THE ROLE OF INSTRUCTIONAL REPRESENTATIONS ON STUDENTS' WRITTEN REPRESENTATIONS AND ACHIEVEMENTS

美国数学教师教学表征对学生数学学习的影响

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Chinese Abstract

中文摘要

这项研究是建立在美国国家自然科学基金会对美国科学促进协会的一个研究中学数学的课题项目的基础之上（#REC－0129398）。作者深入研究、分析了美国德州 14 名中学教师的教学表征和学生的知识表征之间的关系。本书要解决以下几个问题：第一，实际生活、教学模型、图片、口语和书面语言（后两者属于符号表征）这 5 种不同的知识表征的方式是如何反映在这些美国教师的教学过程中的？尤其特别地是，课堂教学互动是如何反映这 5 类知识表征的？这几种知识表征是否和教科书中的知识表征相符合？第二，教师的课堂教学表征和学生的知识表征各呈现出什么样的结构？这些知识表征的结构是如何反映相应的数学知识结构（分数的知识结构）？第三，学生使用的知识表征和他们的学习成绩之间有何种关系？第四，课堂教学的质量和教学时间的长短是如何影响学生学习分数的？课堂教学的质量（使用美国科学促进协会的分类标准）、教学时间以及教师的教学表征结构是否影响到学生使用不同的知识表征，从而影响到学生的学习成绩？

从研究方法上来讲，这本书使用质的研究方法和量的研究方法相结合的方式。首先，研究小组成员确定我们要研究的对象教师，对这些教师作了一次在职培训，重点讲解教师的知识表征，并且在这些教师进行相关的教学之前，对相应的学生做了前测。其次，作者根据这些教师的教学录像（项目组成员对教师的教学进行录像）给这些教师使用的知识表征编码，统计了教师知识表征结构的教学时间，并且记录教师的教学过程；然后，对学生的学习成绩做后测，并且把学生使用的知识表征也进行编码统计。最后，对教师的教学过程进行质的分析，并且应用结构方程和多层次线性模型对教师的教学表征结构和教学时间对学生的知识表征结构的影响进行量化分析。

研究结果表明，第一，教授相同的知识点，不同的教师的教学时间长短不同，他们的课堂的教学质量也存在很大的差异。符号表征（包括口语和书面语言）在这些教师的教学过程中占有统治地位。课堂教学的表征结构呈现出向数学知识的结构聚合的倾向。实际生活、教学模型、图片、口语和书面语言（后两者属于符号表征）这 5 种不同的知识表征无法通过结构方程在本样本中得到应证。9 名教师的教学表征和教科书的教学表征相符合，另外 5 名教师的教学表征没有和教科书的教学表征相符合。第二，作者研究表明教师的教学表征和学生的知识表征都呈现出向数学知识结构聚合的倾向。教学表征反映了美国数学教师知识表征（分数教学）有 5 个亚知识结构，“测量的观点”、“部分和整体”、“交叉相乘”、“商”、“通分（乘 1 法）”。学生的知识表征结构存在一定的落差，只有“部分和整体”、“商”和“通分（乘 1 法）”。第三，学生的知识表征结构中的 3 种表征都很显著（统计上存在显著性）地预测了学生的后测成绩。并且“部分和整体“的知识表征和“和通分（乘 1 法）”之间存在显著的（统计上存在显著性）关系。第四，多层次线性模型从统计学上应证了课堂教学质量显著地（统计上存在显著性）影响学生的后测成绩。教学时间的长短对学生的成绩的影响，在本样本中无法通过多层次线性模型得到确认。

Abstract

This research is based on Middle School Mathematics Project (MSMP) funded by the Interagency Educational Research Initiative through a grant to the American Association for the Advancement of Science (# REC - 0129398). Both teachers' instructional representations and students' written representations were coded and analyzed to investigate the nature and structure of the representations in teaching fractions, decimals and percents in middle school classrooms in Texas. The study explored four questions. First, how do teachers use real-world, manipulatives, pictures, spoken language and written language in classroom teaching? Second, what are the structures of instructional representations and students' written representations? Third, what is the relationship between students' written representations and their achievements? Fourth, what is the relationship between the instructional quality and student achievements?

This study used a mixed approach utilizing both quantitative and qualitative methods. The data was collected in the first two years of a five-year study. A total of 14 sixth grade mathematics teachers from three school districts in Texas were selected from the MSMP project. Before the actual videotaping procedure, a professional development focusing on multiple representations was held for the teachers. Both pre-tests and post-tests were used to examine the relationship between the structure of students' written representations and their achievements. Next, I coded the videotapes and analyzed the teaching procedures. Third, I analyzed students' pre-tests and post-tests in terms of representations. Last, I analyzed the teaching procedures using the instrument developed in the study. I used the Structure Equation Model and Hierarchical Linear Modeling to analyze the data.

The results showed that the both the quantity and quality of teachers' instructional representations varied a lot when teaching the same topic. First,

symbolic representations were the predominant representations in classroom teaching. Structures of instructional representations converge to content sub-constructs rather than format sub-constructs. There are nine teachers use the same types of representations recommended by textbooks. Five teachers use different types of representations suggested by the textbook. Second, both instructional representations and students' written representations converged towards content sub-constructs. The content sub-constructs include part-whole, measure, quotient, multiplication by one and cross product. However, connections between these sub-constructs were not statistically significant. Within the three content sub-constructs (part-whole, quotient, and multiplication by one) that revealed by students' written representations, quotient and multiplication by one significantly predicated the students' post-test scores. Fourth, I found that the teaching comprehensibility score significantly predicated student achievements in the post-tests.

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CHAPTER I INTRODUCTION

Background

In the mathematics education community the use of representations has received researchers' attention since the 1980s. There is more and more agreement on the positive influence of representations. These are useful in developing students' conceptual understanding, mathematic reasoning, problem solving, and communication skills (Ainsworth, 1999; Ball, 1988; Baxter & Glaser, 1998; Hiebert & Wearne, 1986; Kaput, 1989). The National Council of Teachers of Mathematics (NCTM) has had a powerful influence on the mathematics education field by accomplishing several goals. Among these is the promotion of the critical role of representations in teaching and learning mathematics (NCTM, 2000).

A significant amount of literature has been devoted to the role of different representations in assisting students' learning (Chandler & Sweller, 1992; Garrity, 1998; Haas, 1998; Hinzman, 1997; Kalyuga, Chandler & Sweller, 1998; Leinenbach & Raymond, 1996; McClung, 1998; Post, 1981; Sharp, 1995). However, empirical results were inconsistent regarding whether one form of representation was better than another. For example, manipulatives were one of the most controversial forms of representations in public schools. These were reported as both effective and ineffective in the literature. Some empirical studies stated that manipulatives improved students' learning (Garrity, 1998; Haas, 1998; Leinenbach & Raymond, 1996; Post, 1981). In contrast, other studies claimed that there was no significant correlation between manipulatives and students' learning improvements (Hinzman, 1997; McClung, 1998; Sharp, 1995). Another example of conflict in the literature was the debate on multiple representations, i. e., different forms of representations.

Some studies indicated that less effective learning occurred because of increased cognitive load when using multiple representations (Chandler & Sweller, 1992; Kalyuga, Chandler, & Sweller, 1998). In contrast, some other studies showed that students who used multiple representations tended to have a better understanding (Kaput, 1989; Resnick & Omanson, 1987; Schoenfeld, 1986; Sfard, 1991). A third idea claimed that there was no single best representation. It was thought that the effectiveness depended on the properties of the content learned (Bibby & Payne, 1993).

Each kind of representation (e.g., pictures, tables, graphs, and symbolic representations) can promote students' understanding of a certain attribute of a concept. Therefore, it is usually suggested that multiple representations be used in classroom instruction (American Association for the Advancement of Science, 2000; NCTM, 2000; Wood, 1999). Reasons for using multiple representations could be classified into the following three categories:

(1) The first one relates to the nature of the concept in reality. A concept usually consists of several sub-constructs. Using only one form of representation usually limits the meaning of a concept. Thus, utilizing multiple representations will prevent superficial understanding of a certain concept. Kaput (1992) stated, "multiple systems are required for their full expression-meaning that multiple, linked representations will grow in importance as an application of the new, dynamic, interactive media" (p. 530).

(2) Another advantage of multiple representations is that multiple forms of representation are more likely to appeal to the various learning styles of the students (Ainsworth, 1999).

(3) Finally, scientific research indicates that perception and cognition are processes that correlate with each other. Different portions of the brain are associated with comprehending different but related representations. For example, various parts of the brain may process symbolic or pictorial representations, while other parts process various kinds of knowledge presented in different forms. Therefore, the use of multiple representations tends to capture the internal connections between the distributed forms of knowledge according to different brain functions. Thus, knowledge is preserved in a more

comprehensive manner(Gazzaniga,2000).

Researchers have repeatedly reported that middle-grade students have difficulties in developing conceptual understanding of fractions, decimals and percents(Condon & Hilton, 1999; Goldin & Passantino, 1996; Lesh, Post, & Behr, 1987; Post, Cramer, Behr, Lesh, & Harel, 1993; Watanabe, Reynolds, & Lo, 1995). Indeed, even students in junior college have difficulty dealing with fractions. This can be connected to their earlier experiences in elementary school, when they first encountered fractions (Haas, 1998). Haas (1998) reported the reason for the difficulty. It was that instruction regarding fractions was fragmented. It did not link the connections between manipulatives representation and symbolic representations. Taber(2001) also indicated that addressing the connection among different forms of representations is important. This is necessary in order to develop a conceptual understanding of fractions.

In contrast to the whole numbers, there are not as many real world experiences in which students may use fractions to solve problems. Thus the classroom is the major environment wherein students can learn fractions (Streefland, 1991). If students have received inadequate instruction in the early stages of their learning, it is not surprising that they may lag behind as they advance to middle school, or even to adulthood. Students' poor performance with fractions, decimals and percents is a reflection of inadequate instruction.

In teaching fractions, emphasizing the importance and effectiveness of representations is not enough. Teachers should have the corresponding mathematical and pedagogical knowledge to construct an environment that allows students to experience different representations in order to facilitate learning. However, what is the reality of using representations in teaching fractions, decimals and percents in middle school classrooms? What are the concepts that are most commonly taught? How does the quality and quantity of teaching relate to student achievement? There is limited research on how middle school teachers use representations in classroom practice. Neither is there much research examining the impact of teaching quality and quantity on student understanding and achievement.

The Middle School Mathematics Project (MSMP) at Texas A&M University was part of a five-year longitudinal study. It was funded by the Interagency Educational Research Initiative, through a grant to the American Association for the Advancement of Science (Roseman, Kulm, & Manon, 2001). The main goal of the MSMP was to investigate how professional development and textbooks assist teachers' classroom instructional practices. It further investigated how teaching practices influence student achievement. Four professional development workshops were conducted in the first four years. Each year, three to five lessons per teacher were video-taped. Corresponding students were administered a pre-test and a post-test. This research used the data collected by the MSMP project member, including the author. By analyzing the teachers' videotapes and students' pre-tests and post-tests, the role of the quality and quantity of the teachers' instructional representations was investigated. This was done in light of student understanding and achievement regarding fractions, decimals and percents. During a pilot study, some teachers were found to have insufficient knowledge or skills. This may have led to their inability to use representations appropriately in classrooms. Teachers must be aware of the benefits and disadvantages of using different forms of these. They also need to be aware of the effectiveness of representations in improving conceptual understanding. Armed with this knowledge, teachers can apply them effectively in classroom instruction, thus better serving their students.

Statement of the Problem

Researchers in the field of cognitive psychology claimed that there were two categories of representations: external and internal, which were correlated with each other(Kaput, 1999; Goldin, 2003; Zelazo & Lourenco, 2003). Both internal and external representations were critical in developing children's understanding of mathematics(Jonassen, Cole, & Bamford, 1992; Kaput, 2001; Lenze & Dwyer, 1993; Miura, 2001). The visualization aspect of external representations could profoundly illustrate a concept by capturing different characteristics of the concept (Goldin, 2003). Internal representations also played an important role in learning (Hall, Bailey & Tillman, 1997; Hiebert & Carpenter, 1992; Schwartz, 1993). Hiebert and Carpenter (1992) contended that knowledge represented in an internal mental network tended to enhance mathematical conceptual understanding. Zhang (1997) stated that learning occurred during the interaction between the external representations and internal representations. This research aims to investigate the role of teachers' instructional representations (external representations) on students' external representations. Students' external representations are correlated with their internal representations and thus indicate their level of understanding. According to Zelazo and Lourenco (2003), "It has long been assumed that children's understanding and use of external representations, such as drawings and speech, potentially provide insight into the development of internal representations" (p. 55).

The research literature suggests that students' understanding of external symbolic representations of fractions, decimals and percents is one of the most difficult tasks middle school mathematics education faces (Condon & Hilton, 1999; Goldin & Passantino, 1996; Lesh, Post, & Behr, 1987; Post, Cramer, Behr, Lesh, & Harel, 1993; Watanabe, Reynolds, & Lo, 1995). Many middle school students have problems translating between external symbolic representations. They experience difficulty changing from fractions to decimals, and from decimals to percents (Condon & Hilton, 1999; Markovits & Sowder, 1991; Thompson & Walker, 1996; Vance, 1992). They also have problems in translating between external symbolic representations and external pictorial and manipulative representations. For example, it is a challenge for them to find the location of 1/4 on a number line (Vance, 1992), or to use a hundredths grid to represent 0.4 (Hiebert & Wearne, 1986).

As stated previously, students' learning depends on both the quality and quantity of teachers' instruction (Aronson, Zimmerman, & Carlos, 1998; Black, 2002; Carpenter & Fennema, 1991; Simon, 1997; Smith, 2000; Walker, 1976). In terms of deciding which is more important, some researchers argue for quality (Aronson, Zimmerman, & Carlos, 1998; Smith, 2000), while some others argue for quantity (Black, 2002; Walker, 1976). The American Association of Advancement of Science (AAAS) claimed that good representations should be accurate, comprehensible and varied, not allowing students to develop misconceptions (AAAS, 2000). However, few researches have been done to examine the quality of teachers' instructional representations in terms of accuracy and comprehensibility. Black (2002) categorized teaching time into allocated time, engaged instructional time and academic learning time. There have been few empirical studies which investigated the structure of engaged instructional time in teaching and learning fractions. There were not many empirical studies reporting the influence of both the quality and quantity of instructional representations on students' written representations and achievement.

This study investigated both the quality and the quantity of instructional representations of fractions, decimals, and percents. The quality of representations was indicated by whether the instructional representations were accurate, comprehensible and connected. The quantity of instructional representations was investigated as to the extent teachers used symbolic representations, manipulatives, pictures and real world experiences in their instructions. What would be explored as well was the influence of both quality and quantity of instructional representations on students' work with written representations of fractions, decimals and percents. This also involved noting the effect of different forms of written representations on student achievement.

Theoretical Framework

There are two components in the theoretical framework. One aspect involves different forms of representation and the other addresses the sub-constructs of fractions. In 1960, Bruner proposed discovery learning that aimed to provide experiences in order for students to explore and investigate knowledge. During the learning process, understanding of a concept was developed, based on previous knowledge and understanding (Bruner, 1960).