

中国博士后 科学论文集

王志石 等

清华大学出版社



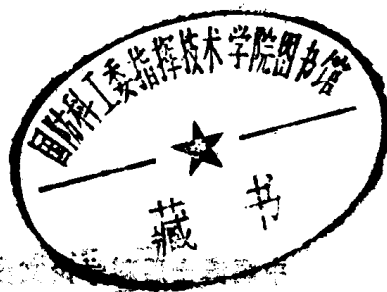
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内 容 简 介

本论文集收集了1988年5月在北京召开的第一届中国博士后学术年会中选出的32篇论文,涉及数学物理、应用化学、生物与地学以及若干应用技术,许多论文反映了多学科、跨学科的特点,具有理论深度和应用价值,并代表了各有关学科的前沿,对高等学校和科研单位的科技人员有参考价值。



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编 者 的 话

本论文集主要收集了1988年5月在北京召开的第一届博士后学术年会提交的部分论文,同时征集了部分国外博士后的文章。反映了自中国建立博士后科研流动站制度以来,博士后科研人员的部分研究成果。

论文集分为四个部分,即数学物理、应用化学、生物与地学以及应用技术。由于许多论文反映了多学科、跨学科的特点,上述划分只是为了读者阅读方便,并不是绝对的。收集的论文力求具有理论深度和应用价值,同时代表了学科的前沿。是否达到了这一目的,敬请广大读者评价。

由于篇幅有限,有些博士后提交的文章没有收入,敬请谅解。

本论文集是由中国北京博士后联谊会第一任干事长王志石博士组织编辑的,并由王枢平同志作了全面整理与校核工作。在此,表示衷心感谢。

本论文集是在人事部专家司关怀下和资金支持下出版的,始终得到庄毅司长、杨铮处长以及李连伟副处长的支持。在此,表示由衷感谢。

中国北京博士后联谊会

1989年2月

目 录

一、数学物理

挤压薄膜分析——本征值问题

- 中国科学院力学研究所 王发民 英国皇家理工大学 J.T.Stuart
新加坡国立大学 T.W.NG (1)

关于最大熵方法谱估计的分辨本领

- 中国科学院北京天文台 吴乃龙 (19)

关于Gould-Hus倒数转换的多维分析

- 中国科学院系统科学研究所 初文昌 (23)

梯度法在线性塑性波方程的反转解上的应用

- 中国科学院力学研究所 丁 华 (29)

位涡度方程的整体光滑解

- 中国科学院大气物理研究所 穆 穆 (36)

在流形上的Weyl变换——定义及同构定理

- 北京大学 钱敏 中国科学院数学研究所 许连超 (39)

原子核高自旋态的宏观模型研究

- 北京大学技术物理系 陈信义 (54)

关于气体分子——固体表面碰撞动力学的理论研究

- 清华大学无线电系 沈学英 (58)

形状记忆合金本构模型

- 清华大学力学系 王志刚 (64)

有限元法对结构碰撞效应的模拟分析

- 北京航空学院固体力学研究中心 李丰年 (73)

二、应用化学

水污染控制的胶体化学基础

- 清华大学环境工程系 王志石 (80)

含氮有机化合物的臭氧氧化

- 清华大学环境工程系 魏 盈 (86)

聚吡咯在水溶液中的电化学特性

- 中国科学院化学研究所 李永舫 钱人元 (92)

肽——化学合成和生物学功能

- 北京医科大学有机化学教研室 李根等 (96)

新型镇痛药物的研究——潜在的激动-拮抗混合型吗啡类似物的合成

- 北京医科大学药学院 张勇民 张孔和 刘维勤 (100)

高聚物在固-液界面的行为及其对于胶体悬浊液体系稳定性的影响

• ■ •

法国巴黎居里大学高分子物理化学实验室 王统宽	(104)
研究固-液界面的新型原位测试技术	
法国巴黎冶金实验室 余 宁	(110)
准晶	
法国巴黎冶金实验室 余 宁 张 葵	(116)
关于基底膜的物理化学性质的研究	
法国巴黎化学物理实验室 范伯涛 G. Lapluye	(122)
三、生物、地学	
在BHA和BZo对抗白内障形成过程中的晶状体生化改变	
北京医科大学生生化教研室 黄莉莉 贾维红 张昌颖	(141)
离体培养条件对百合小孢子减数分裂的影响	
中国科学院植物研究所 刘公社 王伏雄	(146)
二维裸地蒸发的数值分析	
中国科学院地理研究所 陈镜明 北京农业工程大学 杨邦杰	(153)
西准噶尔夏坦河——塘巴勒蛇绿岩带地质特征和岩石成因	
中国科学院地质研究所 马鸿文	(157)
经济——生态体系的研究	
北京大学生物系 张倩文	(160)
四、应用技术	
柴油掺水燃烧节能机理的研究	
中国科学院力学研究所 盛宏至	(165)
用YBa CuO薄膜制备的dc SQUID	
北京大学物理系 王世光等 电子工业部南京固体器件研究所 邵 凯等	(168)
一个新的填料塔传质模型	
清华大学化学工程系 张泽廷	(172)
关于城市下水污泥热处理——厌氧消化法的动力学研究	
清华大学环境工程系 王 伟	(177)
计算机控制光学非球面加工	
北京工业学院光学工程系 冯之敬	(184)
压气机进口畸变流场的主动控制	
北京航空学院 徐力平	(187)
双相钢中马氏相变热力学分析	
冶金部钢铁研究总院 沈显璞 张 杰	(191)
悬链线氧化内应力测量法	
北京钢铁学院表面科学与腐蚀工程系 赵建国 张勇军	(196)
Bi-Sr-Ca-Cu-O系统的高温超导特性和微观构造	
北京大学物理系 朱 星等 北京钢铁研究总院 朱 竟等	(201)

Content

I. Mathematical Physics:

Squeeze Film Analysis I — Eigenvalue Problem

- Wang Famin Institute of Mechanics, Academia Sinica, Beijing, J.T. Stuart
Department of Mathematics, Imperial College of Science and Technology, London, U.K.
T.W. NG Department of Mathematics, National University of Singapore,
Singapore (1)

On the Resolvability of Power Spectrum Estimates in the Maximum Entropy Method

- Wu Nailong Beijing Astronomical Observatory, Academia Sinica (19)

On the Multidimensional Generalizations of Gould-Hsu Reciprocal Transformations

- Chu Wenchang Institute of Systems Science, Academia Sinica, Beijing (23)

Application of Gradient Method in Inverse Problems of Linear Elastic Wave Equation

- Ding Hua Institute of Mechanics, Academia Sinica, Beijing CHINA (29)

A Integral Smooth-solution of the Potential Vortex Equation

- Mu Mu Institute of Atmospherical Physics, Academia Sinica, Beijing, (36)

Weyl Transformation on the Flow Pattern—Definition and Iso-form Therom

- Qian Min Peking University, Beijing
Xu Lianchao Institute of Mathematics, Academia Sinica, Beijing (39)

A Macroscopic Model Study of Nuclear High Spin States

- Chen Xinyi Department of Technical Physics, Peking University (54)

Dynamic Study of Gas Molecule-Solid Surface Collisions with the Theoretical Approaches

- Shen Xueving Department of Radio Electronics, Tsinghua University, Beijing (58)

A Constitutive Model for Shape Memory Alloys

- Wang Zhigang Department of Engineering Mechanics, Tsinghua University, Beijing
..... (64)

An Effective Strategy for Impact Simulation of Structures by Finite Element Method

- Li Fengnian Solid Mechanics Research Centre, Beijing Institute of Aeronautics &
Astronautics (73)

II. Applied Chemistry:

The Colloidal Chemical Basis of Water Pollution Control

Wang Zhishi Department of Environmental Engineering, Tsinghua University	(80)
Ozonization of Nitrogenous Organical Compounds	
Wei Ying Department of Environmental Engineering, Tsinghua University	(86)
Electrochemical Characteristics of Poly-Pyrrole in Aqueous Solutions	
Li Yongfang, Qian Renvuan Institute of Chemistry, Academia Sinica, Beijing	(92)
Peptide—Chemical Synthesis and Biological Function	
Li Gen et.al. Biochemistry Division, Beijing Medical University	(96)
A New Type of Analgesia Medicine—Morphine Analogues	
Zhang Yongmin, Zhang Lihe, Liu Weiqin Medicine School, Beijing Medical University	(100)
Chemical Behavior of High Polymers at Solid/Liquid Interfaces and Their Effects on Colloidal stability of Suspension	
Wang Tongkuan Laboratoire de Physico-Chimie Macromoléculaire, Paris, France	(104)
Recently Developed In Situ Techniques for Studying Solid-Liquid Interface	
Yu Ning Laboratoire de Métallurgie Structurale, Paris, France	(110)
Quasicrystals	
Yu Ning, Zhang Kui Laboratoire de Métallurgie Structurale, Paris, France	(116)
Basement Membranes, Investigation of Their Physicochemical Properties	
Fan Botao, G. Lapluye Laboratoire de Chimie Physique, Paris, France	(122)

I. Biology, Geography and Geology:

Biochemical Variation of Crystalline Lens in Formation of Cataract by Competition between BHA and BZo	
Huang Lili, Jia Weihong, Zhang Cangying Biochemistry Division, Beijing Medical University	(141)
Influence of In Vitro Condition on Meiosis of David Lily	
Lu Gongshè, Wang Fuxiong Institute of Plant, Academia Sinica, Beijing	(146)
Numerical Analysis of Evaporation in Two-dimensional Field	
Chen Jingmin Institute of Geography, Academia Sinica, Beijing	
Yang Bangjie Beijing Agriculture Engineering University	(153)
Geology and Petrogenesis of Xiatahe-Tangbale Ophiolite Belt, Western Zungger	
Ma Hongwén Institute of Geology, Academia Sinica, Beijing	(157)
Study on the System of Econom-ecology	
Zhang Qingwen Department of Biology, Peking University, Beijing	(160)

IV. Applied Techniques

Investigation on Mechanisms of Saving Energy by Combustion of mixture of Diesel Oil with Water	
Shong Hongzhi Institute of Mechanics, Academia Sinica, Beijing	(165)

Preparation of Dc SQUID with YBaCuO Thin Film

Wang Shiguang et.al. Department of Physics, Peking University

Shao Kai Nanjing Solid Device Institute(168)

A New Mass Transfer Model in Packet Columns

Zhang Zeting Department of Chemical Engineering, Tsinghua University.....(172)

Kinetics of Anaerobic Digestion with Heat Pretreatment for Waste Sludge Process

Wang Wei Department of Environmental Engineering, Tsinghua University.....(177)

Optical Non-spherical Surface Processes by Control of Computer

Feng Zhijing Department of Optical Engineering, Beijing Industrial Institute.....(184)

Control of Divergent Flow Field At Inlets of Compressor Engines

Xu Lipin Beijing Institute of Aeronautics & Astronautics(187)

Thermodynamic Analysis of Martensitic Transformation of Dual Phase Steels

Shen Xiampu, Zhang Jie Central Iron and Steel Research Institute(191)

A New Experimental Method for Oxidation Internal Stress Measurement

Zhao Jianguo, Zhang Yongjun Department of Surface Science and Engineering,
BUIST, Beijing(196)

High Tc Superconductivity and Substructure of Bi-Sr-Ca-Cu-O System

Zhu Xing et.al. Department of Physics, Peking University

Zhu Jing et.al. Central Iron and Steel Research Institute(201)

Squeeze Film Analysis

II—Eigenvalue Problem

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Abstract In this paper we present a mathematical study of solving eigenvalue equation obtained in Part I. An informal solution of special initial value problem of a perturbation developing at some given time is obtained by using Jacobi's method. With this solution known the possible instability for a squeeze film is then discussed. The study of Jacobi's method to investigate a class of mathematical instability problem, in which the basic flow varies significantly in two coordinates with time t and the stability equation derived is an ordinary differential equation containing three parameters representing partial derivatives of a phase function, suggested by J.T.Stuart, is a new approach in hydrodynamic stability studies.

1 INTRODUCTION OF JACOBI METHOD

This section is concerned with the discussion of solutions of the first order partial differential equation(3.23), relating to the problem of squeeze film stability. The general procedure of solving this equation is to use the method of W.K.B.J., to get an asymptotic solution for some fixed local point. In this section, following an earlier work of Stuart and Diprima, we study the mathematical analysis of Jacobi's method to infer an informal solution of this eigen

problem.

To illustrate the main idea, we consider a general partial differential equation

$$F(x, y, z, p, q) = 0 \quad (1.1)$$

$$\text{If } U(x, y, z) = 0 \quad (1.2)$$

is a relation between x, y and z , then

$$p = -\frac{U_1}{U_3}, \quad q = -\frac{U_2}{U_3} \quad (1.3)$$

where $U_1 = \frac{\partial U}{\partial x}$, $U_2 = \frac{\partial U}{\partial y}$, $U_3 = \frac{\partial U}{\partial z}$. If we substitute from (1.3) into equation (1.1), we obtain a partial differential equation of the type

$$f(x, y, z, U_1, U_2, U_3) = 0 \quad (1.4)$$

in which the new dependent variable U does not appear.

The fundamental idea of Jacobi's method is the introduction of two further differential equations of the first order:

$$\left. \begin{aligned} g(x, y, z, U_1, U_2, U_3, a) &= 0 \\ h(x, y, z, U_1, U_2, U_3, b) &= 0 \end{aligned} \right\} \quad (1.5)$$

involving two arbitrary constants a and b and such that:

- (a) equations (1.4) and (1.5) can be solved for U_1, U_2, U_3 ;
- (b) the equation

$$dU = U_1 dx + U_2 dy + U_3 dz \quad (1.6)$$

obtained from these values of U_1, U_2, U_3 is integrable

The main difficulty of this approach is in the determination of the auxiliary equations (1.5). We have to find two equations which are compatible with (1.4), such that the equations g and h are solutions of the linear partial differential equation

$$f_{v_1} \frac{\partial g}{\partial x} + f_{v_2} \frac{\partial g}{\partial y} + f_{v_3} \frac{\partial g}{\partial z} - f_z \frac{\partial g}{\partial U_1} - f_y \frac{\partial g}{\partial U_2} - f_x \frac{\partial g}{\partial U_3} = 0 \quad (1.7)$$

which has auxiliary equations

$$\frac{dx}{f_{v_1}} = \frac{dy}{f_{v_2}} = \frac{dz}{f_{v_3}} = \frac{dU_1}{-f_z} = \frac{dU_2}{-f_y} = \frac{dU_3}{-f_x} \quad (1.8)$$

Once the two integrals $g(x, y, z, a)$ and $h(x, y, z, b)$ of this kind have been found, the problem reduces to solving for U_1, U_2, U_3 , and then the solution of equation (1.6) containing three arbitrary constants will be a complete integral of (1.4). The three constants are necessary if the given equation is (1.4). However, when the equation is given in the form (1.1), we need only two arbitrary constants in the final solution. By taking different choices of third arbitrary constant we get different complete integral of the given equation.

In applying this idea in the problem of instability of squeeze film, the main difficulty here is that the eigenvalue function F appeared in equation is

unknown. However, there is a possibility of finding some partial differential equations of first order, independent the unknown function F , by using the method of Forsyth. With this integral and an expansion of equation F given, we can get the solution for U_1, U_3 (U_2, U_4 are two special constant functions). From the values of U_1 , the Pfaffian equation obtained is a non-linear ordinary differential equation. This equation is solved for some special initial conditions of a perturbation developing at some given time. The transition from laminar flow to turbulence is then found by simplifying the solution of this specific initial value problem.

2 INTEGRABILITY OF EIGENVALUE EQUATION

To seek a solution of this eigenvalue equation, which is a non-linear partial differential equation, we need to introduce three further differential equations of the first order, if the Jacobi's method is used. For this purpose, we let

$$U(x, \theta, T, \Theta) = 0$$

be an arbitrary 4th-dimensional surface and denote the derivatives of U as

$$\frac{\partial U}{\partial x} = U_1, \quad \frac{\partial U}{\partial \theta} = U_2, \quad \frac{\partial U}{\partial T} = U_3, \quad \frac{\partial U}{\partial \Theta} = U_4 \quad (2.1)$$

such that, we have,

$$\Theta_x = -\frac{U_1}{U_4}, \quad \Theta_\theta = -\frac{U_2}{U_4}, \quad \Theta_T = -\frac{U_3}{U_4} \quad (2.2)$$

By substituting equation (2.2), the partial differential equation (2.23) then becomes

$$F \left\{ H^2 \left[\frac{U_1^2}{U_4^2} + \frac{1}{X^2} \frac{U_2^2}{U_4^2} \right], \frac{3}{2} H H_T X \frac{U_1}{U_4}, \frac{2}{3} \frac{H}{H_T X} \frac{U_3}{U_1} \right\} = 0 \quad (2.3)$$

which has auxiliary equations

$$\begin{aligned} & \frac{1}{2H^2 \frac{U_1}{U_4} F_1 + \frac{3}{2} H H_T X \frac{1}{U_4} F_2 - \frac{2}{3} \frac{H}{H_T X} \frac{U_3}{U_1} F_3} \\ &= \frac{d\theta}{2H^2 \frac{U_2}{X^2 U_4} F_1} = \frac{dT}{\frac{2}{3} \frac{H}{H_T X U_1} F_3} \\ &= \frac{d\Theta}{-2H^2 \frac{1}{U_4} \left[U_1^2 + \frac{U_2^2}{X^2} \right] \cdot F_1 - \frac{3}{2} H_T \cdot H \cdot X \frac{U_1}{U_4} F_3} \\ &= \frac{dU_1}{\frac{2H^2 U_2^2}{U_1^2 X^3} F_1 - \frac{3}{2} H H_T \frac{U_1}{U_4} F_2 + \frac{2H}{3H_T X^2} \frac{U_3}{U_1} F_3} \end{aligned}$$

$$\begin{aligned}
&= \frac{dU_1}{-2\frac{HH_T}{U_1^2} \left[U_1^2 + \frac{U_2^2}{X^2} \right] F_1 - \frac{3}{2} [H_T^2 + HH_{TT}] X \frac{U_1}{U_2} F_2 - \frac{2H_T^2 - HH_{TT}}{3H_T^2 X} \frac{U_2 F_3}{U_1}} \\
&= \frac{dU_2}{0} \\
&= \frac{dU_3}{0}
\end{aligned}$$

where F_1, F_2 and F_3 denote the derivatives of function F with respect to eigen parameters α^2, α^R and C respectively. To determine unknown functions U_1, U_2, U_3 and U_4 , we need to seek

$$\begin{cases} U_1 = B \\ U_4 = A \end{cases} \quad (2.4a)$$

where A and B are two arbitrary constants.

Our aim then is to find the third partial differential equation of the type (1.5). By linear combination, we obtain an equivalent form of equations (3.2)

$$\begin{aligned}
&\frac{dU_1}{\frac{2H_T^2 B^2}{A^2 X^3} F_1 - \frac{3}{2} HH_T \frac{U_1}{A} F_2 + \frac{2}{3} \frac{HU_3}{U_1 H_T X^2} F_3} \\
&= \frac{d\theta}{-\frac{2H^2}{A^3} \left[U_1^2 + \frac{B^2}{X^2} \right] F_1 - \frac{3}{2} HH_T X \frac{U_1}{A^2} F_2} = \frac{d\theta}{2H^2 \frac{B}{X^2 A^2} F_1} \\
&= \frac{dT}{\frac{2}{3} \frac{H}{H_T U_1} F_3} = \frac{A_d \theta + U_1 dx + B_{d\theta} + U_{3d} T}{0} \\
&= \frac{-dU_3 + A \frac{H_T}{H} d\theta + \frac{H_{TT}}{H_T} [X dU_1 - B d\theta] - \frac{H_T}{H} U_3 dT}{0} \quad (2.4b)
\end{aligned}$$

There are no further combination in the numerator with zero in the denominator, because no linear combination of the three non-zero denominators can be zero.

To obtain an exact integral which is independent of F , we introduce the integrating factors G and Q , which are functions of six variables $x, \theta, \Theta, T, U_1, U_3$ for the two linear combinations

$$-dU_3 + A \frac{H_T}{H} d\theta + \frac{H_{TT}}{H_T} [X dU_1 - B d\theta] - \frac{H_T}{H} U_3 dT \quad (2.5)$$

and

$$Ad\theta + U_1 dx + U_3 dT + Bd\theta \quad (2.6)$$

A partial differential equation of the first order

$$Q \left(-dU_3 + A \frac{H_T}{H} d\theta + \frac{H_{TT}}{H_T} (X dU_1 - B d\theta) - \frac{H_T}{H} U_3 dT \right)$$

$$+ G(Ad\theta + U_dX + Bd\theta + U_sdT) = 0 \quad (2.7)$$

obtained from (2.5) and (2.6) is then expected to be a complete integral. Now we wish to determine the integrating factors G and Q by using the method given by Forsyth (p.17). For this purpose, we let

$$\left. \begin{aligned} x_1 &= U_s, & x_2 &= X, & x_3 &= U_1 \\ x_4 &= \theta, & x_5 &= T, & x_6 &= \theta \end{aligned} \right\} \quad \text{and}$$

$$\left. \begin{aligned} X_1 &= -Q, & X_2 &= G, & X_3 &= x_2 \frac{H_{TT}}{H_T} Q \\ X_4 &= A Q \frac{H_T}{H} + A G, & X_5 &= -\frac{H_T}{H} x_1 Q + x_1 G \\ X_6 &= -\frac{H_{TT}}{H} B Q + B G \end{aligned} \right\} \quad (2.8)$$

If this new variables are substituted, the equation (2.7) then becomes the standard form:

$$\sum_{i=1}^6 X_i dx_i = 0 \quad (2.9)$$

The conditions of integrability of equation (2.9) require that the equations

$$a_{m,n} X_r + a_{n,r} X_m + a_{r,m} X_n = 0 \quad (2.10)$$

to be satisfied identically for all combinations of the indices m, n, r from series 1, 2, 3, 4, 5, 6. The coefficients a_{ij} are evaluated by following formulas:

$$\left. \begin{aligned} a_{m,n} &= \frac{\partial X_m}{\partial x_n} - \frac{\partial X_n}{\partial x_m} \\ a_{n,r} &= \frac{\partial X_n}{\partial x_r} - \frac{\partial X_r}{\partial x_n} \\ a_{r,m} &= \frac{\partial X_r}{\partial x_m} - \frac{\partial X_m}{\partial x_r} \end{aligned} \right\} \quad (2.11)$$

From equations (2.10), we obtain a set of partial differential equations. There are twenty of such equations, but only ten of them are independent. They are given as follows:

$$123: \left(\frac{H_{TT}}{H_T} Q - G \right) F + x_2 x_3 \frac{H_{TT}}{H_T} \left(G \frac{\partial G}{\partial x_1} - F \frac{\partial G}{\partial x_1} \right) + x_3 \left(G \frac{\partial Q}{\partial x_3} - Q \frac{\partial G}{\partial x_3} \right) = 0$$

$$124: A \left(Q \frac{\partial G}{\partial x_2} - G \frac{\partial Q}{\partial x_2} \right) + A x_3 \frac{H_T}{H} \left(G \frac{\partial G}{\partial x_1} - Q \frac{\partial G}{\partial x_1} \right) + x_3 \left(G \frac{\partial G}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) = 0$$

$$125: x_1 \left(Q \frac{\partial G}{\partial x_2} - G \frac{\partial Q}{\partial x_2} \right) + x_3 \left(G \frac{\partial Q}{\partial x_5} - Q \frac{\partial G}{\partial x_5} \right) + x_3 G \left(G - \frac{H_T}{H} Q \right)$$

$$+ x_2 x_3 \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_1} \right) = 0$$

$$\begin{aligned}
126: \quad & B \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_2} \right) + x_1 \left(G \frac{\partial Q}{\partial x_6} - Q \frac{\partial G}{\partial x_5} \right) + B x_3 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_1} \right) = 0 \\
134: \quad & A \left(Q \frac{\partial G}{\partial x_3} - G \frac{\partial Q}{\partial x_3} \right) + A x_2 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial G}{\partial x_1} \right) = 0 \\
135: \quad & x_1 \left(Q \frac{\partial G}{\partial x_3} - G \frac{\partial Q}{\partial x_3} \right) + x_1 x_1 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_1} \right) \\
& - x_2 Q \left\{ \left[\left(\frac{H_{TT}}{H_T} \right)_T + \frac{H_{TT}}{H_T} \right] Q - \frac{H_{TT}}{H_T} G \right\} = 0 \\
136: \quad & B \left(Q \frac{\partial G}{\partial x_3} - G \frac{\partial Q}{\partial x_3} \right) + B x_2 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_1} \right) = 0 \\
145: \quad & A \left(G^2 - \frac{H_{TT}}{H_T} Q^2 \right) + A \left(G \frac{\partial Q}{\partial x_5} - Q \frac{\partial G}{\partial x_5} \right) + x_1 \left(Q \frac{\partial G}{\partial x_4} - G \frac{\partial Q}{\partial x_4} \right) \\
& + 2A x_1 \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_1} \right) = 0 \\
146: \quad & A \left(G \frac{\partial Q}{\partial x_5} - Q \frac{\partial G}{\partial x_5} \right) + B \left(Q \frac{\partial G}{\partial x_4} - G \frac{\partial Q}{\partial x_4} \right) \\
& + A \cdot B \left(\frac{H_{TT}}{H_T} + \frac{H_T}{H} \right) \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial Q}{\partial x_1} \right) = 0 \\
156: \quad & B \left[\left(\frac{H_{TT}}{H_T} + \frac{H_T}{H} \right) G Q - \left[\left(\frac{H_{TT}}{H_T} \right)_T + \frac{H_T}{H} \right] Q^2 - G^2 \right] \\
& + B x_1 \left(\frac{H_{TT}}{H_T} - \frac{H_T}{H} \right) \left(Q \frac{\partial G}{\partial x_1} - G \frac{\partial G}{\partial x_1} \right) \\
& + x_1 \left(G \frac{\partial Q}{\partial x_5} - Q \frac{\partial G}{\partial x_5} \right) + B \left(Q \frac{\partial G}{\partial x_5} - G \frac{\partial Q}{\partial x_5} \right) = 0 \\
234: \quad & A \left(\frac{H_T}{H} Q + G \right) \left(G - \frac{H_{TT}}{H_T} Q \right) + A x_3 \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_2} - G \frac{\partial Q}{\partial x_2} \right) \\
& + A x_2 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_2} - G \frac{\partial Q}{\partial x_2} \right) + x_2 x_3 \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) = 0 \\
235: \quad & x_1 \left(G - \frac{H_T}{H} Q \right) \left(G - \frac{H_{TT}}{H_T} Q \right) + x_2 x_3 \left(\frac{H_{TT}}{H_T} \right)_T G Q + x_1 x_3 \frac{H_T}{H} \left(G \frac{\partial Q}{\partial x_3} \right. \\
& \left. - Q \frac{\partial G}{\partial x_3} \right) + x_1 x_2 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_2} - G \frac{\partial Q}{\partial x_2} \right) + x_2 x_3 \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) = 0 \\
236: \quad & B \left(\frac{H_{TT}}{H_T} Q - G \right)^2 + B x_3 \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_3} - Q \frac{\partial G}{\partial x_3} \right) + B x_1 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_1} \right. \\
& \left. - G \frac{\partial Q}{\partial x_1} \right) + x_2 x_3 \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) = 0
\end{aligned}$$

$$245: \quad A_{x_3} \left(\frac{H_T}{H} \right)_T QG + 2x_1 \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_2} - G \frac{\partial Q}{\partial x_2} \right) + A_{x_3} \frac{H_T}{H} \left(G \frac{\partial Q}{\partial x_5} - Q \frac{\partial G}{\partial x_5} \right) \\ + x_1 x_3 \frac{H_T}{H} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) = 0$$

$$246: \quad B_{x_1} \left(\frac{H_T}{H} - \frac{H_{TT}}{H_T} \right) \left(G \frac{\partial Q}{\partial x_2} - Q \frac{\partial G}{\partial x_2} \right) + B_{x_3} \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) \\ + x_1 x_3 \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_6} - G \frac{\partial Q}{\partial x_6} \right) + B_{x_3} \left(\frac{H_{TT}}{H_T} \right)_T QG = 0$$

$$345: \quad A_{x_1} Q \left\{ \left[\frac{H_{TT}}{H_T} \left(\frac{H_T}{H} \right)_T - \left(\frac{H_{TT}}{H_T} \right) \frac{H_T}{H} \right] Q - \left[\frac{H_{TT}}{H_T} \right]_T G \right\} \\ + 2A_{x_1} \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_3} - G \frac{\partial Q}{\partial x_3} \right) + x_1 x_2 \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) \\ + A_{x_2} \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_5} - G \frac{\partial Q}{\partial x_5} \right) = 0$$

$$346: \quad AB \left(\frac{H_{TT}}{H_T} + \frac{H_T}{H} \right) \left(Q \frac{\partial G}{\partial x_3} - G \frac{\partial Q}{\partial x_3} \right) + B_{x_2} \frac{H_{TT}}{H_T} \left(G \frac{\partial Q}{\partial x_4} - Q \frac{\partial G}{\partial x_4} \right) \\ + A_{x_2} \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_6} - G \frac{\partial Q}{\partial x_6} \right) = 0$$

$$356: \quad B_{x_2} \left(\frac{H_{TT}}{H_T} \right)_T GQ + x_1 x_2 \frac{H_{TT}}{H_T} \left(Q \frac{\partial G}{\partial x_6} - G \frac{\partial Q}{\partial x_6} \right) + B_{x_2} \frac{H_{TT}}{H_T} \\ \left(G \frac{\partial Q}{\partial x_5} - Q \frac{\partial G}{\partial x_5} \right) + B_{x_1} \cdot \left(\frac{H_T}{H} - \frac{H_{TT}}{H_T} \right) \left(G \frac{\partial Q}{\partial x_3} - Q \frac{\partial G}{\partial x_3} \right) = 0$$

$$456: \quad ABQ \cdot \left\{ \left[\frac{H_T}{H} \left(\frac{H_{TT}}{H_T} \right)_T - \left(\frac{H_T}{H} \right)_T \frac{H_{TT}}{H_T} \right] Q + \left[\left(\frac{H_T}{H} \right)_T + \left(\frac{H_{TT}}{H_T} \right)_T \right] G \right\} \\ + B_{x_1} \left(\frac{H_{TT}}{H_T} - \frac{H_T}{H} \right) \left(Q \frac{\partial G}{\partial x_4} - G \frac{\partial Q}{\partial x_4} \right) + AB \left(\frac{H_{TT}}{H_T} + \frac{H_T}{H} \right) \left(G \frac{\partial Q}{\partial x_5} \right. \\ \left. - Q \frac{\partial G}{\partial x_5} \right) + 2A_{x_1} \frac{H_T}{H} \left(Q \frac{\partial G}{\partial x_6} - G \frac{\partial Q}{\partial x_6} \right) = 0$$

From the above set of equations we can infer the following results:

A. The integrating factors G and Q have to satisfy the equation

$$\frac{G}{Q} = \frac{H_{TT}}{H_T} \quad (2.12)$$

B. The function H describing the motion of the oscillating disk is defined by:

$$H(t) = (1 - \sigma T)^a \quad (2.13)$$

and satisfying the initial condition:

$$H(0) = 1 \quad (2.14)$$

where σ and a are two arbitrary constants.

C. If we substitute the functions G , Q and H into equations 123, 124, 125,

126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, we can then see that they are all satisfied.

D. The partial differential equation becomes a complete integral, if the equation (2.12) and (2.13) are substituted. It may be written in the form:

$$\begin{aligned} (1-\sigma T)dU_3 + A(2a-1)\sigma d\theta + (a-1)\sigma \\ (XdU_1 + U_1dx) - U_3dT = 0 \end{aligned} \quad (2.15)$$

Without loss of generality, we choose two constants

$$\left. \begin{aligned} A &= -1 \\ B &= n \end{aligned} \right\} \quad (2.16)$$

i.e.

$$\theta_0 = n$$

as the disturbance has a wave length 2π .

After integrating, we have the following complete integral containing an arbitrary constant k_1

$$-(2a-1)\theta + (a-1)XU_1 + \left(\frac{1}{\sigma} - T\right)U_3 = k_1 \quad (2.17)$$

Now we have a system of four equations:

$$\left\{ \begin{aligned} F\left\{(1-\sigma T)^{2a}\left(\frac{U_1^2}{U_4^2} + \frac{U_2^2}{X^2U_4^2}\right), \frac{3}{2}(1-\sigma T)^{2a-1} \cdot XR\frac{U_1}{U_4}, \frac{-2(1-\sigma T)U_3}{3\sigma XU_1}\right\} &= 0 \\ -(2a-1)\theta + (a-1)XU_1 + \frac{1}{\sigma}(1-\sigma T)U_3 &= k_1 \\ U_2 &= n \\ U_4 &= -1 \end{aligned} \right. \quad (2.18)$$

for four unknown functions U_1 , U_2 , U_3 and U_4 . In the next section, we shall discuss its solutions to get the Pfaffian equation

$$U_4d\theta + U_2d\theta + U_1dX + U_3dT = 0 \quad (2.19)$$

3 COMPLETE INTEGRAL

1. Pfaffian equation

Our objective here is to seek a solution of Pfaffian equation (2.19). As stated in §3.2, we need to solve the system of equations (3.18) first. But the difficulty is that the function F is unknown. However we can use its expansion given in [Wang, Stuart and NG], and then infer some results for some special values of α and σ .

(i) In the case $\sigma = 1$, $\alpha = 1/2$, the moving speed of the upper disk is in the form

$$H(T)_r = \frac{-1}{2}(1-T)^{-1/2} \quad (3.1)$$