ADVANCES IN COUPLED MODELING IN GEOMECHANICS

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前 言

在环境地质工程、地下采矿工程、地热开采工程、水电工程、边坡工程、油气开采工程、地震预测等领域，热—水—力（简称：THM）多场耦合过程及其相互作用问题的研究变得十分重要和迫切；当前，热—水—力—化（THMC）多场耦合作用的研究也是地球力学中学术前沿的重大课题。尤其是放射性废物地质处置工程的安全性评估问题显得更突出，近十年来已经引起人们的极大关注，有力地推动了热—水—力耦合过程的数学模型及数值模拟研究的快速发展，并已取得了瞩目的成果。**《地球力学中耦合数值模拟之进展》**根据作者在该领域做出的若干研究，结合国际上的最新研究成果，主要阐述了地球力学中地下采矿工程、环境地质工程、地热开采工程、放射性废物地质处置工程中耦合数值模拟研究的新进展。

本书共分7章，第1章绪论：1.1 地球力学中岩体的特性；1.2 地球力学中耦合数值模拟。第2章采矿工程中的耦合数值模拟：2.1 引言；2.2 三轴压缩下耦合效应的装置与方法；2.3 煤样的力学性质；2.4 煤样有效应力的实验结果；2.5 有效应力的分析与讨论；2.6 煤样渗透性的实验结果；2.7 渗透性的分析与讨论；2.8 煤大分子与瓦斯相互作用的实验研究；2.9 瓦斯渗流 Darcy 动力学模型与数值模拟；2.10 径向流场中孔隙压力分布的解析解；2.11 瓦斯渗流非 Darcy 动力学模型与数值模拟；2.12 块裂介质中瓦斯渗流与固体变形的非线性耦合模型。第 3 章煤层气越流系统中的耦合数值模拟：3.1 引言；3.2 模型之假设；3.3 固体弹性变形与瓦斯越流的耦合模型；3.4 固体粘弹性变形与瓦斯越流的耦合模型；3.5 固气耦合数学模型的数值解法；3.6 固体弹性变形与瓦斯越流耦合模型的可视化数值模拟；3.7 小结。第 4 章地热能工程中的耦合数值模拟：4.1 引言；4.2 地热能工程的耦合数值模拟。第 5 章环境地质工程中的耦合数值模拟：5.1 地震的耦合数值模拟；5.2 地表沉降的耦合数值模拟。第 6 章放射性废物处置工程中的耦合数值模拟：6.1 引言；6.2 DECOVALEX I 的耦合模型；6.3 DECOVALEX III 的研究进展。第7章煤层气越流与固体变形耦合数值模拟的源程序（C++）。

本专著反映了国内外地球力学耦合数值模拟领域的发展现状和趋势，适于从事环境地质工程、采矿工程、地热开采工程、水电工程、岩土工程、油气开采工程、地震预测等模型化和数值模拟技术的研究、教学、培训和管理等人员阅读参考。书中对专业词汇进行了中文注译，增加了专业英语的可读性，兼顾了研究生或本科生英文教学的需要。

在编著过程中，引用了瑞典皇家理工学院井兰如（Jing Lanru）教授，太原理工大学赵阳升（Zhao Yangsheng）教授，以及 Michael J.O'Sullivan, Tsang C.F., Hazzard J.F., Hu R.L 等人的研究成果；浙江工商大学05级硕士研究生王如意参加了大量的文字录入和插图工作，在此一并表示感谢！

限于学术水平，而且时间仓促，书中一定存在不足甚至错误之处，敬请读者批评指正。

著 者

2005年10月

目　录

Chapter 1

Introduction[1]

1.1 Special Nature of Rock Mass in Geomechanics

The reason for the general difficulty in modeling rock masses①, by whatever numerical method, is that rock is a natural geological material, and so the physical or engineering properties have to be established, rather than to be defined through a manufacturing process. The rock mass is largely discontinuous, anisotropic②, inhomogeneous③ and non-elastic (DAINE) [1]. Rock masses are under stress and continuously loaded by dynamic movements of the upper crust of the Earth, such as tectonic movements, earthquakes, land uplifting/subsidence, glaciation's cycles and tides. A rock mass is also a fractured porous medium④ containing fluids in either liquid or gas phases, e.g. water, oil, natural gases⑤ and air, under complex in situ⑥ conditions of stresses, temperature and fluid pressures. The complex combination of constituents and its long history of formation make rock masses a difficult material for mathematical representation via numerical modeling.

In relation to the generally discontinuous nature of rock masses, the photograph of a blasted rock surface in Fig. 1.1 highlights the fact that rock masses contain through-going pre-existing fractures, as well as fractures introduced by the excavation process.

Most of the fractures visible in Fig. 1.1 are pre-existing natural fractures. Although these rock fractures have occurred naturally through geological processes, their formation is governed by mechanical principles, as illustrated by the three main sets of fractures that, in this case, are mutually orthogonal and divide the rock mass into cuboids. The fractures are often clustered in certain directions resulting from their geological modes and histories of formation. One of the main tasks of numerical modeling⑦ in rock mechanics is to be able to characterize such mechanical discontinuities in a computer model—either explicitly or implicitly—the so-called ‘material conceptualization⑧’. Additionally, the interaction between the rock mass and the engineering structure has to be incorporated in the modeling procedure for design, so that consequences of the construction process have also to be characterized.

1 This chapter was written based on this paper: Jing Lanru. A review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock engineering. Int J Rock Mech Min Sci. 2003, 40:283-353; ① rock masse：岩体；②anisotropic：各向异性的；③inhomogeneous：非均质的；④fractured porous medium：裂隙孔隙介质；⑤natural gases：天然气；⑥in situ：在现场；⑦numerical modeling：数值模拟；⑧material conceptualization：材料概念化

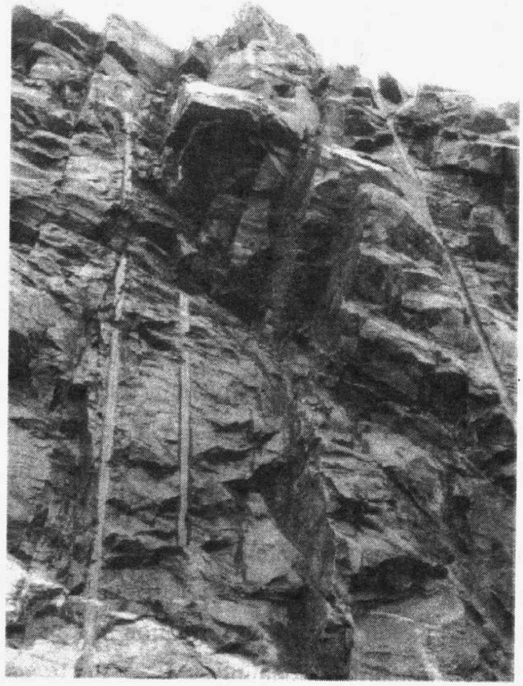

Fig. 1.1 Surface of a blasted rock mass, illustrating that pre-existing fractures can divide the rock mass into discrete blocks, and that the interaction between the rock mass and the engineering processes also needs modeling for the engineer to have a predictive capability for design purposes. Note the 'half-barrels' of the blasting boreholes[774].

To adequately represent the rock mass in computational models, capturing such fracturing and the complete DAINE nature of the rock mass, plus the consequences of engineering, it is necessary to be able to include the following features during model conceptualization:

① The relevant physical processes and their mathematical by partial differential equations (PEDs)[①], especially when coupled thermal, hydraulic and mechanical processes need to be considered simultaneously;

② The relevant mechanisms and constitutive laws with the associated variables and parameters;

③ The pre-existing state of rock stress[②] (the rock mass being already under stress);

④ The pre-existing state of temperature and water pressure (the rock mass is porous, fractured; and heated by a natural geothermal heat gradient[③] or manmade heat sources);

⑤ The presence of natural fractures (the rock mass is discontinuous);

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①partial differential equation（PDE）：偏微分方程；②rock stress：岩石应力；③geothermal heat gradient：地热梯度

⑥ Variations in properties at different locations (the rock mass is inhomogeneous);

⑦ Variations of properties in different directions (the rock mass is anisotropic);

⑧ Time/rate-dependent behavior (the rock mass is not-elastic and may undergo creep or plastic deformation);

⑨ Variations of properties at different scales (the rock mass is scale-dependent);

⑩ The effects resulting from the engineering perturbations (the geometry is altered).

How these features can actually be incorporated into a computer model will depend on the physical processes involved and the modeling technique used; hence, both the modeling and any subsequent rock engineering design will contain subjective judgments.

Rock engineering① projects are becoming larger and more demanding in terms of the modeling requirements, one of which, for example, may be to include coupled thermo-hydro-mechanical (THM)② behavior into the model. A truly fully coupled model (including extra processes, such as chemistry) requires complete knowledge of the geometrical and physical properties and parameters of the fractured rock masses. Thus, the challenge is to know how to develop an adequate model. The model does not have to be complete or perfect, it just has to be adequate for the purpose.

For these reasons, rock mechanics③ modeling and rock engineering design are both a science and an art. They rest on a scientific foundation but require empirical judgments supported by accumulated experiences through long-term practices. This is the case because the quantity and quality of the supporting data for rock engineering design and analysis can never be complete, even though they can be perfectly defined in models.

1.2 Numerical Modelling in Geomechanics

Some form of predictive capability is necessary in order to coherently design an engineered structure, whether it is on the rock mass surface or within the underground rock mass, and whether it is for civil engineering④ addressed in this civil zone⑤ review or for mining, petroleum or environmental engineering. The predictive capability is achieved through a variety of modeling methods. Even if one simply adopts the same design as a previously constructed structure, the rock mass condition is generally site-specific and one should use a computer model adopted for the specific site conditions to ensure that the rock mass is likely to behave in similar fashion.

As rock mechanics modeling has developed for the design of rock engineering structures with widely different purposes, and because different modeling methods have been developed, we now have a wide spectrum of modeling approaches. These can be presented in different ways: the categorization into eight approaches based on four methods and two levels, as illustrated in Fig. 1.2, is from (Hudson, 2001) [2].

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① rock engineering：岩石工程；②thermo-hydro-mechanical：地温场—渗流场—应力场；③rock mechanics：岩石力学；④civil engineering：土木工程；⑤Civil Zone：居民区

The modeling and design work starts with the objective, the top box in Fig. 1.2. Then there are the eight modeling and design methods in the main central box. The four columns represent the four main modeling methods:

① Method A: Design based on previous design experiences;

② Method B: Design based on simplified models[1];

③ Method C: Design based on modeling which attempts to capture most relevant mechanisms;

④ Method D: Design based on "all-encompassing" modeling[2].

There are two rows in the large central box in Fig. 1.2 The top row, Level 1, includes methods in which there is an attempt to achieve one-to-one mechanism mapping in the model. In other words, a mechanism which is thought to be occurring in the rock reality and which is to be included in the model is modeled directly, such as explicit stress–strain relations. Conversely, the lower row, Level 2, includes methods in which such mechanism mapping is not direct. The consequences of, for example, the constitutive models and associated parameters may well be contained within the four modeling and design methods in Level 2, but one cannot explicitly identify the relation within the methodologies, e.g. in the rock mass classification techniques[3].

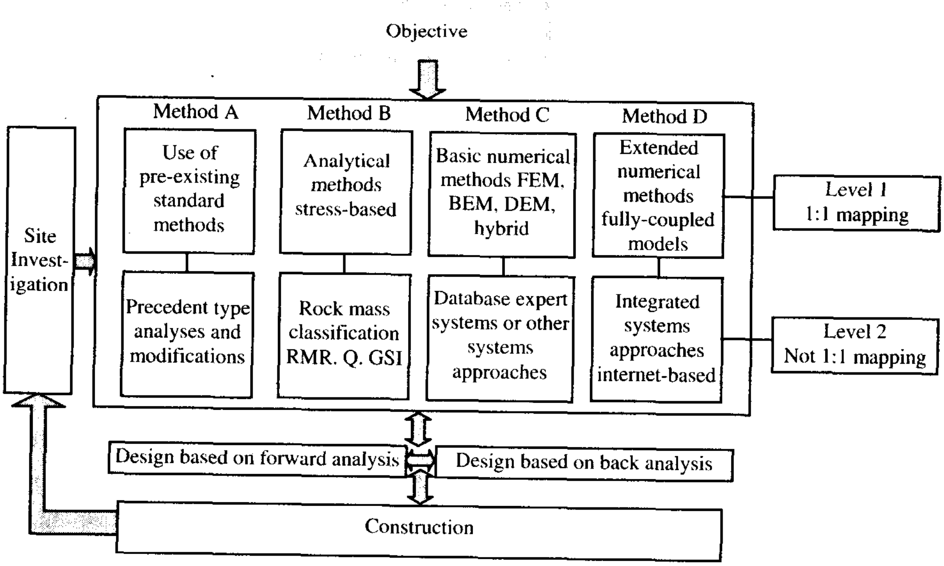

Fig. 1.2 Four basic methods, two levels and hence eight different approaches to rock mechanics modeling and rock engineering design, from Hudson [2].

Some supporting rock mass characterization parameters will be obtained from site investigation,

①simplified models：单一化模型；②'all-encompassing'modeling：全因素模型；③rock mass classification techniques：岩体分类技术

the left-hand box. Then the rock engineering design and construction proceeds, with a feedback loop to the modeling from construction.

An important point is that in rock mechanics and engineering design, having insufficient data is a way of life, rather than a simple local difficulty, and those is why the empirical approaches (i.e. classification systems) have been developed and are still required. Therefore, we will also be discussing the subject of parameter represent ability associated with sample size, representative elemental volume (REV) ①, homogenization/up scaling, because these are fundamental problems associated with modeling, and are relevant to the ABCD method categories in Fig 1.2.

The use of computers makes significant contributions to all the eight modeling and design methods in Fig. 1.2; however, the specific numerical methods and approaches that are being reviewed here are used directly in Methods 1C and 1D. Also, there is concentration on the actual numerical methods (rather than computing per se or design per se) and discussion on the rock mass characterization issues related to the numerical methods. Highlighted are the techniques, advances, coupled mechanisms, technical auditing and the ability to present the content of the modeling, the outstanding issues, and the future of this type of modeling. In short, highlighted is the special contribution that numerical models are currently making to rock mechanics.

Because the focus of this Review is on the modeling concepts, the associated special features of modeling rock fractures, the main development milestones, typical application requirements, development trends, and outstanding issues of importance and difficulty, special attention is paid to Section 3 for alternative formulations in each of the modeling methods, noting the potentials for rock mechanics problems. It is hoped that this treatment will provide readers with a comprehensive presentation of the state-of-the-art ② of papers[1-774] on numerical analysis in rock mechanics in general, and civil engineering applications in particular—in terms of historical background, presents status and likely future trends.

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①representative elemental volume：表征单元体；②state-of-the-art：技术现状,综述

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