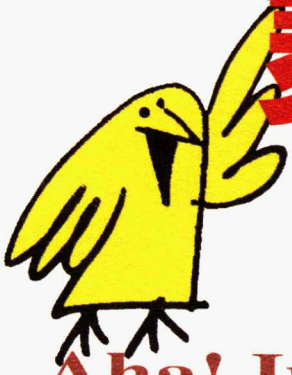
20世纪
科普经典
特藏

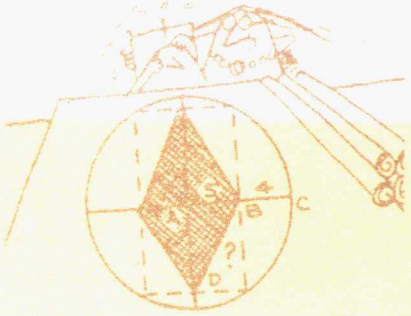
aha!

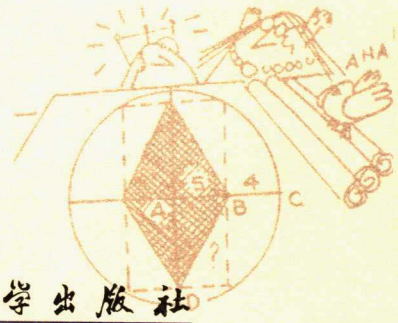
啊哈，灵机一动

〔美〕马丁·伽德纳 著
胡作玄 评点



Aha! Insight





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序

20世纪在科学发展史上是一个辉煌的世纪，以物理学和生物学的创新性成果为标志的科学成就，极大地改变了世界的面貌，改变了人类的认知水平、生产方式和生活方式。20世纪也是科学史上的一个英雄世纪，一大批别具一格的科学大师风云际会，相继登场，使科学的舞台展现出前所未有的绚丽风采。20世纪发生了两次世界大战，二战催生的原子弹，使社会公众了解了科学的巨大威力，也促使人类认真地审视科学，了解到科学必须要与人类的良知，与人文精神结合在一起，只有合理地利用，才能造福于人类，才能有利于和平，有利于人类社会的可持续发展。进入20世纪80年代，人类更进一步认识到必须携起手来保护生态，控制环境污染，探索可持续发展的道路。可持续发展理念的形成，是20世纪阶级社会发展观进步的一个重大的事件。

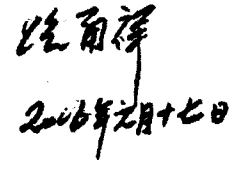
回顾20世纪科学走过的道路，从突飞猛进的科学创造，到科学与人文伦理的深度撞击，形成与人文精神交融并进的局面，最终在人类文明史上留下了不同寻常的篇章。

20世纪诞生的科学和思想大师所取得的非凡的科学成就、创造的充足科学和思想养分，孕育了一批优秀的科普作品，为公众提供了丰富的精神食粮。人们可以跟着爱因斯坦、薛定谔、伽莫夫、沃森、温伯格、霍金等等科学大师的生花妙笔去领略科学创造的历程、登攀一个个科学顶峰的征程和科学高峰的神奇景观；可以跟着卡逊在寂静的春天里思考知更鸟的命运；可以跟着萨根去观察宇宙和生命……。今天这些科学大

师和思想大师大部分都已离开了我们，但那些优秀科普作品是他们留给后代的不朽的精神财富。

20世纪已经过去，21世纪已经肯定是一个全球化、知识化的世纪，也是科技国际化、网络化的一个时代。可持续发展依然是人类唯一的发展道路，自然科学、社会科学、人文精神将交叉融合，世界的文化环境会发生很大的变化，东西方文化将会在激荡过程中进一步融合升华，创造出具有国际化，又有民族特色的新文化。在未来15年，中国要基本完成向一个创新型国家过渡。建立创新体系、创新机制配套的基础是要大幅度提高国民的文化教育水平和科学素质，把我国庞大的人口负担真正转化为无可比拟的创新人力资源。

在中国这样一个大国传播普及科技知识、科学精神是一个宏大的系统工程，需要政府组织倡导和社会各界的积极努力。中国科学院也承担着光荣而艰巨的任务，我们有义务整合全院资源努力把科普工作做大、做好，为国家和社会发挥更大的作用。科学出版社是科普图书出版的一支战略方面军，应该大有作为。《20世纪科普经典特藏》把原汁原味的经典科普大餐奉献给新时代读者，辅之以中文点评是一个很好的尝试。希望这些经典著作能给读者以启发，开拓读者的科学视野，更希望这些经典著作能起到示范的作用，推进我们自己的原创科普和科学文化作品的创作和出版。



马丁·伽德纳
Martin Gardner

本书作者马丁·伽德纳（Martin Gardner）是当代最著名的数学科普作家之一。他于1914年生于美国俄克拉荷马州的塔尔萨，中学时代就对数学深感兴趣，并一直保持至今。由于他想成为一位物理学家，因此，他没有接受过正式的高等数学教育。他进入芝加哥大学之后，逐渐对科学哲学产生了兴趣，因此放弃了物理学，专攻哲学，并于1936年获得学士学位。毕业后，他从事新闻工作。1941年美国参战，他应征入海军服役4年。战后，他回到芝加哥大学读哲学研究生，但未取得学位。其后8年他主要是自由撰稿人，特别是为儿童杂志“Humpty Dumpty”（这是在英美众所周知的矮胖子的形象）撰稿来维持生计。

1956年，数学界出现两件大事改变了他的一生。当时由纽曼（Newman）主编的四大卷“The world of Mathematics”（数学世界）成为英美的畅销书。也正是在这件事的影响下，有着110年历史的著名科普杂志“Scientific American”（科学美国人）的主编皮尔（Gerard Piel）看到了数学科普的商机，决定创办《数学游戏》专栏。由于此前伽德纳曾写过一两篇数学方面的文章给《科学美国人》，因此，皮尔邀请他主持这个专栏。后来他回忆说，他根本就没有准备好，当时他连一本有关数学游戏的书都没有。于是，他跑到纽约，买下所有有关书籍。事情就这样开始了，这成为他的终身事业。

他说，他喜欢写这个专栏是因为他热爱数学。但万万没有想到的是，《数学游戏》专栏受到广泛的欢迎，成为《科学美国人》的招牌产品。他本人也出了

名，结交了许多大数学家，也受到许多业余数学爱好者的注意。从1956年到1981年，他几乎每月一篇，连续不断写了25年。1981年底他退休后，每年还偶尔写上一两篇。这些文章现在大都收集在一起，形成了十几本单行本。另外，他又写了十几本书，例如《数学狂欢节》（Mathematical Carnival）等，而本书“Aha! Insight”（1978）是其中最著名的，曾被译成法文、德文、俄文、日文等多种外文，中文译本《啊哈！灵机一动》（白英彩、崔良沂译）于1981年由上海科学技术文献出版社出版。除了数学科普之外，他还写了许多一般科普著作，值得注意的是，他与伪科学进行不懈的斗争，特别是那些打着科学名义贩卖伪科学私货的东西。

由于他在数学科普方面的贡献，他荣获1987年美国数学会斯蒂尔（Steele）奖和1994年数学交流奖。

胡作玄

数学游戏帮你学数学

在漫长的学数学的过程中，写作业已经压得学生喘不过气来，哪还有时间读点伽德纳关于数学游戏的书呢？大多数人不去读，他们在老师、家长的督促下的确可以继续走下去。可是，多数人没有兴趣，没有主动性，更不会问学数学究竟为什么，自己究竟有什么提高？

学过9年或12年数学之后，也许能够解许多练习题，甚至考一个好分数，但是，思想上有什么提高呢？有人说，数学是思想的体操，学过数学之后，思想上应该有个飞跃，可是媒体上不时报道这样的消息，大学生积极参加骗人的传销，也有人相信以前的彩票结果对今后的彩票有影响等等。在日常生活中，更多的人不知如何理财使自己获益最大。这些都显示，数学还要从教科书以外去学习，教科书与数学游戏书是互补的。

伽德纳的书对于读者有什么好处呢？我想首先是激发你的兴趣。很少人会对数学教科书有兴趣，但多数人会对数学游戏感兴趣，上个世纪末，匈牙利人的魔方风靡世界就是一个证明。魔方的数学比较复杂，伽德纳的书则十分简单，容易入手。大多数人对下棋、扑克牌有兴趣，而数学游戏则更为简单，而且可以一个人玩。这要比沉溺于电脑游戏更有利于身心。其次，数学游戏激发你的主动性。课本甚至奥校的题目，想出来正确的答案也就完了，但是许多数学游戏可以永远地做下去，只要你还有兴趣。中国发明的幻方就是一种。幻方在中国叫“纵横图”，也就是把1，2，3，…，

n个数排列在$n \times n$个方格中，使得n行，n列以及两条对角线上数码之和都相等。这样得到的结果称为n阶幻方。3阶幻方本质上只有一种，4阶幻方就有880种，因此造出幻方对每个有兴趣的人都是一个挑战。直到最近，仍有许多大数学家研究各种巧妙的构造n阶幻方的方法。因此，只要去做，永远有做不完的事。第三，数学游戏帮助思想方法的提高。许多数学游戏并不是一下子就能想到的，一旦想到之后，对于思想会有很大的促进，在切蛋糕与火柴拼图的游戏中，我们往往只考虑平面的情形，而一旦想到3维空间去，问题就迎刃而解。还有许多问题试来试去解不出，这时就要考虑它是否没有解，当然这是需要证明的。在数学史上这种“眼前无路想回头”的事屡见不鲜，而且都形成划时代的成就，例如古希腊三大作图问题，5次方程一般不能根式解，非欧几何等等。当然历史上也有少数人顽固不化，死不回头，等待他们的也只能是失败。

有了这么多好处，快来看这本书吧！除了上面的好处之外，还有一个，那就是同时可以学不少英文！

胡作玄

Introduction

The creative act owes little to logic or reason. In their accounts of the circumstances under which big ideas occurred to them, mathematicians have often mentioned that the inspiration had no relation to the work they happened to be doing. Sometimes it came while they were traveling, shaving or thinking about other matters. The creative process cannot be summoned at will or even cajoled by sacrificial offering. Indeed, it seems to occur most readily when the mind is relaxed and the imagination roaming freely.

Morris Kline, *Scientific American*, March 1955.

引言是全书的核心，在日常生活中，在科学、艺术、商业、政治等活动中更多需要直觉，更多需要灵机一动，而这是从教科书中和数学练习中难以学到的。而这正是本书的用处。它能教会你 aha!灵机一动。

Experimental psychologists like to tell a story about a professor who investigated the ability of chimpanzees to solve problems. A banana was suspended from the center of the ceiling, at a height that the chimp could not reach by jumping. The room was bare of all objects except several packing crates placed around the room at random. The test was to see whether a lady chimp would think of first stacking the crates in the center of the room, and then of climbing on top of the crates to get the banana.

The chimp sat quietly in a corner, watching the psychologist arrange the crates. She waited patiently until the professor crossed the middle of the room. When he was directly below the fruit, the chimp suddenly jumped on his shoulder, then leaped into the air and grabbed the banana.

The moral of this anecdote is: A problem that seems

difficult may have a simple, unexpected solution. In this case the chimp may have been doing no more than following her instincts or past experience, but the point is that the chimp solved the problem in a direct way that the professor had failed to anticipate.

At the heart of mathematics is a constant search for simpler and simpler ways to prove theorems and solve problems. It is often the case that a first proof of a theorem is a paper of more than fifty pages of dense, technical reasoning. A few years later another mathematician, perhaps less famous, will have a flash of insight that leads to a proof so simple that it can be expressed in just a few lines.

Sudden hunches of this sort—hunches that lead to short, elegant solutions of problems—are now called by psychologists "aha! reactions." They seem to come suddenly out of the blue. There is a famous story about how William Rowan Hamilton, a famous Irish mathematician, invented quaternions while walking across a stone bridge. His aha! insight was a realization that an arithmetic system did not have to obey the commutative law. He was so staggered by this insight that he stopped and carved the basic formulas on the bridge, and it is said that they remain there in the stone to this day.

Exactly what goes on in a creative person's mind when he or she has a valuable hunch? The truth is that nobody knows. It is some kind of mysterious process that no one has so far been able to teach to, or store in, a computer. Computers solve problems by mechanically going step-by-step through a program that tells them exactly what to do. It is only because computers can perform these steps at such incredible speeds that computers can solve certain problems that a human mathematician cannot solve because it might take him or her several thousand years of nonstop calculation.

The sudden hunch, the creative leap of the mind that "sees" in a flash how to solve a problem in a simple way, is something quite different from general intelligence. Recent studies show that persons who possess a high aha! ability are all intelligent to a moderate level, but beyond that level there seems to be no correlation between high intelligence and aha! thinking. A person may have an extremely high I. Q., as measured by standardized tests,yet rate low in aha! ability. On the other hand, people who are not particularly brilliant in other ways may possess great aha! ability. Einstein, for instance, was not particularly skillful in traditional mathematics, and his records in school and college were mediocre. Yet the insights that produced his general theory of relativity were so profound that they completely revolutionized physics.

获得灵感的心态最主要是：先设法找出一个答案，然后看是否能够把方法简化。当动了脑筋之后，也就会使你的解题能力大大提高。

This book is a careful selection of problems that seem difficult, and indeed are difficult if you go about trying to solve them in traditional ways. But if you can free your mind from standard problem solving techniques, you may be receptive to an aha! reaction that leads immediately to a solution. Don't be discouraged if, at first, you have difficulty with these problems. Try your best to solve each one before you read the answer. After a while you will begin to catch the spirit of offbeat, nonlinear thinking, and you may be surprised to find your aha! ability improving. If so, you will discover that this ability is useful in solving many other kinds of problems that you encounter in your daily life. Suppose, for instance, you need to tighten a screw. Is it necessary to go in search of a screwdriver? Will a dime in your pocket do the job just as well?

The puzzles in this collection are great fun to try on friends. In many cases, they will think for a long time about a problem, and finally give it up as too difficult. When you tell them the simple answer, they will usually laugh. Why do they laugh? Psychologists are not sure,

but studies of creative thinking suggest some sort of relationship between creative ability and humor. Perhaps there is a connection between hunches and delight in play. The creative problem solver seems to be a type of person who enjoys a puzzling challenge in much the same way that a person enjoys a game of baseball or chess. The spirit of play seems to make him or her more receptive for that flash of insight that solves a problem.

Aha! power is not necessarily correlated with quickness of thought. A slow thinker can enjoy a problem just as much, if not more, than a fast thinker, and he or she may be even better at solving it in an unexpected way. The pleasure in solving a problem by a shortcut method may even motivate one to learn more about traditional solving techniques. This book is intended for any reader, with a sense of humor, capable of understanding the puzzles.

There certainly is a close connection, however, between aha! insights and creativity in science, in the arts, business, politics, or any other human endeavor. The great revolutions in science are almost always the result of unexpected intuitive leaps. After all, what is science if not the posing of difficult puzzles by the universe? Mother Nature does something interesting, and challenges the scientist to figure out how she does it. In many cases the solution is not found by exhaustive trial and error, the way Thomas Edison found the right filament for his electric light, or even by a deduction based on the relevant knowledge. In many cases the solution is a Eureka insight. Indeed, the exclamation "Eureka!" comes from the ancient story of how Archimedes suddenly solved an hydraulic problem while he was taking a bath. According to the legend, he was so overjoyed that he leaped out of the tub and ran naked down the street shouting "Eureka! Eureka!" (I have found it!)

We have classified the puzzles of this book into six categories: combinatorial, geometric, number, logic, procedural and verbal. These are such broad areas that there is a certain amount of unavoidable overlap, and a problem in one category could just as well be regarded as in one of the others. We have tried to surround each puzzle with a pleasant, amusing story line intended to put you in a playful mood. Our hope is that this mood will help you break away from standard problem solving routines. We urge you, each time you consider a new puzzle, to think about it from all angles, no matter how bizarre, before you spend unnecessary time trying to solve it the long way.

Gardner 教你 6 个办法去解题。当然这需要灵活运用。

Gardner 的问题共有 65 组，可以分为 6 类。但分类不一定很严格，差不多所有题都是综合的。这使你要灵活运用所学过的东西。

After each problem, with its delightful illustrations by the Canadian graphic artist Jim Glen, we have added some notes. These comments discuss related problems, and indicate how, in many cases, the puzzles lead into significant aspects of modem mathematics. In some cases, they introduce problems that are not yet solved.

We have also tried to give some broad guidelines for the channels along which aha! thinking sometimes moves:

1. Can the problem be reduced to a simpler case?
2. Can the problem be transformed to an isomorphic one that is easier to solve?
3. Can you invent a simple algorithm for solving the problem?
4. Can you apply a theorem from another branch of mathematics?
5. Can you check the result with good examples and counterexamples?
6. Are aspects of the problem given that are actually irrelevant for the solution, and whose presence in the story serves to misdirect you?

Gardner 问题中有些是很简单甚至无聊的问题，如第 6 组一些问题；也有一些解决了就完了的，缺少变化的问题，如火柴图形变化问题；真正好的问题是能够继续发展下去能激发你更进一步探索的问题，如幻方、几何剖分等。许多数学家都是由此起步得出重要结果的。

We are rapidly entering an age in which there will be increasing temptation to solve all mathematical

problems by writing computer programs. The computer, making an exhaustive trial-and-error search, may solve a problem in just a few seconds, but sometimes it takes a person hours, even days, to write a good program and remove all its bugs. Even the writing of such a program often calls for aha! insights. But with the proper aha! thinking, it may be possible to solve the same problem without writing a program at all.

It would be a sad day if human beings, adjusting to the Computer Revolution, became so intellectually lazy that they lost their power of creative thinking. The central purpose of this collection of puzzles is to exercise and improve your ability in this technique of problem solving.

Contents 目录