中国博士后科学论文集

王志石 等

清华大学出版社

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内 容 简 介

本论文集收集了1988年5月在北京召开的第一届中国博士后学术年会中选出的32篇论文，涉及数学物理、应用化学、生物与地学以及若干应用技术，许多论文反映了多学科、跨学科的特点，具有理论深度和应用价值，并代表了各有关学科的前沿，对高等学校和科研单位的科技人员有参考价值。

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编 者 的 话

本论文集主要收集了1988年5月在北京召开的第一届博士后学术年会提交的部分论文，同时征集了部分国外博士后的文章。反映了自中国建立博士后科研流动站制度以来，博士后科研人员的部分研究成果。

论文集分为四个部分，即数学物理、应用化学、生物与地学以及应用技术。由于许多论文反映了多学科、跨学科的特点，上述划分只是为了读者阅读方便，并不是绝对的。收集的论文力求具有理论深度和应用价值，同时代表了学科的前沿。是否达到了这一目的，敬请广大读者评价。

由于篇幅有限，有些博士后提交的文章没有收入，敬请谅解。

本论文集是由中国北京博士后联谊会第一任干事长王志石博士组织编辑的，并由王枢平同志作了全面整理与校核工作。在此，表示衷心感谢。

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中国北京博士后联谊会

1989年2月

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Squeeze Film Analysis II—Eigenvalue Problem

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Abstract In this paper we present a mathematical study of solving eigenvalue equation obtained in Part I. An informal solution of special initial value problem of a perturbation developing at some given time is obtained by using Jacobi's method. With this solution known the possible instability for a squeeze film is then discussed. The study of Jacobi's method to investigate a class of mathematical instability problem, in which the basic flow varies significantly in two coordinates with time t and the stability equation derived is an ordinary differential equation containing three parameters representing partial derivatives of a phase function, suggested by J.T.Stuart, is a new approach in hydrodynamic stability studies.

1 INTRODUCTION OF JACOBI METHOD

This section is concerned with the discussion of solutions of the first order partial differential equation(3.23), relating to the problem of squeeze film stability. The general procedure of solving this equation is to use the method of W.K.B.J., to get an asymptotic solution for some fixed local point. In this section, following an earlier work of Stuart and Diprima, we study the mathematical analysis of Jacobi's method to infer an informal solution of this eigen problem.

To illustrate the main idea, we consider a general partial differential equation

$$F(x,y,z,p,q)=0 \tag{1.1}$$

If

$$U(x,y,z)=0 \tag{1.2}$$

is a relation between x,y and z, then

$$p=-\frac{U_1}{U_3}, \qquad q=-\frac{U_2}{U_3} \tag{1.3}$$

where $U_1=\frac{\partial U}{\partial x}$, $U_2=\frac{\partial U}{\partial y}$, $U_3=\frac{\partial U}{\partial z}$. If we substitute from (1.3) into equation (1.1), we obtain a partial differential equation of the type

$$f(x,y,z,U_1,U_2,U_3)=0 \tag{1.4}$$

in which the new dependent variable U does not appear.

The fundamental idea of Jacobi's method is the introduction of two further differential equations of the first order.

$$\left.\begin{aligned} g(x,y,z,U_1,U_2,U_3,a)=0 \\ h(x,y,z,U_1,U_2,U_3,b)=0 \end{aligned}\right\} \tag{1.5}$$

involving two arbitrary constants a and b and such that:

(a) equations (1.4) and (1.5) can be solved for U_1, U_2, U_3;

(b) the equation

$$\mathrm{d}U=U_1\mathrm{d}x+U_2\mathrm{d}y+U_3\mathrm{d}z \tag{1.6}$$

obtained from these values of U_1, U_2, U_3 is integrable

The main difficulty of this approach is in the determination of the auxiliary equations (1.5). We have to find two equations which are compatible with (1.4), such that the equations g and h are solutions of the linear partial differential equation

$$f_{U_1}\frac{\partial g}{\partial x}+f_{U_2}\frac{\partial g}{\partial y}+f_{U_3}\frac{\partial g}{\partial z}-f_x\frac{\partial g}{\partial U_1}-f_y\frac{\partial g}{\partial U_2}-f_z\frac{\partial g}{\partial U_3}=0 \tag{1.7}$$

which has auxiliary equations

$$\frac{\mathrm{d}x}{f_{U_1}}=\frac{\mathrm{d}y}{f_{U_2}}=\frac{\mathrm{d}z}{f_{U_3}}=\frac{\mathrm{d}U_1}{-f_x}=\frac{\mathrm{d}U_2}{-f_y}=\frac{\mathrm{d}U_3}{-f_z} \tag{1.8}$$

Once the two integrals $g(x,y,z,a)$ and $h(x,y,z,b)$ of this kind have been found, the problem reduces to solving for U_1, U_2, U_3, and then the solution of equation (1.6) containing three arbitrary constants will be a complete integral of (1.4). The three constants are necessary if the given equation is (1.4). However, when the equation is given in the form (1.1), we need only two arbitrary constants in the final solution. By taking different choices of third arbitrary constant we get different complete integral of the given equation.

In applying this idea in the problem of instability of squeeze film, the main difficulty here is that the eigenvalue function F appeared in equation is unknown. However, there is a possibility of finding some partial differential equations of first order, independing the unknown function F, by using the method of Forsyth. With this integral and an expansion of equation F given, we can get the solution for U_1, U_3(U_2, U_4 are two special constant functions). From the values of U_i, the Pfaffian equation obtained is a non-linear ordinary differential equation. This equation is solved for some special initial conditions of a perturbation developing at some given time. The transition from laminar flow to turbulence is then found by simplifying the solution of this specific initial value problem.

2 INTEGRABILITY OF EIGENVALUE EQUATION

To seek a solution of this eigenvalue equation, which is a non-linear partial differential equation, we need to introduce three further differential equations of the first order, if the Jacobi's method is used. For this purpose, we let

$$U(x, \theta, T, \Theta) = 0$$

be an arbitrary 4th-dimensional surface and denote the derivatives of U as

$$\frac{\partial U}{\partial x} = U_1, \quad \frac{\partial U}{\partial \theta} = U_2, \quad \frac{\partial U}{\partial T} = U_3, \quad \frac{\partial U}{\partial \Theta} = U_4 \tag{2.1}$$

such that, we have,

$$\Theta_x = -\frac{U_1}{U_4}, \quad \Theta_\theta = -\frac{U_2}{U_4}, \quad \Theta_T = -\frac{U_3}{U_4} \tag{2·2}$$

By substituting equation (2.2), the partial differential equation (2.23) then becomes

$$F\left\{H^2\left[\frac{U_1^2}{U_4^2} + \frac{1}{X^2}\,\frac{U_2^2}{U_4^2}\right], \frac{3}{2}H\;H_T X\frac{U_1}{U_4}, \frac{2}{3}\;\frac{H}{H_T X}\;\frac{U_3}{U_1}\right\} = 0 \tag{2·3}$$

which has auxiliary equations

$$\frac{1}{2H^2\frac{U_1}{U_2}\;F_1 + \frac{3}{2}H\;H_T X\frac{1}{U_4}\;F_2 - \frac{2}{3}\;\frac{H}{H_T X}\;\frac{U_3}{U_1}F_3}$$

$$= \frac{\mathrm{d}\theta}{2H^2\frac{U_2}{X^2U_4^2}\;F_1} = \frac{\mathrm{d}T}{\frac{2}{3}\;\frac{H}{H_T X U_1}F_3}$$

$$= \frac{\mathrm{d}\Theta}{-2H^2\frac{1}{U_4^3}\left[U_1^2 + \frac{U_2^2}{X^2}\right]\cdot F_1 - \frac{3}{2}H_T\cdot H\cdot X\;\frac{U_1}{U_4^2}F_2}$$

$$= \frac{\mathrm{d}U_1}{\frac{2H^2U_2^2}{U_4^2X^3}\;F_1 - \frac{3}{2}\;H H_T\frac{U_1}{U_4}\;F_2 + \frac{2H}{3H_T X^2}\;\frac{U_3}{U_1}F_3}$$

$$= \frac{dU_3}{-2\frac{HH_T}{U_4^2}\left[U_1^2+\frac{U_3^2}{X^2}\right]F_1-\frac{3}{2}[H_T^2+HH_{TT}]X\frac{U_1}{U_4}F_2-\frac{2H_T^2-HH_{TT}}{3H_T^2X}\frac{U_3F_3}{U_1}}$$

$$= \frac{dU_2}{0}$$

$$= \frac{dU_4}{0}$$

where F_1, F_2 and F_3 denote the derivatives of function F with respect to eigen parameters α^2, α^R and C respectively. To determine unknown functions U_1, U_2, U_3 and U_4, we need to seek

$$\left.\begin{array}{l} U_3 = \mathrm{B} \\ U_4 = \mathrm{A} \end{array}\right\} \tag{2.4a}$$

where A and B are two arbitrary constants.

Our aim then is to find the third partial differential equation of the type (1.5). By linear combination, we obtain an equivalent form of equations (3.2)

$$\frac{dU_1}{\frac{2H_T^2B^2}{A^2X^3}F_1-\frac{3}{2}HH_T\frac{U_1}{A}F_2+\frac{2}{3}\frac{HU_3}{U_1H_TX^2}F_3}$$

$$= \frac{d\Theta}{-\frac{2H^2}{A^3}\left[U_1^2+\frac{B^2}{X^2}\right]F_1-\frac{3}{2}HH_TX\frac{U_1}{A^2}F_2} = \frac{d\theta}{2H^2\frac{B}{X^2A^2}F_1}$$

$$= \frac{dT}{\frac{2}{3}\frac{H}{H_TU_1}F_3} = \frac{A_d\Theta+U_1dx+B_{d\theta}+U_{3d}T}{0}$$

$$= \frac{-dU_3+A\frac{H_T}{H}d\Theta+\frac{H_{TT}}{H_T}[X_dU_1-B_{d\theta}]-\frac{H_T}{H}U_3dT}{0} \tag{2.4b}$$

There are no further combination in the numerator with zero in the denominator, because no linear combination of the three non-zero denominators can be zero.

To obtain an exact integral which is independent of F, we introduce the integrating factors G and Q, which are functions of six variables x, θ, Θ, T, U_1, U_3 for the two linear combinations

$$-dU_3+A\frac{H_T}{H}d\Theta+\frac{H_{TT}}{H_T}[XdU_1-Bd\theta]-\frac{H_T}{H}U_3dT \tag{2.5}$$

and

$$Ad\Theta+U_1dx+U_3dT+Bd\theta \tag{2.6}$$

A partial differential equation of the first order

$$Q\left(-dU_3+A\frac{H_T}{H}d\Theta+\frac{H_{TT}}{H_T}(XdU_1-Bd\theta)-\frac{H_T}{H}U_3dT\right)$$

$$+G(Ad\Theta+U_1 dX+Bd\theta+U_3 dT)=0 \tag{2.7}$$

obtained from (2.5) and (2.6) is then expected to be a complete integral. Now we wish to determine the integrating factors G and Q by using the method given by Forsyth (p.17). For this purpose, we let

$$\left.\begin{aligned}
&x_1=U_3, \quad x_2=X, \quad x_3=U_1 \\
&x_4=\Theta, \quad x_5=T, \quad x_6=\theta \\
&\text{and} \\
&X_1=-Q, X_2=Gx_3, X_3=x_2\frac{H_{TT}}{H_T}Q \\
&X_4=AQ\frac{H_T}{H}+AG, X_5=-\frac{H_T}{H}x_1Q+x_1G \\
&X_6=-\frac{H_{TT}}{H}BQ+BG
\end{aligned}\right\} \tag{2.8}$$

If this new variables are substituted, the equation (2.7) then becomes the standard form:

$$\sum_{i=1}^{6} X_i dx_i=0 \tag{2.9}$$

The conditions of integrability of equation (2.9) require that the equations

$$a_{m,n}X_r+a_{n,r}X_m+a_{r,m}X_n=0 \tag{2.10}$$

to be satisfied identically for all combinations of the indices m, n, r from series 1,2,3,4,5,6. The coefficients a_{ij} are evaluated by following formulas:

$$\left.\begin{aligned}
a_{m,n}&=\frac{\partial X_m}{\partial x_n}-\frac{\partial X_n}{\partial x_m} \\
a_{n,r}&=\frac{\partial X_n}{\partial x_r}-\frac{\partial X_r}{\partial x_n} \\
a_{r,m}&=\frac{\partial X_r}{\partial x_m}-\frac{\partial X_m}{\partial x_r}
\end{aligned}\right\} \tag{2.11}$$

From equations (2.10), we obtain a set of partial differential equations. There are twenty of such equations, but only ten of them are independent. They are given as follows:

123: $\left(\frac{H_{TT}}{H_T}Q-G\right)F+x_2x_3\frac{H_{TT}}{H_T}\left(G\frac{\partial G}{\partial x_1}-F\frac{\partial G}{\partial x_1}\right)+x_3\left(G\frac{\partial Q}{\partial x_3}-Q\frac{\partial G}{\partial x_3}\right)=0$

124: $A\left(Q\frac{\partial G}{\partial x_2}-G\frac{\partial Q}{\partial x_2}\right)+Ax_3\frac{H_T}{H}\left(G\frac{\partial G}{\partial x_1}-Q\frac{\partial G}{\partial x_1}\right)+x_3\left(G\frac{\partial G}{\partial x_4}-Q\frac{\partial G}{\partial x_4}\right)=0$

125: $x_1\left(Q\frac{\partial G}{\partial x_2}-G\frac{\partial Q}{\partial x_2}\right)+x_3\left(G\frac{\partial Q}{\partial x_5}-Q\frac{\partial G}{\partial x_5}\right)+x_3G\left(G-\frac{H_T}{H}Q\right)$

$$+x_2x_3\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_1}-G\frac{\partial Q}{\partial x_1}\right)=0$$

126: $B\left(Q\frac{\partial G}{\partial x_2} - G\frac{\partial Q}{\partial x_2}\right) + x_2\left(G\frac{\partial Q}{\partial x_6} - Q\frac{\partial G}{\partial x_6}\right) + Bx_3\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial Q}{\partial x_1}\right) = 0$

134: $A\left(Q\frac{\partial G}{\partial x_3} - G\frac{\partial Q}{\partial x_3}\right) + Ax_2\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial G}{\partial x_1}\right) = 0$

135: $x_1\left(Q\frac{\partial G}{\partial x_3} - G\frac{\partial Q}{\partial x_3}\right) + x_1x_1\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial Q}{\partial x_1}\right)$

$$- x_2Q\left\{\left[\left(\frac{H_{TT}}{H_T}\right)_T + \frac{H_{TT}}{H_T}\right] Q - \frac{H_{TT}}{H_T}G\right\} = 0$$

136: $B\left(Q\frac{\partial G}{\partial x_3} - G\frac{\partial Q}{\partial x_3}\right) + Bx_2\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial Q}{\partial x_1}\right) = 0$

145: $A\left(G^2 - \frac{H_{TT}}{H_T}Q^2\right) + A\left(G\frac{\partial Q}{\partial x_5} - Q\frac{\partial G}{\partial x_5}\right) + x_1\left(Q\frac{\partial G}{\partial x_4} - G\frac{\partial Q}{\partial x_4}\right)$

$$+ 2A_{x_1}\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial Q}{\partial x_1}\right) = 0$$

146: $A\left(G\frac{\partial Q}{\partial x_6} - Q\frac{\partial G}{\partial x_6}\right) + B\left(Q\frac{\partial G}{\partial x_4} - G\frac{\partial Q}{\partial x_4}\right)$

$$+ A\cdot B\left(\frac{H_{TT}}{H_T} + \frac{H_T}{H}\right)\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial Q}{\partial x_1}\right) = 0$$

156: $B\left[\left(\frac{H_{TT}}{H_T} + \frac{H_T}{H}\right)GQ - \left[\left(\frac{H_{TT}}{H_T}\right)_T + \frac{H_T}{H}\right]Q^2 - G^2\right]$

$$+ B_{x_1}\left(\frac{H_{TT}}{H_T} - \frac{H_T}{H}\right)\left(Q\frac{\partial G}{\partial x_1} - G\frac{\partial G}{\partial x_1}\right)$$

$$+ x_1\left(G\frac{\partial Q}{\partial x_6} - Q\frac{\partial G}{\partial x_6}\right) + B\left(Q\frac{\partial G}{\partial x_5} - G\frac{\partial Q}{\partial x_5}\right) = 0$$

234: $A\left(\frac{H_T}{H}Q + G\right)\left(G - \frac{H_{TT}}{H_T}Q\right) + A_{x_3}\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_3} - G\frac{\partial Q}{\partial x_3}\right)$

$$+ A_{x_2}\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_2} - G\frac{\partial Q}{\partial x_2}\right) + x_2x_3\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_4} - Q\frac{\partial G}{\partial x_4}\right) = 0$$

235: $x_1\left(G - \frac{H_T}{H}Q\right)\left(G - \frac{H_{TT}}{H_T}Q\right) + x_2x_3\left(\frac{H_{TT}}{H_T}\right)_T GQ + x_1x_3\frac{H_T}{H}\left(G\frac{\partial Q}{\partial x_3}\right.$

$$\left. - Q\frac{\partial G}{\partial x_3}\right) + x_1x_2\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_2} - G\frac{\partial Q}{\partial x_2}\right) + x_2x_3\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_5} - Q\frac{\partial G}{\partial x_5}\right) = 0$$

236: $B\left(\frac{H_{TT}}{H_T}Q - G\right)^2 + B_{x_3}\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_3} - Q\frac{\partial G}{\partial x_3}\right) + B_{x_2}\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_2}\right.$

$$\left. - G\frac{\partial Q}{\partial x_2}\right) + x_2x_3\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_6} - Q\frac{\partial G}{\partial x_6}\right) = 0$$

245: $$A_{x_3}\left(\frac{H_T}{H}\right)_T QG + 2_{x_1}\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_2} - G\frac{\partial Q}{\partial x_2}\right) + A_{x_3}\frac{H_T}{H}\left(G\frac{\partial Q}{\partial x_5} - Q\frac{\partial G}{\partial x_5}\right)$$
$$+ x_1 x_3\frac{H_T}{H}\left(G\frac{\partial Q}{\partial x_4} - Q\frac{\partial G}{\partial x_4}\right) = 0$$

246: $$B_{x_1}\left(\frac{H_T}{H} - \frac{H_{TT}}{H_T}\right)\left(G\frac{\partial Q}{\partial x_2} - Q\frac{\partial G}{\partial x_2}\right) + B_{x_3}\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_4} - Q\frac{\partial G}{\partial x_4}\right)$$
$$+ x_1 x_3\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_6} - G\frac{\partial Q}{\partial x_6}\right) + Bx_3\left(\frac{H_{TT}}{H_T}\right)_T QG = 0$$

345: $$A_{x_2} Q\left\{\left[\frac{H_{TT}}{H_T}\left(\frac{H_T}{H}\right)_T - \left(\frac{H_{TT}}{H_T}\right)\frac{H_T}{H}\right]Q - \left[\frac{H_{TT}}{H_T}\right]_T G\right\}$$
$$+ 2A_{x_1}\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_3} - G\frac{\partial Q}{\partial x_3}\right) + x_1 x_2\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_4} - Q\frac{\partial G}{\partial x_4}\right)$$
$$+ A_{x_2}\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_5} - G\frac{\partial Q}{\partial x_5}\right) = 0$$

346: $$AB\left(\frac{H_{TT}}{H_T} + \frac{H_T}{H}\right)\left(Q\frac{\partial G}{\partial x_3} - G\frac{\partial Q}{\partial x_3}\right) + B_{x_2}\frac{H_{TT}}{H_T}\left(G\frac{\partial Q}{\partial x_4} - Q\frac{\partial G}{\partial x_4}\right)$$
$$+ A_{x_2}\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_6} - G\frac{\partial Q}{\partial x_6}\right) = 0$$

356: $$B_{x_2}\left(\frac{H_{TT}}{H_T}\right)_T GQ + x_1 x_2\frac{H_{TT}}{H_T}\left(Q\frac{\partial G}{\partial x_6} - G\frac{\partial Q}{\partial x_6}\right) + B_{x_2}\frac{H_{TT}}{H_T}$$
$$\left(G\frac{\partial Q}{\partial x_5} - Q\frac{\partial G}{\partial x_5}\right) + B_{x_1}\cdot\left(\frac{H_T}{H} - \frac{H_{TT}}{H_T}\right)\left(G\frac{\partial Q}{\partial x_3} - Q\frac{\partial G}{\partial x_3}\right) = 0$$

456: $$ABQ\cdot\left\{\left[\frac{H_T}{H}\left(\frac{H_{TT}}{H_T}\right)_T - \left(\frac{H_T}{H}\right)_T\frac{H_{TT}}{H_T}\right]Q + \left[\left(\frac{H_T}{H}\right)_T + \left(\frac{H_{TT}}{H_T}\right)_T\right]G\right\}$$
$$+ B_{x_1}\left(\frac{H_{TT}}{H_T} - \frac{H_T}{H}\right)\left(Q\frac{\partial G}{\partial x_4} - G\frac{\partial Q}{\partial x_4}\right) + AB\left(\frac{H_{TT}}{H_T} + \frac{H_T}{H}\right)\left(G\frac{\partial Q}{\partial x_5}\right.$$
$$\left.- Q\frac{\partial G}{\partial x_5}\right) + 2A_{x_1}\frac{H_T}{H}\left(Q\frac{\partial G}{\partial x_6} - G\frac{\partial Q}{\partial x_6}\right) = 0$$

From the above set of equations we can infer the following results:

A. The integrating factors G and Q have to satisfy the equation

$$\frac{G}{Q} = \frac{H_{TT}}{H_T} \tag{2.12}$$

B. The function H describing the motion of the oscillating disk is defined by:

$$H(t) = (1 - \sigma T)^a \tag{2.13}$$

and satisfying the initial condition:

$$H(0) = 1 \tag{2.14}$$

where σ and a are two arbitrary constants.

C. If we substitute the functions G, Q and H into equations 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, we can then see that they are all satisfied.

D. The partial differential equation becomes a complete integral, if the equation (2.12) and (2.13) are substituted. It may be written in the form:

$$(1-\sigma T)dU_3 + A(2a-1)\sigma d\Theta + (a-1)\sigma (XdU_1 + U_1 dx) - U_3 dT = 0 \tag{2.15}$$

Without loss of generality, we choose two constants

$$\left.\begin{array}{l} A = -1 \\ B = n \end{array}\right\} \tag{2.16}$$

i.e. $\Theta_\theta = n$

as the disturbance has a wave length 2π.

After integrating, we have the following complete integral containing an arbitrary constant k_1

$$-(2a-1)\Theta + (a-1)XU_1 + \left(\frac{1}{\sigma} - T\right)U_3 = k_1 \tag{2.17}$$

Now we have a system of four equations:

$$\begin{cases} F\left\{(1-\sigma T)^{2a}\left(\frac{U_1^2}{U_4^2} + \frac{U_2^2}{X^2 U_4^2}\right),\ \frac{3}{2}(1-\sigma T)^{2a-1} \cdot XR\frac{U_1}{U_4},\ \frac{-2(1-\sigma T)U_3}{3\sigma X U_1}\right\} = 0 \\ -(2a-1)\Theta + (a-1)XU_1 + \frac{1}{\sigma}(1-\sigma T)U_3 = k_1 \\ U_2 = n \\ U_4 = -1 \end{cases} \tag{2.18}$$

for four unknown functions U_1, U_2, U_3 and U_4. In the next section, we shall discuss its solutions to get the Pfaffian equation

$$U_4 d\Theta + U_2 d\theta + U_1 dX + U_3 dT = 0 \tag{2.19}$$

3 COMPLETE INTEGRAL

I. Pfaffian equation

Our objective here is to seek a solution of Pfaffian equation (2.19). As stated in §3.2, we need to solve the system of equations (3.18) first. But the difficulty is that the function F is unknown. However we can use its expansion given in [Wang, Stuart and NG], and then infer some results for some special values of a and σ.

(i) In the case $\sigma = 1$, $a = 1/2$, the moving speed of the upper disk is in the form

$$H(T)_T = \frac{-1}{2}(1-T)^{-1/2} \tag{3.1}$$