王国俊 等著

拓扑分子格理论

陕西师范大学出版社

TUOPU FENZIGE LILUN

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序

从拓扑分子格理论提出到现在已经整整10年了. 让我们回顾一下它产生的背景、与其它学科的关系、自身理论框架的拓广过程以及当前有待解决的问题. 这或许会对它的进一步发展起到一些作用.

1965年,美国控制论专家 L. A. Zadeh 教授提出了模糊集的概念[1]. 所谓集 X 上的一个模糊子集 A, 实际上是从 X 到单位区间的一个映射 $A:X\to[0,1]$. 我们把这种模糊子集的全体记作 $[0,1]^X$. 如果把 X 的分明子集(即通常意义下的子集) B 与其特征函数等同看待的话,那么 X 的一个分明子集 B 实际上是从 X 到 $\{0,1\}$ 的一个映射 $B:X\to\{0,1\}$. 这种分明子集的全体是 $\{0,1\}^X$. 因为 $\{0,1\}\subset[0,1]$,所以从纯数学的角度看,模糊子集的概念是分明子集概念的推广. 1968年,C. L. Chang 提出了模糊拓扑空间的概念[2],他称 $[0,1]^X$ 的一个子族 δ 为 X 上的模糊拓扑,如果 $0,1\in\delta$ 且 δ 对有限交与任意并运算关闭, 这里 $0,1$ 分别表示 X 上最小与最大的映射, 而交与并分别指映射取下确界与上确界的运算. 称序对 (X,δ) 为模糊拓扑空间. 有了这个模糊拓扑 δ, 一系列的基本概念就可完全仿效点集拓扑学中的相应概念而得出. 如, 称 δ 中的模糊集 G 为开集;若 $A\in[0,1]^X$,则称包含于 A 的最大开集为 A 的开核,记作 A^0;若 $H'=1-H\in\delta$,则称 H 为闭集;称包含

A 的最小闭集为 A 的闭包，记作 A^-；称 (X,δ) 为紧空间，若 X 的每个开覆盖 $\mathscr{U}$（即 $\mathscr{U}\subset\delta$ 且 $\sup\mathscr{U}=1$）都有有限子覆盖；等等。看来还有很多点集拓扑学中的概念和命题都可以不费力气地推广到模糊拓扑空间中来。其实，点集拓扑是在 $\{0,1\}^X$ 上展开的拓扑理论，模糊拓扑则是在 $[0,1]^X$ 上展开的拓扑理论，由于 $\{0,1\}^X$ 与 $[0,1]^X$ 有许多相似之处（如，De Morgan 对偶律都成立，联系两个空间的基本映射有许多共同的性质等），点集拓扑学的那些立足于这些共同基础的命题自然也就在模糊拓扑空间中成立而无需再进行任何证明（参看[3]）。但是 $[0,1]^X$ 与 $\{0,1\}^X$ 毕竟有许多区别，如，排中律不成立（即，$A\wedge A'=0$ 与 $A\vee A'=1$ 不再成立），择一原则不成立（即，从 $x_\lambda\in\bigvee_i A_i$ 推不出存在 i_0 使 $x_\lambda\in A_{i_0}$，参看[4]）等。这些都是重大的原则性差异，以致使点集拓扑学中传统的邻域方法在模糊拓扑学中无法适用（参看[5]）。70 年代初的一些工作遇到的困难正是这一事实的反映。随后以 B. Hutton 等人为代表的无点化学派兴起，他们关于拟一致结构与正规性的研究是相当漂亮的成果，无疑可作为无点化学派的代表作。但从另一角度去看，无点化研究的兴起恐怕也是由于有点化研究没有出路带来的结果。在此情况下，我国学者刘应明教授提出的重域理论就具有特别重要的意义（参看[6]），它为重振有点化研究打开了新的局面。

从格论的角度来看，无论是在 $L_1=\{0,1\}^X$ 上展开的点集拓扑还是在 $L_2=[0,1]^X$ 上展开的模糊拓扑，都是某种格 L 上的拓扑，从而也都可以纳入拓扑格理论之中。G. Nöbeling 的书[7]对此早有论述，只是[7]的框架过于广泛以致所得出的具体结论过于

贫乏，特别是缺少点的概念及其相应的邻近结构理论，从而象仿紧性等这样重要的局部性质、象嵌入理论等这样基本的研究课题等都无法讨论。因此，我们自然想到去构造一种新的拓扑格理论，使它一方面具有相当的广泛性，至少能把点集拓扑学与模糊拓扑学二者都作为特殊情况而包含在内，同时又保留点集拓扑学的点式风格和丰茂的研究成果。正是基于这种考虑，我们于 1979 年提出了《拓扑分子格 (I)》的理论(参看[8])。我们提出了这样一种格 L，其中含有充分多的类似于点的元素，叫做分子，它是点集拓扑学中点的概念以及模糊拓扑学中模糊点概念的抽象化。它一方面起着单位的作用，任何元素都可表示为分子之并，另一方面它又不是最小的和不可分的。这也正是我们称之为分子而不称其为原子的原因所在。对于分子，我们提出了它的远域系的概念，这是对点集拓扑学中邻域概念以及模糊拓扑学中重域概念进行变革与抽象而得出的概念。利用远域这一工具，我们成功地建立起了分子的 Moore-Smith 收敛理论。同时，在拓扑分子格范畴中我们还提出了一种新的态射——序同态，它是从通常映射与 Zadeh 型函数的众多性质中抓住其正向保并与逆向保对合这两条性质并加以抽象化而得出的。事实证明正是那两条性质起着关键的作用，整个理论的展开也表明把序同态作为拓扑分子格范畴中的态射是恰当的。这样，分子、远域与序同态就成了拓扑分子格理论的三个支柱。

自《拓扑分子格 (I)》提出后，许多学者在充分肯定这一理论的同时也提出了如何进一步拓宽其理论框架的问题。1983 年，我们又提出了《广义拓扑分子格》理论（参看[9]，即《拓扑分子格 (II)》），对分子的概念作了大幅度的推广，而远域与序同态这两个概念则保持不变。不久我们就发现 Fuzzy 格 L（即具有逆序对合

对应的完全分配的完备格）是满足《广义拓扑分子格》中那类格的条件的，从而《广义拓扑分子格》理论已经把 L-fuzzy 拓扑学作为特例而包含在内了．此后，我们又注意到逆序对合对应这一条件对于拓扑分子格理论的展开并不起什么重要作用，于是，1985 年我们又提出了《完全分配格上的点式拓扑理论》[10]，这里已不再要求格 L 上具有逆序对合对应存在，L 可以是不对称的．这时基于闭元概念的远域工具照样适用，但重域工具已无法应用，同时我们也就更清楚地看到，就邻近结构而言，远域结构具有最广泛的适用性．与之对应的一个事实是：闭的概念比之于开的概念更为重要！再者，由于一个分子的远域系构成一个理想，因而在广泛的框架之下理想的收敛理论就要取代与邻域系方法相配套的渗透的收敛理论．关于序同态，由于逆序对合对应已不复存在，我们把逆向保对合的要求改为逆向保并，提出了广义序同态理论（参看[11]）．至此，文[10]成为到目前为止在最广泛的意义下系统论述的拓扑分子格理论．

拓扑分子格理论发展到今天一方面已经具有了一个初步的理论框架，另一方面却远谈不上完整．以[10]为例，关于乘积理论、子拓扑分子格理论乃至重要的紧性理论等都未论及，拟一致结构理论也有局限性．这些方面是随后由我的学生赵东升、杨忠强、樊太和、孙叔豪和孙国正等同志分别进行研究和补足的，但也只是一个开端．比如，虽有了乘积理论，但有哪些性质是可乘的？目前尚讨论不多．有了子分子格理论，但有哪些性质是遗传的？目前讨论也不多．何况这些理论自身是否理想也有待进一步的探讨．特别是紧性，赵东升虽已把良紧性推广到了 L-fuzzy 拓扑空间的情形[12]，但它毕竟不是对于一般拓扑分子格而言的紧性．另一方面，

以有限覆盖性质作为拓扑分子格中的紧性显然有诸多弊病，所以至今紧性理论在拓扑分子格中仍是一个空白. 这里顺便谈一下拓扑分子格与 L-fuzzy 拓扑空间的关系问题. 后者是前者的特例是显见的，但却不能笼统地说前者也是后者的特例. 从表面上看，设 (L^X,δ) 是一个 L-fuzzy 拓扑空间，那么令 X 为单元素集，比如设 $X=\{x_0\}$，则 (L^X,δ) 就成为拓扑分子格 (L,δ). 这无疑是对的. 但问题在于这时对 (L^X,δ) 所引入的许多性质却不能转化为拓扑分子格理论中相应的性质. 象 Hausdorff 分离性就是明显的例子，当 $X=\{x_0\}$ 时，(L^X,δ) 毫无例外的是 Hausdorff 的，但显然并非所有的拓扑分子格都是 Hausdorff 的. 对于紧性也一样，[12]中提出的良紧性是不适用于一般的拓扑分子格理论的. 此外，如理想的收敛理论，杨忠强在[13]中虽已有很好的讨论，但毕竟还有不少问题有待解决. 再如关于拟一致结构理论，孙叔豪在[14]中虽已有研究，但也只是初步的结果. 象是否存在具有对称性的一致结构以及如何考虑分离性问题等都未论及. 至于局部概念的建立，仿紧性问题的讨论等都是有待进一步研究的课题. 总之，拓扑分子格的框架是有了，一些最基本的理论也已建立，特别是分子、远域和序同态这些基本工具已被越来越多的学者所接受，但毕竟拓扑分子格理论还很不完整，若干课题尚有待于进行开发性的研究. 兄弟院校的许多学者在这方面已有不少好的研究成果(参看[15]). 毫无疑问，大家的共同关心必将开创拓扑分子格理论研究的新局面. 现在我们把陕西师范大学基础数学研究室的同志们的有关研究成果汇集成此书，希望它能反映出拓扑分子格理论的一个概貌，从而也向有志于此的海内学人提供研究的方便. 本书中的大多数论文虽已正式发表，但各种疏漏乃至错误

恐怕仍然不少，敬请各位学者不吝赐教。

王 国 俊

1989年7月于西安

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PREFACE

It has been 10 years since the theory of topological molecular lattices (TMLs for short) was first introduced. Let us now look at the following aspects of this theory: the background based on which the theory was introduced; the relations between the theory of TMLs and other branches of mathematics; the development of its research field and some crucial problems to be solved in order to get further progress.

In 1965, Professor L. A. Zadeh, an American expert in control theory, introduced the concept of fuzzy set[1], A fuzzy subset A on a set X is in fact a mapping $A: X \to [0,1]$. We denote by $[0,1]^X$ the set of all such fuzzy subsets. If we identify a crisp subset B of X (i. e., a subset in ordinary sense) with the characteristic function of B, then a crisp subset B of X is in fact a mapping $B: X \to \{0, 1\}$. The set of all such mappings is just $\{0,1\}^X$. As $\{0,1\} \subset [0,1]$, hence the concept of fuzzy subset is a generalization of the concept of crisp subset. In 1968 C. L. Chang introduced the concept of fuzzy topological spaces[2]. He called a subfamily δ of $[0,1]^X$ a fuzzy topology on X if $0, 1 \in \delta$, and δ is closed under finite intersections and arbitrary unions, here 0, 1 are respectively the least and the greatest mappings on X,and the intersections and unions are the operations by taking infima and suprema of mappings respectively. The pair (X, δ) is called a fuzzy topological space. After the concept of fuzzy topological spaces was introduced, a series of basic concepts in fuzzy topology can be defined by imitating the corresponding concepts in general topology. E. g., a fuzzy set G is defined to be an open set if G is in δ; if $A\in[0,1]^X$, the greatest open set contained in A is called the open kernel of A, denoted by A^0; if $H'=1-H\in\delta$, then H is called closed; the least closed set containing A is called the closure of A, denoted by A^-; (X,δ) is called compact, if each open cover $\mathscr{U}$ (i.e., $\mathscr{U}\subset\delta$ and $\sup\mathscr{U}=1$) of X has a finite subcover. It seems that many more concepts and theorems in general topology can be generalized into fuzzy topological spaces without difficulty. The fact is that point set topology is the topological theory developed on $\{0,1\}^X$, fuzzy topology is the topological theory developed on $[0,1]^X$, as $\{0,1\}^X$ and $[0,1]^X$ have many properties in common (e.g., the De Morgan law holds in both cases and the mappings connecting two spaces have many common properties etc.), hence those theorems in point set topology based on these common properties are naturally true in fuzzy topological spaces (see [3]). On the other hand, $[0,1]^X$ and $\{0,1\}^X$ are quite different, for example, The Law of Exclude the Middle does not hold (that is, $A\wedge A'=0$ and $A\vee A'=1$ does not hold in general), The Multiple Choice Principle does not hold $\left(\text{that is, } x_\lambda\in\bigvee_i A_i \text{ does not imply that there is } i_0 \text{ such that } x_\lambda\in A_{i_0}, \text{ see [4]}\right)$. It is just these great differences that make the traditional neighborhood method no longer applicable in fuzzy topology (see [5]). This fact was reflected in the difficulties met in the early 1970's. Then the pointless school evolved, B. Hutton was one of the pioneers in this direction.

The pointless school did much excellent work on quasi uniformities and normalities, and this can be regarded as the representative work of the pointless school. On the other hand,the evolution of the pointless school maybe is due to the fact that the pointwise study did not find a way out. In this situation,the theory of Q-neighborhood introduced by professor Liu Yingming turned out to be very important (see [6]), it paved a new way for the bracing up of the pointwise school.

Lattice-theoretically,both the topological theory developed on $L_1 = \{0,1\}^X$ and the fuzzy topological theory developed on $L_2 = [0, 1]^X$ are certain topological theories on certain lattice L, hence they can all be regarded as part of the topological lattice theory. G. Nobeling's book [7] had detailed illustrations about this fact, only the framework of [7] was so wide that many basic concepts in general topology can not be discussed in this wide framework,in particular, because of the lack of the concept of points and the corresponding concept of neighborhood structures, many important concepts such as paracompactness, embedding theory can not be discussed. Thus, we need a new topological lattice theory that is on one hand wide enough to include fields such as point set topology and fuzzy topology as special cases and on the other hand it can retain the pointwise characteristic and the aboundant results of point set topology. It is just based on this consideration that in 1979 we introduced the theory of "topological molecular lattices(I)" (see [8]). We introduced such a kind of lattice, it contains enough point like elements called molecules, the molecules are the abstraction of the concept of points in point set topology and the concept of fuzzy points in fuzzy topology. On one hand molecules play the role of points, each element can be represented as the union of molecules, on the other hand molecules are not the smallest units, they are decomposable. This is why we call them molecules rather than atoms. For molecules, we introduced the concept of remote neighborhood systems, this concept is anabstraction of the concept of neighborhood in point set topology and the concept of Q-neighborhood in fuzzy topology. By using remote neighborhood, we established successfully the Moore-Smith convergence theory. At the same time, in the category of TMLs we introduced a new kind of morphisms…… order homomorphisms, this was got by abstracting the fact that ordinary mappings and Zadeh's functions are union preserving and their inverses are involution preserving. It turns out that the above two properties are essential, and play a key role in developing the whole theory of TMLs, order homomorphisms are just the proper morphisms in the category of TMLs, as is shown by the development of the theory. Thus, molecules, remote neighborhoods and order homomorphisms are the three kinds of pillars of TMLs theory.

After "topological molecular lattices (I)" was introduced, they were affirmed widely, and, at the same time, the question that how to widen the framework was raised. In 1983 we introduced the theory of "generalized topological molecular lattices" (see [9], i.e., "topological molecular lattices (II)"), we generalized greatly the concept of molecules, but remote neighborhoods and order homomorphisms were the same as before. Shortly after we found that a Fuzzy lattice (i.e., a completely distributive lattice with order reversing involution) are special kind of "generalized topological molecular lattices", thus the theory of "generalized topological molecular lattices" includes L-fuzzy topology as special case. After some time we further found that the order reversing involution is not essential to the theory of topological molecular lattices, thus in 1985 we introduced "the theory of pointwise topology on completely distributive lattices"[10], this theory does not require that the lattices discussed have order reversing involution, and the lattices involved may not be symmetric. In this theory remote neighborhood method based on closed elements can still be used, but the Q-neighborhood method is not valid. We can also see clearly that, as to the adjacent structures, the remote neighborhood structures are the most acceptable structures. Thus the concept of closed elements is much more important than the concept of open elements! As the remote neighborhood system of a molecule forms an ideal, thus in this wide framework the ideal convergence theory can be used to replace the filter convergence theory based on the neighborhood system method. As to order homomorphisms, because there exist no order reversing involutions in this wide framework, we replace the requirement that the inverse mappings preserve involutions by the requirement that they preserve arbitrary unions, thus the theory of generalized order homomorphisms was introduced (see [11]). Up to the present, [10] is the paper that discusses systematically in the widest framework the theory of TMLs.

Up to the present the theory of TMLs on one hand has formed an initial theoretic framework, but on the other hand it is far from complete. For example, in [10] concepts such as product, sub-topological molecular lattice and the very important concept of compactness were not mentioned at all, and the quasi-uniformities defined there have many shortcomings. These are studied afterwards by my students Zhao Dongsheng, Yang Zhongqiang, Fan Taihe, Sun Shuhao,Sun Guozheng etc., but it is only a beginning. For example, there have not much discussions about the productivity and hereditarity of topological properties. As to compactness, though Zhao Dongsheng has generalized the nice compactness to the case of L-fuzzy topological spaces[12], it is very difficult to generalize it further into the case of TMLs. On the other hand, the compactness defined by using the finite cover property in TMLs has many deficiencies, so, the theory of compactness in TMLs remains a blank space. Now let us turn to the relation between TMLs and L-fuzzy topological spaces, obviously the later are special cases of the former. But the study of L-fuzzy topological spaces can not be all reduced to the study of TMLs. To be exact,if (L^X,δ) is an L-fuzzy topological space, let X be the set of all singletons, say, if $X=\{x_0\}$, then (L^X, δ) is a TML (L,δ), but the question now is, many properties introduced in (L^X,δ) can not be transformed into the corresponding properties in TMLs. For example, as to Hausdörff separation axiom, when $X=\{x_0\}$, each L-fuzzy topological space (L^X,δ) is Hausdörff, but not all TMLs are Hausdorff. There also exist some problems in compactness, N-compactness introduced in [12] can not be used in the theory of TMLs. As to the ideal convergence theory, though Yang Zhongqiang discussed it in detail in [13], but there still exist many related problems to be solved. As to quasi-uniformities, Sun Shuhao's study in [14] is only a beginning, questions such as whether there exist symmetric uniform structures and how to discuss separation axioms by virtue of quasi-uniformities were not dealt with in [14] at all. As to local properties, problems such as paracompactness are to be further studied. In summary, though the most fundamental structure of the theory of topological molecular lattices has been laid, the concepts of molecules, remote neighborhoods and order homomorphisms have been widely accepted, the theory is far from complete. There exist many problems to be further exploited. Apart from the work in our laboratory, there have been many deep and interesting results on TMLs (see [15]). Now we collect in this book some of the work done by the research laboratory of pure mathematics in Shaanxi Normal University. We hope that this book can reflect the general picture of the theory of TMLs and will be beneficial to those who want to study the theory of TMLs. Though most of the papers in this book have been published officially, there will certainly be some careless omissions or even mistakes which are all due to the authors.

Xi'an, July 1989 Wang Guojun

目　　录