走向代数表示论

刘绍学文集

北京師範大學出版社

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▶ 1956年6月8日，在莫斯科大学刚答辩完副博士论文后摄于校园内的一个花坛边

▼ 1991年刘绍学和他的学生们首次秋游，〔左起：王志玺、章璞、刘绍学、肖杰、林亚南、彭联刚〕

▼ 1991年10月，在参加广西师范大学举行的第一次中日环论国际会议时摄于桂林漓江的游船上。（第一排左起：林亚南、张英伯、刘绍学、C.M.Ringel、章璞、惠昌常、彭联刚；第二排：郭晋云）

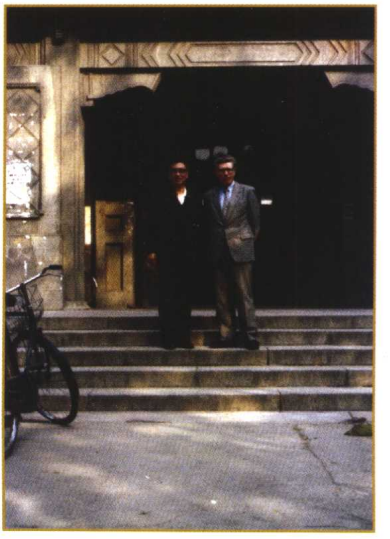
▶1994年8月，在南开大学举行中国代数年时摄于南开大学数学研究所楼前。〔第一排左起：C.M.Ringel、惠昌常、邓邦明、张顺华；第二排左起：刘绍学、郭晋云、M.Auslander、I.Reiten、张英伯、张跃辉；第三排左起：李思泽、张卫、吴求先、姚海楼、杜先能〕

◀ 1993年学生们祝贺刘绍学64岁生日，〔左起：朱彬、杜先能、刘绍学、李思泽、张顺华、黄兆泳、姚海楼、张跃辉〕

▼1999年,学生们为刘绍学先生过70岁生日摄于北京师范大学。〔第一排左起：肖杰、张英伯、刘绍学、肖元正、曹文龄、罗运纶、郭晋云；第二排左起：彭联刚、李思泽、林亚南、惠昌常、朱彬、章璞、王志玺、周梦、刘海霞〕

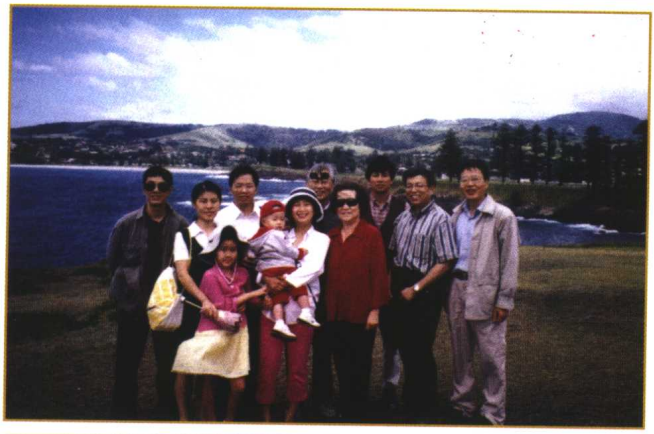
▲ 1999年10月，在北京师范大学举行全国代数会，于天伦王朝饭店的晚宴上还为曹锡华、周伯壎、刘绍学三位先生祝寿。〔图为刘绍学在晚宴上助兴〕

▲ 1999年5月，北京师范大学数学系《五教授执教五十周年庆祝会》上向刘绍学等献花〔右起：刘绍学、王梓坤〕



▶ 1999年10月，俄罗斯科学院院士、莫斯科大学代数教研室主任A.I.Kostrinkin应刘绍学邀请来北京师范大学讲学期间，摄于北京师范大学数学楼前。

◀ 2000年6月访问比利时期间，刘绍学一家人在Antwerp大学教授F.Van. Oystaeyen家做客。曹文龄摄〔自右逆时针转：刘绍学、Fred、其夫人、其女儿、其儿子、刘绍学之大女儿刘艺冰〕



◀ 2001年10月，刘绍学夫妇去澳大利亚探亲，摄于卧龙岗附近海边。〔第一排左起：徐文、刘路、其女儿徐堪、华九召夫人及其子、曹文龄、杜杰、彭联刚；第二排左起：华九召、刘绍学、邓邦明〕

▼ 2004年10月6日，在扬州大学举行的《全国代数表示论高级讨论班》的晚宴上祝贺刘绍学75岁诞辰〔站立讲话者是郭晋云，其前面左起是曹文龄，刘绍学，王宏玉（扬州大学数学科学学院院长）〕

序　言

Preface

On the occasion of the 75th birthday of Professor Shao-xue Liu, Beijing Normal University is publishing a volume containing a collection of his papers. In this preface, I want to focus the attention to his initiative to study problems in representation theory. I will outline the development of a very fruitful cooperation between China and the West concerning the representation theory of finitedimensional algebras, along the line of our mutual contacts. During the years, we became true friends, and I will add some personal comments about my introduction to China and the Chinese culture. Professor Shao-xue Liu has to be considered as the Nestor of algebra in China and we are very happy that he shifted the interest of many of his students and collaborators to algebras and their representations. Indeed, he is the founder of a large and very successful Chinese school in representation theory of finitedimensional algebras which covers a broad range of topics and deals with many different applications.

This school has its origin at BNU and now has strong outlets at many other scientific institutions not only in Beijing (one has to mention at least Tsinghua University and the Academy of Science), but also Shanghai, Chengdu, Hefei, Xiamen, Changsha, and there are many more places.

It does not seem to be necessary to comment on the individual papers in this collection, but one should look also at Liu's book on Rings and Algebras which had and still has a tremendous influence. As the title indicates, it is the structure theory for rings and algebras which is the central theme, questions concerning representations are treated only marginal (still, it deals with semisimple modules and the corresponding double centralizer property). In particular, the Wedderburn-Malcev theory, as developed in the book, has to be considered also as starting point for dealing with representations of algebras. The book has served as a basic reference not only for his own students, but seems to have been used throughout China. I tried to persuade Springer to publish a translation into English because it could have been an important and very useful addition to the Western market, but the publishing company did not want to get competition to its other books addressed to the same audience.

1985

In 1985, Professor Liu visited several European universities, including Bielefeld—this was our first contact. The aim of his travel was to get in touch with some of the active research centers in the West dealing with rings and modules. He had carefully analyzed the present state of research in algebra. Apparently, he was impressed by the stormy development of the representation theory of finitedimensional algebras and he had decided that China should get involved in such investigations. He wanted to start to cooperate with all the relevant schools, hoping that after some time the Chinese algebraists would be able to participate in the worldwide research competition. It was obvious that he had a clear vision of the role China should play in the future: to become a leading contributor to the research.

It was August, the typical holiday time in Germany, so not many

mathematicians were around, and usually no lectures are scheduled at this time. Still I felt that I should ask Professor Liu whether he would like to present some results, but with the warning that the audience should be rather small. Yes, he said, he would be happy to give a lecture, or better: to give four short lectures, on four different topics, each one of 15 minutes. I was uncertain that this could work, but what could I do? The lecture given by him was a real surprise and it was very well arranged: It was the precision of his thinking, the perfect presentation and the well-chosen motivation which impressed us very much. In any one of the four parts, he first outlined in detail the context, with all the necessary definitions, specified a problem and finally showed the solution (or a partial solution) which had been obtained by him and his students during the last years. In this way we obtained an impressive insight on the scientific interests and the research power of his working group. The broad range of topics was remarkable: T-ideals, nilpotency of radicals, algebras of finite representation type, and non-associative rings (Jordan algebras). At that time, Skowronski (from Torun, Poland) was also in Bielefeld, and the part which was of most interest to Skowronski and me was the third part, dealing with algebras of finite representation type (Xiao had shown that the module category of such an algebra is always perfect). I still remember that after the lecture there was a long (and fruitful) discussion on possible generalizations and on related questions such as the pure-semisimplicity conjecture. In this way, Liu's lecture has to be considered as the starting point for a long cooperation.

This was his plan: he wanted to send students to all the relevant universities in the world in order to obtain there a Ph.D., and conversely to invite some of the experts for a stay at BNU in order to give series of lectures. As first measure, Deng was sent to Zurich where Gabriel was working, Luo to Carleton University (Dlab), Shiping Liu to Liverpool (Brenner and Butler), and Chang-chang Xi to Bielefeld. At that time, M. Auslander who was also approached saw problems to accept a student, but later Luo moved to Brandeis University. After my visit to China in 1987, Ying-bo Zhang

came as second Ph.D. student to Bielefeld and then later Ya-nan Lin as a third one. Furthermore, Guo obtained a Humboldt fellowship for working with me. Later several other Humboldt fellowships were awarded to members of the Chinese representation theory group (to Pu Zhang, Lian-gang Peng and to Bang-ming Deng). This, of course, increased the mutual contacts between Germany and China considerably.

Most of the students mentioned had been for some time at BNU, but actually they were selected from all over the country. In fact, it seemed to me that Professor Liu and Professor Cao (from ECNU, the East China Normal University) felt their responsibility to care for all the young Chinese algebraists, at least those studying at "Normal Universities" (those devoted to the training of future teachers). The best of these students were invited to come to BNU or to ECNU, to get there their education, so that later they could go back to their home university with a proper training. And some of these were selected to obtain a Ph.D. from abroad, or they were asked to apply for other support such as a Humboldt fellowship.

Shao-xue Liu had brought several photos with him in order to present a more vivid picture of BNU, and it was clear that he was proud of his university. Most of the pictures were very convincing, so for example I learned that Xun Lu had been a member of BNU (at that time, a nice volume with some of the most famous stories and texts of him had appeared as a volume of Die Andere Bibliothek). But there was one picture which irritated us. It showed the gate of BNU: it neither looked old (that would be a convincing argument in the West for presenting it), nor very fancy. Why does someone care about a gate? Are there actually still walls around a university or other institutions (indeed, these are!)? And could it be that the doors are closed in the night, as it was customary in the middle age in European towns? It took me quite a while (and several trips to China) to get some understanding of the importance of the walls around a danwei. A university in China is more than a place to study and to do research. Professors and students, as well as all the subsidiary staff do not only work together but really

live together, in a well-defined area, surrounded by a wall. In some sense, such a university also resembles a monastery, well-shielded from the outside, with nice gardens and not too many distractions. These units (which may be a university, or a factory, or some administration…) have their own schools, their hospitals, their shops and restaurants, they are nearly self-sustaining, something like small towns in themselves. A city such as Beijing breaks up into thousands of independent units—and the gates symbolize the importance of the units.

During Liu's stay at Bielefeld, we discussed various topics, ranging from mathematics to culture and politics. It was of interest to see his rating of different mathematical subjects, and his clear assessment of the relevance of notions and results. For example, I remember that he raised the question why I follow the tradition to speak of "torsion theories"? Is this a theory? Or is it not just a typical mathematical object, namely a pair of subcategories, and thus should be named "torsion pair"? And he was right. So, in later publications I changed the terminology.

1987

My first visit to China was in March and April 1987, being invited by BNU and ECNU. The German research council sponsored the flight and I remember that my application was commented by one of the officials by saying that recently there were many similar applications: Apparently, it is easier to invite lecturers than to buy books.

My lectures were scheduled in the morning, usually for three or four hours. Three days lecture were followed by a free day. The graduate students who attended my lectures were Guo, Xiao, Ying-bo Zhang and Tang, thus two male and two female students, and there were around ten undergraduate students, also here the proportion of female students was high. This was quite strange for me, since in Germany mathematics is considered in public as a "male" discipline, and unfortunately female students are often scared away. In later years, it seemed to me that a similar tendency was observed in China — a very unfortunate development. All the students were

very active, some came in the afternoon or even late in the evening to ask questions and to discuss with me partial solutions of their home work. And they were very well prepared for my lecture, even showing me copies of original papers related to the content of my teaching. They had seen already quite a lot of the relevant definitions and had read theorems and proofs, however they had not worked through any example at all, so they had severe difficulties to see the relevance of the results.

My lectures followed notes which I had made for a similar series of lectures at Antwerp, however the reaction of the audience was quite different: the atmosphere at BNU was very enthusiastic, but I had to spend much more time on some of the details when dealing with specific applications. So in later lectures, I changed the presentation and shifted all the attention to a better understanding of relevant examples. It was clear that it was important for Chinese students to get personal instructions, not just to obtain books to be read.

One has to recall the hardship at that time: it was very cold, the students were wrapped in heavy coats in order to survive. When giving lectures, I was happy of being able to move around, and I would have been afraid to sit there for three or four hours (also, as lecturer, I had the privilege to obtain all the time hot tea, a very kind gesture). It was end of March, and the official winter period was over (the weather did not care about that), so there was no heating anywhere. An exception was made in the building for foreign experts (where I stayed), and for Professor Tuan whom we visited at Beijing University. He was a student of R. Brauer and it was of interest to listen to his recollection.

To work with Chinese students was usually a pleasure. But sometimes there were surprises. I remember that Chang-chang Xi, being new at Bielefeld, once contacted me claiming that he did not understand some proof of mine (in the Springer Lecture Notes Volume 1099). Without looking at the corresponding pages, I tried to explain him the principles of the methods used; still he insisted that he had problems with the proof. So finally we

went through the text, line by line. When I mentioned in by passing that apparently there was a misprint, a t was typed instead of an n, he immediately responded: yes, if one uses there n instead of t, then the proof works well. Obviously, he never would have dared to believe that a written text could contain misprints!

And most of the Chinese students had a forceful desire to solve problems, if possible to solve a new problem every month. Such an attitude will not allow any polishing of proofs, no reflection on the best possible way for writting up the results. And I guess that Professor Liu himself, whose writting always was very refined, could not be pleased in this way.

During the first days of my stay, I had a cook for my own. He prepared every day a kind of Western dish just for me, but I wanted to have Chinese food. Thus, after some days, I took all my courage and copied some Chinese characters from my travel guide, telling him that I would not come back the next day. After that, I learned about the refectory for foreign students, where I enjoyed various kinds of Chinese food. But I was astonished to see the strict separation between Chinese and non-Chinese students (whether from Europe, America or say Vietnam) at that time. The food was fine, but of course it couldn't be compared to the jao-zi which I tasted when visiting the home of Professor Liu: they were prepared by his wife and were really delicious. Once, also Ying-bo Zhang invited me and cooked herself: I was allowed to watch her and to follow all the steps. Clearly, Chinese cooking is an art in itself. And there are many specialities like sea cucumbers, rice field snakes, or also dogs ("The yellow dogs taste best" —a sentence which brings every German table conversation to a sudden end), and astonishing variations of classical European ingrediants (for example potatoes cut into tiny sticks).

Shao-xue Liu also introduced me to Beijing operas. At that time, I did not even appreciate the European operas (in contrast to all kinds of other theater plays): They seemed to me too artificial, too far away from any reality (just imagine someone standing there and singing "I run away, I run

away?" Why doesn't he do what he asserts?), and the Chinese operas obviously looked even worse. The first one I saw was the White Snake, and this was a great shock for me. It took me some while to realize the essence of such operas: the perfectness of the presentation and the virtuosity, but also the symbolic reference, the refinement of style, the precise interplay between music and movement. Beijing operas were however popular only for elder people. Indeed, when I mentioned my interest to some of the students, they were surprised and told me: You are young, why do you care about such things? There was already concern about the survival of traditional operas, Guo accompanied me to a corresponding meeting. Coming back to Germany, I tried to learn more about operas, both the European and the Chinese ones. Yes, I have to admit that my understanding of the European operas relies on seeing Beijing operas.

After my visit to Beijing, I took a train in order to stay at ECNU for another week. On the way I wanted to stop over in order to climb the mount Tai Shan. Professor Liu accompanied me to Qufu (the home town of Kong Fu-Tse, and near the Tai Shan). However, it was not possible to preorder tickets, thus to continue for Shanghai, I had to stay at Qufu for three days. During these days, I gave some lectures at Qufu Normal University, apparently as one of the first visitors after the cultural revolution.

1991 and 1994

A further visit of me to China was planned for 1989. I had managed to make a booking for the Transsiberian Railway from Novosibirsk to Beijing, which was not easy at all-usually you had to use the Transsib all the way, but there was the Malcev conference at Novosibirsk which I wanted to use as a start. Quite a while after June, Ying-bo Zhang asked me whether I would cancel the trip to China. I replied: No, I will go. She said: You don't hesitate to go to China this year? I told her: No. She told me: You will not go. But I wanted. However, some weeks later, I obtained a letter from Professor Liu, telling me that he had heard that I had hesitations and that in this case I should not feel obliged to come even having made the

promise. I interpreted this as the message to cancel my plan——which I did with quite regret. At Novosibirsk, I met Professor Liu and found out that we both were unhappy about the cancelation of my visit, but is was too late to renew the arrangements.

I was excited to hear that there was the plan to hold a Chinese-Japanese ring theory conference in 1991. I liked the idea very much, because both countries have a long tradition in ring theory and I felt that despite of possible political concerns scientists should always cooperate. By that time I had strong contacts both to Japan and China, and so I was very pleased to be invited (as one of a small number of Westerners) to this conference, which was held at Guilin. The Chinese contributions showed in which way algebra had survived (and even flourished) during the cultural revolution, in complete isolation, without proper access to books or journals. Without much prerequisites, general properties of rings and modules had been studied. Most of the talks focused the attention to properties which hold for the vast majority say of rings, without any discussion of examples. Special properties of those rings which arise in nature, those which are of interest in other parts of mathematics, were not considered as being of greater interest. As a counter balance, I decided to speak on finitedimensional hereditary algebras, their properties and the relationship of this class of algebras to other parts of mathematics. My aim was to stress the importance of detailed studies of very concrete mathematical objects.

It is customary that during a mathematical conference one half day or even one day is reserved for some excursion. The organizers of the Guilin conference had planned even two such occasions: the visit of a marvellous stalactite cave and a boat trip on the river Li. The landscape around the river is very special, with mountains which look like sugar loafs. Some of these mountains are plain rocks, and looking at the stone one may envision hidden pictures. Liu challenged the participants to use their imagination and to explain what they see. For example, one of these peeks is famous for its "twelve horses". It was interesting to observe the different reactions to this

game. One of the mathematicians from Japan proposed instead to evaluate the phallic shape of the peaks, but Liu condemned this as too obvious and blatant.

Europeans are less enthusiastic about stones and the miracles they hide. Of course, the so-called precious stones are admired and are used for jewellery, and one finds stones like marble in entrance halls, as floor covering or table tops, but never you will take an individual stone as a decoration like a painting or a sculpture. I learned from Liu to follow the flow of lines and to interprete the colors in order to see the inherent pictures. Such a perception yields a transfer from nature to art.

The Nankai Institute in Tianjin devoted the academic year 1993/1994 to Representation Theory and it was a pleasure for me to stay there for several weeks. For the same period, also Auslander and Reiten had been invited, and later Puig arrived. The discussions with all the participants were very fruitful. I presented the idea of adding infinitedimensional indecomposables to the Auslander-Reiten quiver (which is made from the finitedimensional indecomposables) in order to sew together different components. What one obtains in this way may be called an Auslander-Reiten quilt.

I was happy to have again a bicycle at my disposal and I used much of the free time to cycle around. Once Auslander was asking me where I had been. I told him of all the temples I had visited, the old traditional houses I had seen and so on. He was amazed to hear about temples and pagodas—— he had asked Guo about the city and was told there would not be any attraction. So when he spoke to Guo again, Guo replied: There is nothing special, it is just like any other Chinese city. What seems to be the usual thing for the local people may be of great interest, even surprising, for foreigners: I remember, when I went with Professor Liu to one of the cellars in Antwerp which serve a big variety of beer, with wooden tables and candle light (Antwerp is very proud of these places), he told me that it is very difficult for him to understand why it should be nice to have candle light, when one could have a proper electric illumination!