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内潮的一种分层三维数值模式*

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在中国近海的不少海域曾经发现相当强的内潮现象，例如赵俊生(1992)报道了渤海海峡的内潮现象，Yamashiro(1988)在东海陆坡处亦发现内潮的存在。特别是 Amoco 石油公司在南海陆坡处进行了一年多的海流测量，测得最大的全日潮流振幅可超过 60 cm/s，这里正压潮流小于 10cm/s，故观测到的潮流主要与内潮有关。迄今关于内潮的数值模式大多将表面潮作为已知输入条件，从而使模式的应用受到较多限制(Jiang and Fang, 1992)。Heaps(1983)曾发展了一种自由海面多层模式，但他们忽略了非线性平流项，这无疑也会损失许多有用信息。本文将叙述一种自由海面的多层模式，它能够同时模拟表面潮和内潮，并包含了非线性平流项。

一、基本方程

我们把海洋分成 K 层，用 $k=1,2,\cdots,K$ 记之。每层具有相同的海水密度 ρ_k，即 ρ_k 与

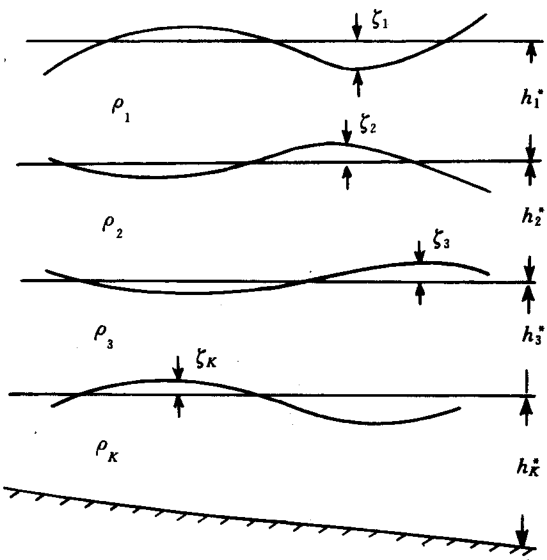

图 1　垂向分层

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层号 k 有关，与水平位置(x,y)无关。各层静止时的厚度除底层外也只与 k 有关，与(x,y)无关，记为 $h_1^*, h_2^*, \cdots$，各层上界面的扰动高度记为ζ_k(如图 1)。以 u_k 和 v_k 记各层水平流速的 x 和 y 方向分量；τ_{kx}和 τ_{ky}记第 k 层与第 $k+1$ 层之间摩擦力的 x 和 y 方向分量，特别地，τ_{sx},τ_{sy}和 τ_{Kx},τ_{Ky}分别为海面风应力和海底处摩擦力；h 为未扰动总水深，f 和 g 分别为 Coriolis 参量和重力加速度，P_a 为海面大气压，A 为水平湍粘系数，则基本方程可写为：

$$\begin{cases}\dfrac{\partial u_1}{\partial t}=-L_1(u_1)+fv_1-g\dfrac{\partial \zeta_1}{\partial x}-\dfrac{1}{\rho_1}\dfrac{\partial p_a}{\partial x}+\dfrac{\tau_{sx}-\tau_{1x}}{h_1+\zeta_1-\zeta_2}+A\Delta u_1\\ \dfrac{\partial v_1}{\partial t}=-L_1(v_1)-fu_1-g\dfrac{\partial \zeta_1}{\partial y}-\dfrac{1}{\rho_1}\dfrac{\partial p_a}{\partial y}+\dfrac{\tau_{sy}-\tau_{1y}}{h_1+\zeta_1-\zeta_2}+A\Delta v_1\\ \dfrac{\partial \zeta_1}{\partial t}=\dfrac{\partial \zeta_2}{\partial t}-\dfrac{\partial[(h_1+\zeta_1-\zeta_2)u_1]}{\partial x}-\dfrac{\partial[(h_1+\zeta_1-\zeta_2)v_1]}{\partial y}\end{cases} \tag{1.1}$$

$$\begin{cases}\dfrac{\partial u_2}{\partial t}=-L_2(u_2)+fv_2-g\dfrac{\rho_1}{\rho_2}\dfrac{\partial \zeta_1}{\partial x}-g\dfrac{\rho_2-\rho_1}{\rho_2}\dfrac{\partial \zeta_2}{\partial x}-\dfrac{1}{\rho_2}\dfrac{\partial p_a}{\partial x}+\dfrac{\tau_{1x}-\tau_{2x}}{h_2+\zeta_2-\zeta_3}+A\Delta u_2\\ \dfrac{\partial v_2}{\partial t}=-L_2(v_2)-fu_2-g\dfrac{\rho_1}{\rho_2}\dfrac{\partial \zeta_1}{\partial y}-g\dfrac{\rho_2-\rho_1}{\rho_2}\dfrac{\partial \zeta_2}{\partial y}-\dfrac{1}{\rho_2}\dfrac{\partial p_a}{\partial y}+\dfrac{\tau_{1y}-\tau_{2y}}{h_2+\zeta_2-\zeta_3}+A\Delta v_2\\ \dfrac{\partial \zeta_2}{\partial t}=\dfrac{\partial \zeta_3}{\partial t}-\dfrac{\partial[(h_2+\zeta_2-\zeta_3)u_2]}{\partial x}-\dfrac{\partial[(h_2+\zeta_2-\zeta_3)v_2]}{\partial y}\end{cases} \tag{1.2}$$

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$$\begin{cases}\dfrac{\partial u_K}{\partial t}=-L_K(u_K)+fv_K-g\dfrac{\rho_1}{\rho_K}\dfrac{\partial \zeta_1}{\partial x}-g\dfrac{\rho_2-\rho_1}{\rho_K}\dfrac{\partial \zeta_2}{\partial x}-\cdots\\ \qquad -g\dfrac{\rho_{K-1}-\rho_{K-2}}{\rho_K}\dfrac{\partial \zeta_{K-1}}{\partial x}-g\dfrac{\rho_K-\rho_{K-1}}{\rho_K}\dfrac{\partial \zeta_K}{\partial x}\\ \qquad -\dfrac{1}{\rho_K}\dfrac{\partial p_a}{\partial x}+\dfrac{\tau_{K-1x}-\tau_{Kx}}{h_K+\zeta_K}+A\Delta u_K\\ \dfrac{\partial v_K}{\partial t}=-L_K(v_K)-fu_K-g\dfrac{\rho_1}{\rho_K}\dfrac{\partial \zeta_1}{\partial y}-g\dfrac{\rho_2-\rho_1}{\rho_K}\dfrac{\partial \zeta_2}{\partial y}-\cdots\\ \qquad -g\dfrac{\rho_{K-1}-\rho_{K-2}}{\rho_K}\dfrac{\partial \zeta_{K-1}}{\partial y}-g\dfrac{\rho_K-\rho_{K-1}}{\rho_K}\dfrac{\partial \zeta_K}{\partial y}\\ \qquad -\dfrac{1}{\rho_K}\dfrac{\partial p_a}{\partial y}+\dfrac{\tau_{K-1y}-\tau_{Ky}}{h_K+\zeta_K}+A\Delta v_K\\ \dfrac{\partial \zeta_K}{\partial t}=-\dfrac{\partial[(h_K+\zeta_K)u_K]}{\partial x}-\dfrac{\partial[(h_K+\zeta_K)v_K]}{\partial y}\end{cases} \tag{1.3}$$

其中，

$$L_k(a)=\frac{\partial}{\partial x}(u_k a)+\frac{\partial}{\partial y}(v_k a),\qquad \Delta a=\frac{\partial^2 a}{\partial x^2}+\frac{\partial^2 a}{\partial y^2} \tag{1.4}$$

由上列方程可得全流方程如下:

$$\begin{cases}\dfrac{\partial u}{\partial t}=-L(u)+fv-g\dfrac{\partial \zeta_1}{\partial x}-\dfrac{1}{\rho}\dfrac{\partial p_a}{\partial x}+\dfrac{\tau_{sx}-\tau_{Kx}}{h+\zeta_1}+A\Delta u\\ \dfrac{\partial v}{\partial t}=-L(v)+fv-g\dfrac{\partial \zeta_1}{\partial y}-\dfrac{1}{\rho}\dfrac{\partial p_a}{\partial y}+\dfrac{\tau_{sy}-\tau_{Ky}}{h+\zeta_1}+A\Delta v\\ \dfrac{\partial \zeta}{\partial t}=-\dfrac{\partial[(h+\zeta_1)u]}{\partial x}-\dfrac{\partial[(h+\zeta_1)v]}{\partial y}\end{cases} \tag{1.5}$$

其中,

$$L(a)=\frac{1}{h+\zeta_1}\sum_{k=1}^{K}[H_kL_k(a_k)] \tag{1.6}$$

$$H_k=\begin{cases}h_k+\zeta_k-\zeta_{k+1}, & \text{当 } k=1,2,\cdots,K-1\\ h_K+\zeta_K, & \text{当 } k=K\end{cases} \tag{1.7}$$

相应的海区侧边界条件是闭边界处法向流速为零,即

$$u,u_k=0 \qquad \text{或 } v,v_k=0 \tag{1.8}$$

开边界各网格点处一般取表面潮高为时间的已知函数,各界面垂直位移由表面潮高计算,即

$$\begin{cases}\zeta_1=\Sigma A_m\cos(\omega_m t-\theta_m)\\ \zeta_2=(1-h_1/h)\zeta_1\\ \cdots\cdots\cdots\cdots\cdots\cdots\\ \zeta_K=[1-(h_1+h_2+\cdots h_{K-1})/h]\zeta_1\end{cases} \tag{1.9}$$

上式中 A 为分潮振幅,ω 为角频率,θ 为迟角;$\zeta_2,\cdots,\zeta_K$ 的公式隐含开边界处无斜压潮这个条件。当然,如不采用此条件也可另外给定。在全部侧边界取滑动(无粘)条件,即切向流速分量沿法向的梯度为零。

二、水平网格及有关的变量和参数

这里采用 Arakawa-C 网格,计算点的配置如图 2 所示。下面我们以 3 层模式为例说明计算过程。各网格点有下列各变量:

ζ 点:$\zeta_1(i,j),\zeta_2(i,j),\zeta_3(i,j),\tilde{p}(i,j)$

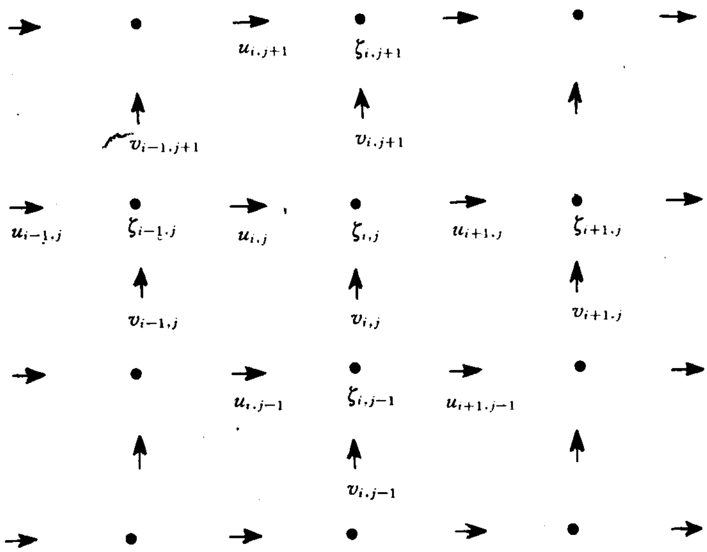

图 2 水平网格点配置图

u 点：$h'(i,j),K'(i,j),h'_1(i,j),h'_2(i,j),h'_3(i,j),\beta'(i,j),g'_2(i,j),g'_3(i,j),\tau_{sx}(i,j),$
$u(i,j),u_1(i,j),u_2(i,j),u_3(i,j),Q_x(i,j)$

v 点：$h''(i,j),K''(i,j),h''_1(i,j),h''_2(i,j),h''_3(i,j),\beta''(i,j),g''_2(i,j),g''_3(i,j),\tau_{sy}(i,j),$
$v(i,j),v_1(i,j),v_2(i,j),v_3(i,j),Q_y(i,j)$

此外，在模式中包含有下列参数：
$\rho_1,\rho_2,\rho_3,h_1^*,h_2^*,g,f,A,C,\Delta x,\Delta y,\Delta t,T_x(=\Delta t/(2\Delta x)),T_y(=\Delta t/(2\Delta y)),C_r(=f\Delta t/2),v_2,v_3$

其中 v_2 和 v_3 为第一、二层之间及第二、三层之间的垂直湍粘系数。上列变量和参数的计算方法如下：

1. $\tilde{p}\equiv p_a/\rho_1$，若 p_a 单位取 mb(即 hPa)，则 $\tilde{p}=p_a\div 102.5$，单位：m.

2. 如水深 $h_{i,j}$ 在水平网格空格处(即 $v_{i-1,j}$ 和 $v_{i,j}$ 之间，亦即 $u_{i,j-1}$ 和 $u_{i,j}$ 之间)给出，则可事先算出 $h'_{i,j}$ 和 $h''_{i,j}$，算完后 $h_{i,j}$ 即可不必保留。计算公式为

$h'_{i,j}=(h_{i,j}+h_{i,j+1})/2,\qquad h''_{i,j}=(h_{i,j}+h_{i+1,j})/2$

3. K',h'_1,h'_2,h'_3 的确定方法如下：

若 $h''_{i,j}\le h_1^*$，则 $K'_{i,j}=1,h'_{1i,j}=h'_{i,j}$

若 $h_1^*<h'_{i,j}\le h_1^*+h_2^*$，则 $K'_{i,j}=2,h'_{1i,j}=h_1^*,h'_{2i,j}=h'_{i,j}-h_1^*$

若 $h_1^*+h_2^*<h'_{i,j}$，则 $K'_{i,j}=3,h'_{1i,j}=h_1^*,h'_{2i,j}=h_2^*,h'_{3i,j}=h'_{i,j}-h_1^*-h_2^*$

K'',h''_1,h''_2,h''_3 确定方法一样。

4. $g'_{2i,j}=g[h'_{2i,j}(\rho_2-\rho_1)/\rho_2+h'_{3i,j}(\rho_2-\rho_1)/\rho_3]/h'_{i,j}$

$g'_{3i,j}=g[h'_{3i,j}(\rho_3-\rho_1)/\rho_3]/h'_{i,j}$

$\beta'_{i,j}=[h'_{1i,j}+h'_{2i,j}(\rho_1/\rho_2)+h'_{3i,j}(\rho_1/\rho_3)]/h'_{i,j}$

g''_2,g''_3,β''算法一样,仅将h',h'_k换成h'',h''_k。

三、有限差分方程

设$n\Delta t$时刻ζ_1,u,v已知,$n\Delta t$时刻$u_k,v_k(k=1,2,3)$已知,$(n+1/2)\Delta t$时刻$\zeta_k(k=2,3)$已知,$(n+1/2)\Delta t$时刻$\widetilde{p},\tau_{sx},\tau_{sy}$已算出,则下一个$\Delta t$时刻内的计算过程如下:

1. ζ_1,u,v的计算

ζ_1,u,v为与表面潮有关的量,我们采用Leendertse(1967)的交替方向隐格式,具体计算步骤如下:

(1) 第一个半步

(a) ζ_1,u,用隐式计算,方程组为

$$\begin{cases}A'_{i,j}\zeta_{1i-1,j}^{n+1/2}+B'_{i,j}u_{i,j}^{n+1/2}+C'_{i,j}\zeta_{1i,j}^{n+1/2}=F'_{i,j}\\ A_{i,j}u_{i,j}^{n+1/2}+B_{i,j}\zeta_{1i,j}^{n+1/2}+C_{i,j}u_{i+1,j}^{n+1/2}=F_{i,j}\end{cases}\tag{3.1}$$

其中

$$\begin{cases}A'_{i,j}=-T_xg\beta'_{i,j}\\ B'_{i,j}=1\\ C'_{i,j}=-A'_{i,j}\\ F'_{i,j}=u_{i,j}^n+(\Delta t/2)\{f\overline{v}_{i,j}^n-Q_{xi,j}+A[(u_{i+1,j}^n+u_{i+1,j}^n-2u_{i,j}^n)/\Delta x^2\\ \quad+(u_{i,j-1}^n+u_{i,j+1}^n-2u_{i,j}^n)/\Delta y^2]\}\end{cases}\tag{3.2}$$

$$\begin{cases}A_{i,j}=-T_xh'_{i,j}\\ B_{i,j}=1\\ C_{i,j}=T_xh'_{i+1,j}\\ F_{i,j}=\zeta_{1i,j}^n-T_y(h''_{i,j+1}v_{i,j+1}^n-h''_{i,j}v_{i,j}^n)\end{cases}\tag{3.3}$$

式(3.2)中

$$\begin{aligned}Q_{xi,j}=&[g'_{2i,j}(\zeta_{2i,j}^{n+1/2}-\zeta_{2i-1,j}^{n+1/2})+g'_{3i,j}(\zeta_{3i,j}^{n+1/2}-\zeta_{3i-1,j}^{n+1/2})\\ &+\beta'_{i,j}(\widetilde{p})_{i,j}^{n+1/2}-\widetilde{p}_{i-1,j}^{n+1/2})]/\Delta x\\ &-\{\tau_{sxi,j}^{n+1/2}-C[(u_{K'i,j}^n)^2+(\overline{v}_{K'i,j}^n)^2]^{1/2}u_{K'i,j}^n\}/h'_{i,j}\end{aligned}\tag{3.4}$$

(b) v 用显式计算，公式为

$$v_{i,j}^{n+1/2}=v_{i,j}^{n}-(\Delta t/2)\{Q_{yi,j}+g\beta''_{i,j}(\zeta_{1i,j}^{n}-\zeta_{1i,j-1}^{n})/\Delta y +A[(v_{i-1,j}^{n}+v_{i+1,j}^{n}-2v_{i,j}^{n})/\Delta x^2 +(v_{i,j-1}^{n}+v_{i,j+1}^{n}-2v_{i,j}^{n})/\Delta y^2]\} \tag{3.5}$$

式中

$$Q_{yi,j}=f\bar{u}_{i,j}^{n+1/2}+[g''_{2i,j}(\zeta_{2i,j}^{n+1/2}-\zeta_{2i,j-1}^{n+1/2})+g''_{3i,j}(\zeta_{3i,j}^{n+1/2}-\zeta_{3i,j-1}^{n+1/2})+\beta''_{i,j}(\tilde{p}_{i,j}^{n+1/2}-\tilde{p}_{i,j-1}^{n+1/2})]/\Delta y -\{\tau_{syi,j}^{n+1/2}-C[(\bar{u}_{K''i,j}^{n})^2+(v_{K''i,j}^{n})^2]^{1/2}v_{K''i,j}^{n}\}/h''_{i,j} \tag{3.6}$$

(2) 第二半步

(a) ζ_1, v 用隐式计算，方程组为

$$\begin{cases} A'_{i,j}\zeta_{i,j-1}^{n+1}+B'_{i,j}v_{i,j}^{n+1}+C'_{i,j}\zeta_{i,j}^{n+1}=F'_{i,j} \\ A_{i,j}v_{i,j}^{n+1}+B_{i,j}\zeta_{i,j}^{n+1}+C_{i,j}v_{i,j+1}^{n+1}=F_{i,j} \end{cases} \tag{3.7}$$

其中

$$\begin{cases} A'_{i,j}=-T_y g\beta''_{i,j} \\ B'_{i,j}=1 \\ C'_{i,j}=-A'_{i,j} \\ F'_{i,j}=v_{i,j}^{n+1/2}-(\Delta t/2)\{Q_{yi,j}-A[(v_{i-1,j}^{n+1/2}+v_{i+1,j}^{n+1/2}-2v_{i,j}^{n+1/2})/\Delta x^2 \\ \qquad +(v_{i,j-1}^{n+1/2}+v_{i,j+1}^{n+1/2}-2v_{i,j}^{n+1/2})/\Delta y^2]\} \end{cases} \tag{3.8}$$

$$\begin{cases} A_{i,j}=-T_y h''_{i,j} \\ B_{i,j}=1 \\ C_{i,j}=T_y h''_{i,j+1} \\ F_{i,j}=\zeta_{1i,j}^{n+1/2}-T_x(h'_{i+1,j}u_{i+1,j}^{n+1/2}-h'_{i,j}u_{i,j}^{n+1/2}) \end{cases} \tag{3.9}$$

式(3.8)中 $Q_{yi,j}$ 见式(3.6)。

(b) u 用显式计算，公式为

$$u_{i,j}^{n+1}=u_{i,j}^{n+1/2}+(\Delta t/2)\{f\bar{v}_{i,j}^{n+1}-g\beta'_{i,j}(\zeta_{1i,j}^{n+1/2}-\zeta_{1i-1,j}^{n+1/2})-Q_{xi,j} +A[(u_{i-1,j}^{n+1/2}+u_{i+1,j}^{n+1/2}-2u_{i,j}^{n+1/2})/\Delta x^2+(u_{i,j-1}^{n+1/2}+u_{i,j+1}^{n+1/2}-2u_{i,j}^{n+1/2})/\Delta y^2]\} \tag{3.10}$$

上面各式中 $\bar{u}, \bar{v}$ 定义如下：

$$\bar{u}_{i,j}=(u_{i,j}+u_{i+1,j}+u_{i+1,j-1}+u_{i,j-1})/4,\bar{v}_{i,j}=(v_{i,j}+v_{i-1,j}+v_{i-1,j+1}+v_{i,j+1})/4 \tag{3.11}$$

2. $u_k,v_k(k=1,2,3),\zeta_k(k=2,3)$的计算

(1) u_k,v_k 的计算

(a) u_k 的计算

若第(i,j)个 u 点的 $K'_{i,j}=1$,可直接取

$$u_{1i,j}^{n+1}=u_{i,j}^{n+1} \tag{3.12}$$

若 $K'_{i,j}=2$,则

$$\begin{cases}\hat{u}_{1i,j}^{n+1}=(B_2F_1-C_1F_2)/(1-A_2-C_1)\\ \hat{u}_{2i,j}^{n+1}=(B_2F_2-A_2F_1)/(1-A_2-C_1)\end{cases} \tag{3.13}$$

式中

$$\begin{cases}C_1=-2v_2\Delta t/[h'_{1i,j}(h'_{1i,j}+h'_{2i,j})]\\ B_1=1-C_1\\ A_2=C_1h'_{1i,j}/h'_{2i,j}\\ B_2=1-A_2\\ F_1=u_{1i,j}^n+\Delta t\{f\bar{v}_{1i,j}^n-G_1+\tau_{sxi,j}^{n+1/2}/h'_{1i,j}+A[(u_{1i-1,j}^n+u_{1i+1,j}^n-2u_{1i,j}^n)/\Delta x^2\\ \qquad +(u_{1i,j-1}^n+u_{1i,j+1}^n-2u_{1i,j}^n)/\Delta y^2]\}\\ F_2=u_{2i,j}^n+\Delta t\{f\bar{v}_{2i,j}^n-G_2-C[(u_{2i,j}^n)^2+(\bar{v}_{2i,j}^n)^2]^{1/2}u_{2i,j}^n/h'_{2i,j}\\ \qquad +A[(u_{2i-1,j}^n+u_{2i+1,j}^n-2u_{2i,j}^n)/\Delta x^2+(u_{2i,j-1}^n+u_{2i,j+1}^n-2u_{2i,j}^n)/\Delta y^2]\}\end{cases} \tag{3.14}$$

其中

$$\begin{cases}G_1=[g(\zeta_{1i,j}^{n+1/2}-\zeta_{1i-1,j}^{n+1/2})+(\tilde{p}_{i,j}^{n+1/2}-\tilde{p}_{i-1,j}^{n+1/2})]/\Delta x\\ G_2=(\rho_1/\rho_2)G_1+g(1-\rho_1/\rho_2)(\zeta_{2i,j}^{n+1/2}-\zeta_{2i-1,j}^{n+1/2})/\Delta x\end{cases} \tag{3.15}$$

最后取

$$u_{1i,j}^{n+1}=\hat{u}_{1i,j}^{n+1}+\Delta u,\qquad u_{2i,j}^{n+1}=\hat{u}_{2i,j}^{n+1}+\Delta u \tag{3.16}$$

其中

$$\Delta u=u_{i,j}^{n+1}-(\hat{u}_{1i,j}^{n+1}h'_{1i,j}+\hat{u}_{2i,j}^{n+1}h'_{2i,j})/h'_{i,j} \tag{3.17}$$

应注意的是，在计算 u_k, v_k 时用到 $\zeta_1^{n+1/2}$，故对 $\zeta_{1(i,j)}$ 需两个数组。在计算出 $u^{n+1/2}$ 和 $\zeta_1^{n+1/2}$ 后，ζ_1 仍要保留，用于计算 $v^{n+1/2}$[见式(3.5)]；在计算出 $v^{n+1/2}$ 和 ζ_1^{n+1} 后 ζ_1^n 可不要，但 ζ_1^{n+1} 仍要保留，用于计算 u_k, v_k。

若 $K'_{i,j}=3$，则需解下列三对角系数矩阵的方程组：

$$\begin{cases} B_1\hat{u}_{1i,j}^{n+1}+C_1\hat{u}_{2i,j}^{n+1} & =F_1 \\ A_2\hat{u}_{1i,j}^{n+1}+B_2\hat{u}_{2i,j}^{n+1}+C_2\hat{u}_{3i,j}^{n+1} & =F_2 \\ \qquad A_3\hat{u}_{2i,j}^{n+1}+B_3\hat{u}_{3i,j}^{n+1} & =F_3 \end{cases} \tag{3.18}$$

其中

$$\begin{cases} C_1=-2v_2\Delta t/[h'_{1i,j}(h'_{1i,j}+h'_{2i,j})] \\ B_1=1-C_1 \\ A_2=C_1h'_{1i,j}/h'_{2i,j} \\ C_2=-2v_3\Delta t/[h'_{2i,j}(h'_{2i,j}+h'_{3i,j})] \\ B_2=1-A_2-C_2 \\ A_3=C_2h'_{2i,j}/h'_{3i,j} \\ B_3=1-A_3 \\ F_1=u_{1i,j}^n+\Delta t\{f\bar{v}_{1i,j}^n-G_1+\tau_{sxi,j}^{n+1/2}/h'_{1i,j}+A[(u_{1i-1,j}^n+u_{1i+1,j}^n-2u_{1i,j}^n)/\Delta x^2 \\ \qquad +(u_{1i,j}^n+u_{1i,j+1}^n-2u_{1i,j}^n)/\Delta y^2]\} \\ F_2=u_{2i,j}^n+\Delta t\{f\bar{v}_{2i,j}^n-G_2+A[(u_{2i-1,j}^n+u_{2i+1,j}^n-2u_{2i,j}^n)/\Delta x^2 \\ \qquad +(u_{2i,j}^n+u_{2i,j+1}^n-2u_{2i,j}^n/\Delta y^2)]\} \\ F_3=u_{3i,j}^n+\Delta t\{f\bar{v}_{3i,j}^n-G_3-C[(u_{3i,j}^n)^2+(\bar{v}_{3i,j}^n)^2]^{1/2}u_{3i,j}^{n'}/h'_{3i,j} \\ \qquad +A[(u_{3i-1,j}^n+u_{3i+1,j}^n-2u_{3i,j}^n)/\Delta x^2+(u_{3i,j-1}^n+u_{3i,j+1}^n-2u_{3i,j}^n)/\Delta y^2]\} \end{cases} \tag{3.19}$$

其中

$$\begin{cases} G_1=[g(\zeta_{1i,j}^{n+1/2}-\zeta_{1i-1,j}^{n+1/2})+(\tilde{p}_{i,j}^{n+1/2}-\tilde{p}_{i-1,j}^{n+1/2})]/\Delta x \\ G_2=(\rho_1/\rho_2)G_1+g(1-\rho_1/\rho_2)(\zeta_{2i,j}^{n+1/2}-\zeta_{2i-1,j}^{n+1/2})/\Delta x \\ G_3=(\rho_2/\rho_3)G_2+g(1-\rho_2/\rho_3)(\zeta_{3i,j}^{n+1/2}-\zeta_{3i-1,j}^{n+1/2})/\Delta x \end{cases} \tag{3.20}$$

最后取

$$u_{1i,j}^{n+1}=\hat{u}_{1i,j}^{n+1}+\Delta u, \qquad u_{2i,j}^{n+1}=\hat{u}_{2i,j}^{n+1}+\Delta u, \qquad u_{3i,j}^{n+1}=\hat{u}_{3i,j}^{n+1}+\Delta u \tag{3.21}$$

其中

$$\Delta u=u_{i,j}^{n+1}-(\hat{u}_{1i,j}^{n+1}h'_{1i,j}+\hat{u}_{2i,j}^{n+1}h'_{2i,j}+\hat{u}_{3i,j}^{n+1}h'_{3i,j})/h'_{i,j} \tag{3.22}$$

(b) v_k 的计算

若 $K''_{i,j}=1$,可直接取

$$v_{1i,j}^{n+1}=v_{i,j}^{n+1} \tag{3.23}$$

若 $K''_{i,j}=2$,则

$$\begin{cases}\hat{v}_{1i,j}^{n+1}=(B_2F_1-C_1F_2)/(1-A_2-C_1)\\ \hat{v}_{2i,j}^{n+1}=(B_1F_2-A_2F_1)/(1-A_2-C_1)\end{cases} \tag{3.24}$$

其中

$$\begin{cases}C_1=-2v_2\Delta t/[h''_{1i,j}(h''_{1i,j}+h''_{2i,j})]\\ B_1=1-C_1\\ A_2=C_1h''_{1i,j}/h''_{2i,j}\\ B_2=1-A_2\\ F_1=v_{1i,j}^n+\Delta t\{-f\overline{u}_{1i,j}^n-G_1+\tau_{syi,j}^{n+1/2}/h''_{1i,j}+A[(v_{1i-1,j}^n+v_{1i+1,j}^n-2v_{1i,j}^n)/\Delta x^2\\ \qquad +(v_{1i,j-1}^n+v_{1i,j+1}^n-2v_{1i,j}^n)/\Delta y^2]\}\\ F_2=v_{2i,j}^n+\Delta t\{-f\overline{u}_{2i,j}^n-G_2-C[(\overline{u}_{2i,j}^n)^2+(v_{2i,j}^n)^2]^{1/2}v_{2i,j}^n/h''_{2i,j}\\ \qquad +A[(v_{2i-1,j}^n+v_{2i+1,j}^n-2v_{2i,j}^n)/\Delta x^2+(v_{2i,j-1}^n+v_{2i,j+1}^n-2v_{2i,j}^n)/\Delta y^2]\}\end{cases} \tag{3.25}$$

式中

$$\begin{cases}G_1=[g(\zeta_{1i,j}^{n+1/2}-\zeta_{1i,j-1}^{n+1/2})+(\widetilde{p}_{i,j}^{n+1/2}-\widetilde{p}_{i,j-1}^{n+1/2})]/\Delta y\\ G_2=(\rho_1/\rho_2)G_1+g(1-\rho_1/\rho_2)(\zeta_{2i,j}^{n+1/2}-\zeta_{2i,j-1}^{n+1/2})\Delta y\end{cases} \tag{3.26}$$

最后取

$$v_{1i,j}^{n+1}=\hat{v}_{1i,j}^{n+1}+\Delta v,\qquad v_{2i,j}^{n+1}=\hat{v}_{2i,j}^{n+1}+\Delta v \tag{3.27}$$

其中

$$\Delta v=v_{i,j}^{n+1}-(\hat{v}_{1i,j}^{n+1}h''_{1i,j}+\hat{v}_{2i,j}^{n+1}h''_{2i,j})/h''_{i,j} \tag{3.28}$$

若 $K''_{i,j}=3$,则 $\hat{v}_k^{n+1}$ 由下列方程解出

$$\begin{cases} B_1\hat{v}_{1i,j}^{n+1}+C_1\hat{v}_{2i,j}^{n+1} & =F_1 \\ A_2\hat{v}_{1i,j}^{n+1}+B_2\hat{v}_{2i,j}^{n+1}+C_2\hat{v}_{3i,j}^{n+1} & =F_2 \\ \quad A_3\hat{v}_{2i,j}^{n+1}+B_3\hat{v}_{3i,j}^{n+1} & =F_3 \end{cases} \tag{3.29}$$

其中

$$\begin{cases} C_1=-2v_2\Delta t/[h''_{1i,j}(h''_{1i,j}+h''_{2i,j})] \\ B_1=1-C_1 \\ A_2=C_1h''_{1i,j}/h''_{2i,j} \\ C_2=-2v_3\Delta t/[h''_{2i,j}(h''_{2i,j}+h''_{3i,j})] \\ B_2=1-A_2-C_2 \\ A_3=C_2h''_{2i,j}/h''_{3i,j} \\ B_3=1-A_3 \\ F_1=v_{1i,j}^n+\Delta t\{-f\overline{u}_{1i,j}^n-G_1+\tau_{syi,j}^{n+1/2}/h''_{1i,j}+A[(v_{1i-1,j}^n+v_{1i+1,j}^n-2v_{1i,j}^n)/\Delta x^2 \\ \qquad +(v_{1i,j-1}^n+v_{1i,j+1}^n-2v_{1i,j}^n)/\Delta y^2]\} \\ F_2=v_{2i,j}^n+\Delta t\{-f\overline{u}_{2i,j}^n-G_2+A[(v_{2i-1,j}^n+v_{2i+1,j}^n-2v_{2i,j}^n)/\Delta x^2 \\ \qquad +(v_{2i,j-1}^n+v_{2i,j+1}^n-2v_{2i,j}^n)/\Delta y^2]\} \\ F_3=v_{3i,j}^n+\Delta t\{-f\overline{u}_{3i,j}^n-G_3-C[(\overline{u}_{3i,j}^n)^2+(v_{3i,j}^n)^2]^{1/2}v_{3i,j}^n/h''_{3i,j} \\ \qquad +A[(v_{3i-1,j}^n+v_{3i+1,j}^n-2v_{3i,j}^n)/\Delta x^2+(v_{3i,j-1}^n+v_{3i,j+1}^n-2v_{3i,j}^n)/\Delta y^2]\} \end{cases} \tag{3.30}$$

式中

$$\begin{cases} G_1=[g(\zeta_{1i,j}^{n+1/2}-\zeta_{1i,j-1}^{n+1/2})+(\widetilde{p}_{i,j}^{n+1/2}-\widetilde{p}_{i,j-1}^{n+1/2})]/\Delta y \\ G_2=(\rho_1/\rho_2)G_1+g(1-\rho_1/\rho_2)(\zeta_{2i,j}^{n+1/2}-\zeta_{2i,j-1}^{n+1/2})/\Delta y \\ G_3=(\rho_2/\rho_3)G_2+g(1-\rho_2/\rho_3)(\zeta_{3i,j}^{n+1/2}-\zeta_{3i,j-1}^{n+1/2})/\Delta y \end{cases} \tag{3.31}$$

解出 $\hat{v}_k$ 后，v_k 值由 $\hat{v}_k$ 加上一个订正得到：

$$v_{1i,j}^{n+1}=\hat{v}_{1i,j}^{n+1}+\Delta v,\qquad v_{2i,j}^{n+1}=\hat{v}_{2i,j}^{n+1}+\Delta v,\qquad v_{3i,j}^{n+1}=\hat{v}_{3i,j}^{n+1}+\Delta v \tag{3.32}$$

其中

$$\Delta v=v_{i,j}^{n+1}-(\hat{v}_{1i,j}^{n+1}h''_{1i,j}+\hat{v}_{2i,j}^{n+1}h''_{2i,j}+\hat{v}_{3i,j}^{n+1}h''_{3i,j})/h''_{i,j} \tag{3.33}$$

从式(3.30)可知，$u_1(i,j)$，$u_2(i,j)$，$u_3(i,j)$，$v_1(i,j)$，$v_2(i,j)$，$v_3(i,j)$在计算过程中需使用两个数组。

(2) ζ_k 的计算

ζ_2 和 ζ_3 可用简单的显式计算:

$$\left\{\begin{aligned}\zeta_{3i,j}^{n+3/2}=&\zeta_{3i,j}^{n+1/2}-\Delta t\{(h'_{3i+1,j}u_{3i+1,j}^{n+1}-h'_{3i,j}u_{3i,j}^{n+1})/\Delta x\\&+(h''_{3i,j+1}v_{3i,j+1}^{n+1}-h''_{3i,j}v_{3i,j}^{n+1})/\Delta y\}\\\zeta_{2i,j}^{n+3/2}=&\zeta_{2i,j}^{n+1/2}+\zeta_{3i,j}^{n+3/2}-\zeta_{3i,j}^{n+1/2}-\Delta t\{(h'_{2i+1,j}u_{2i+1,j}^{n+1}-h'_{2i,j}u_{2i,j}^{n+1})/\Delta x\\&+(h''_{3i,j+1}v_{3i,j+1}^{n+1}-h''_{3i,j}v_{3i,j}^{n+1})/\Delta y\}\end{aligned}\right. \tag{3.34}$$

由上式知 $\zeta_k(k=2,3)$ 只要一个数组,其中第二式用到 $\zeta_{3i,j}^{n+3/2}$,可用一个单元暂存。

四、初步应用结果

作为对模式可用性的一次检验,我们选取了南海西北部潮波进行试算。计算区域包括 15°N 以北,115°E 以西的海域,网格距离大约 1/8 度,即取 $\Delta x=13.132\text{km}$,$\Delta y=13.889\text{km}$,$\Delta t=89.4283\text{s}$。其他参数为:$h_1=200\text{m}$,$h_2=200\text{m}$,$\rho_1=1021\text{kg/m}^3$,$\rho_2=1024\text{kg/m}^3$,$\rho_3=1027\text{kg/m}^3$,$A=100\text{m}^2/\text{s}$,$v_2=v_3=0.05\text{m}^2/\text{s}$,$C=0.0025$。由于只模拟潮波,海面风应力忽略,因而开边界以 m_1 和 M_2 分潮潮汐高度输入,m_1 的定义为 K_1 和 O_1 的平均值。

模拟得到的 m_1 和 M_2 见图 3 和图 4,将这些图与由观测得到的同潮图(Fang,1986)相

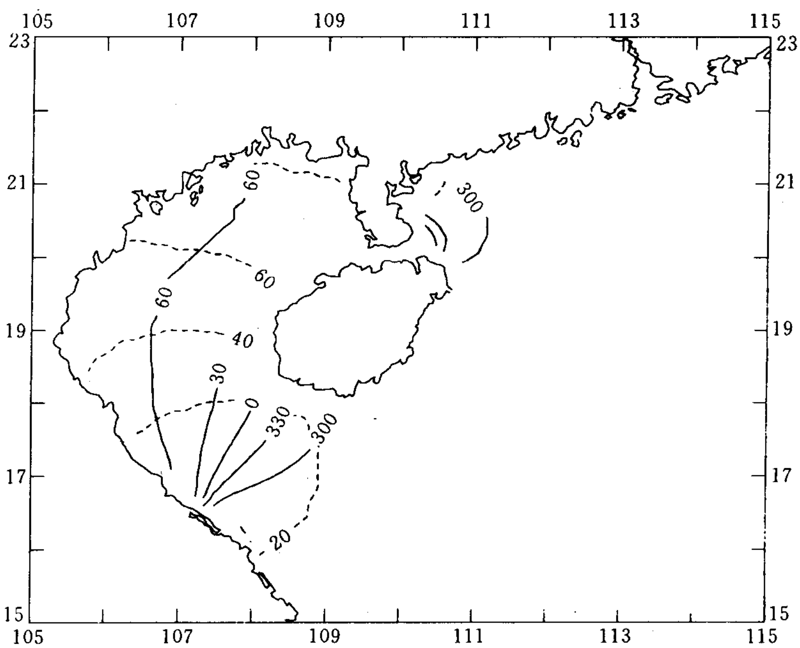

图 3 模拟得到的 m_1 表面同潮图

----振幅(单位为 cm);——迟角(单位为度)

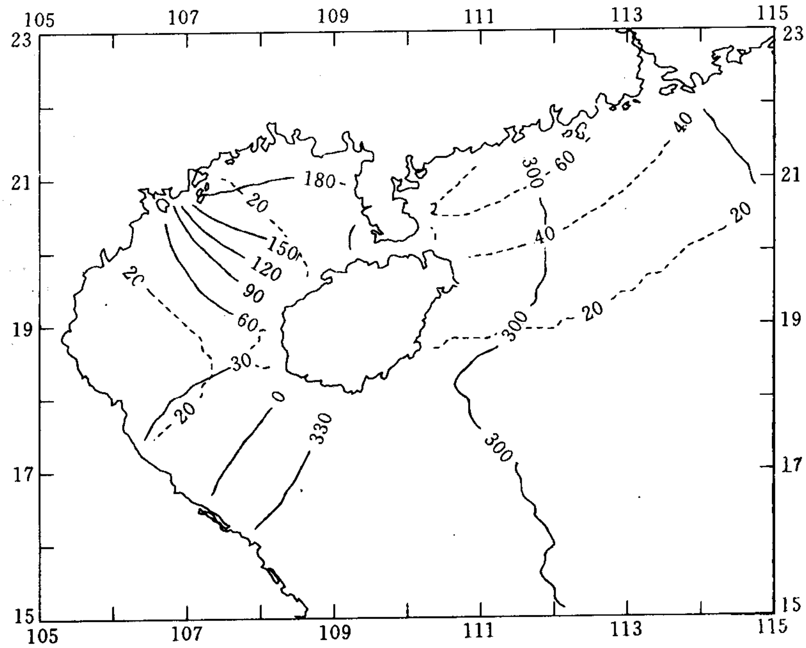

图 4 模拟得到的 M_2 表面同潮图

-----振幅(单位为 cm)；——迟角(单位为度)

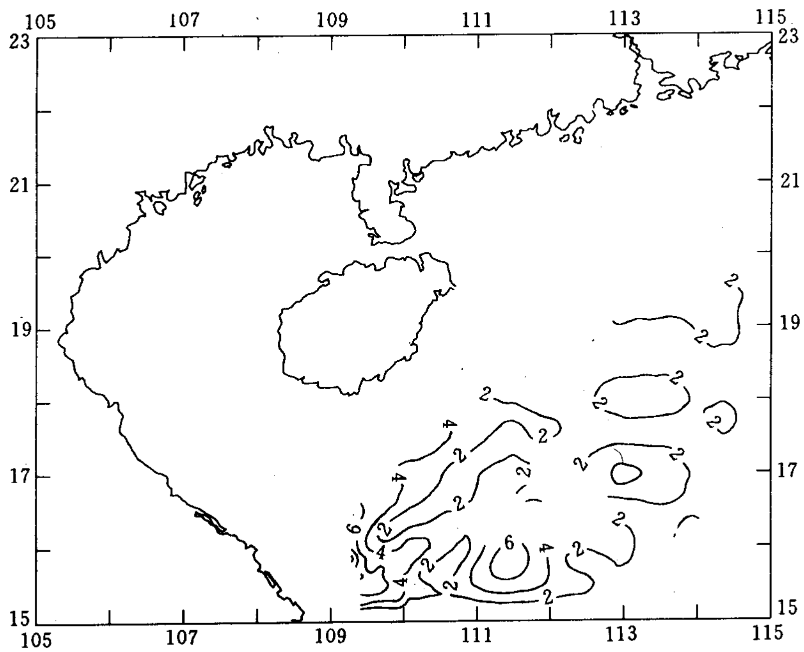

图 5 第一与第二界面处的 m_1 内潮振幅分布(m)