signs multiplied together give a negative result, while an even number of terms with minus signs multiplied together give a positive result. If an incorrect solution to a problem is obtained it is always worth checking the signs of the constituent terms since this is one of the most common sources of error.