

PHY

国外物理名著系列 15

(注释版)

Gauge Theory of
Elementary Particle
Physics

基本粒子物理学的
规范理论

T.P.Cheng L.F.Li



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Particle Physics

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T. A. Cheng, L. F. Li

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北 京

国外物理名著系列序言

对于国内的物理学工作者和青年学生来讲，研读国外优秀的物理学著作是系统掌握物理学知识的一个重要手段。但是，在国内并不能及时、方便地买到国外的图书，且国外图书不菲的价格往往令国内的读者却步，因此，把国外的优秀物理原著引进到国内，让国内的读者能够方便地以较低的价格购买是一项意义深远的工作，将有助于国内物理学工作者和青年学生掌握国际物理学的前沿知识，进而推动我国物理学科研究和教学的发展。

为了满足国内读者对国外优秀物理学著作的需求，科学出版社启动了引进国外优秀著作的工作，出版社的这一举措得到了国内物理学界的积极响应和支持，很快成立了专家委员会，开展了选题的推荐和筛选工作，在出版社初选的书单基础上确定了第一批引进的项目，这些图书几乎涉及了近代物理学的所有领域，既有阐述学科基本理论的经典名著，也有反映某一学科专题前沿的专著。在选择图书时，专家委员会遵循了以下原则：基础理论方面的图书强调“经典”，选择了那些经得起时间检验、对物理学的发展产生重要影响、现在还不“过时”的著作（如：狄拉克的《量子力学原理》）。反映物理学某一领域进展的著作强调“前沿”和“热点”，根据国内物理学研究发展的实际情况，选择了能够体现相关学科最新进展，对有关方向的科研人员和研究生有重要参考价值的图书。这些图书都是最新版的，多数图书都是2000年以后出版的，还有相当一部分是2006年出版的新书。因此，这套丛书具有权威性、前瞻性和应用性强的特点。由于国外出版社的要求，科学出版社对部分图书进行了少量的翻译和注释（主要是目录标题和练习题），但这并不会影响图书“原汁原味”的感觉，可能还会方便国内读者的阅读和理解。

“他山之石，可以攻玉”，希望这套丛书的出版能够为国内物理学工作者和青年学生的工作和学习提供参考，也希望国内更多专家参与到这一工作中来，推荐更多的好书。



中国科学院院士
中国物理学会理事长

前言

Elementary particle physics has made remarkable progress in the past ten years. We now have, for the first time, a comprehensive theory of particle interactions. One can argue that it gives a complete and correct description of all non-gravitational physics. This theory is based on the principle of gauge symmetry. Strong, weak, and electromagnetic interactions are all gauge interactions. The importance of a knowledge of gauge theory to anyone interested in modern high energy physics can scarcely be overstated. Regardless of the ultimate correctness of every detail of this theory, it is the framework within which new theoretical and experimental advances will be interpreted in the foreseeable future.

The aim of this book is to provide student and researcher with a practical introduction to some of the principal ideas in gauge theories and their applications to elementary particle physics. Wherever possible we avoid intricate mathematical proofs and rely on heuristic arguments and illustrative examples. We have also taken particular care to include in the derivations intermediate steps which are usually omitted in more specialized communications. Some well-known results are derived anew, in a way more accessible to a non-expert.

The book is not intended as an exhaustive survey. However, it should adequately provide the general background necessary for a serious student who wishes to specialize in the field of elementary particle theory. We also hope that experimental physicists with interest in some general aspects of gauge theory will find parts of the book useful.

The material is based primarily on a set of notes for the graduate courses taught by one of us (L.F.L.) over the past six years at the Carnegie-Mellon University and on lectures delivered at the 1981 Hefei (China) Summer School on Particle Physics (Li 1981). It is augmented by material covered in seminars given by the other author (T.P.C.) at the University of Minnesota and elsewhere. These notes have been considerably amplified, reorganized, and their scope expanded. In this text we shall assume that the reader has had some exposure to quantum field theory. She or he should also be moderately familiar with the phenomenology of high energy physics. In practical terms we have in mind as a typical reader an advanced graduate student in theoretical physics; it is also our hope that some researchers will use the book as a convenient guide to topics that they wish to look up.

Modern gauge theory may be described as being a 'radically conservative theory' in the sense used by J. A. Wheeler (see Wilczek 1982*b*). Thus, one extrapolates a few fundamental principles as far as one can, accepting some 'paradoxes' that fall short of contradiction. Here we take as axioms the principles of *locality*, *causality*, and *renormalizability*. We discover that a

certain class of relativistic quantum field theory, i.e. the gauge theory, contains unexpected richness (Higgs phenomena, asymptotic freedom, confinement, anomalies, etc.), which is necessary for an understanding of elementary particle interactions. And yet, this does not occasion any revision of the basic principles of relativity and quantum mechanics. Thus the prerequisite for the study of gauge theory is just the traditional preparation in advanced quantum mechanics and quantum field theory, especially the prototype gauge theory of quantum electrodynamics (QED).

The book is organized in two parts. Part I contains material that can be characterized as being ‘pre-gauge theory’. In Chapters 1, 2, and 3 the basics of relativistic quantum field theory (quantization and renormalization) are reviewed, using the simple $\lambda\phi^4$ theory as an illustrative example. In Chapters 4 and 5 we present the elements of group theory, the quark model, and chiral symmetry. The interrelationship of the above main topics—renormalization and symmetry—is then studied in Chapter 6. The argument that quarks are the basic constituents of hadrons is further strengthened by the discovery of Bjorken scaling. Scaling and the quark–parton model are described in Chapter 7. These results paved the way for the great synthesis of particle interaction theories in the framework of the non-Abelian gauge theories, which is treated in Part II. After the classical and quantized versions of gauge theories are discussed in Chapters 8 and 9, we are then ready for the core chapters of this book—Chapters 10–14—where gauge theories of quantum chromodynamics (QCD), quantum flavourdynamics (QFD), and grand unification (GUT) are presented. As a further illustration of the richness of the gauge theory structure we exhibit its nonperturbative solutions in the form of magnetic monopoles and instantons in Chapters 15 and 16.

We have also included at the end of the book two appendices. In Appendix A one can find the conventions and normalizations used in this book. Appendix B contains a practical guide to the derivation of Feynman rules as well as a summary of the propagators and vertices for the most commonly used theories—the $\lambda\phi^4$, Yukawa, QCD, and the (R_ξ gauge) standard model of the electroweak interaction.

In the table of contents we have marked sections and chapters to indicate whether they are an essential part (unmarked), or details that may be omitted upon a first reading (marked by an asterisk), or introductions to advanced topics that are somewhat outside the book’s main line of development (marked by a dagger). From our experience the material covered in the unmarked sections is sufficient for a one-semester course on the gauge theory of particle physics. Without omitting the marked sections, the book as a whole is adequate for a two-semester course. It should also be pointed out that although we have organized the sections according to their logical interconnection there is no need (it is in fact unproductive!) for the reader to strictly follow the order of our presentation. For example, §1.2 on path integral quantization can be postponed until Chapter 9 where it will be used for the first time when we quantize the gauge theories. As we anticipate a readership of rather diverse background and interests, we urge each reader to study the table of contents carefully before launching into a study pro-

gramme. A certain amount of repetition is deliberately built into the book so that the reader can pick and choose different sections without any serious problems. An experimentally inclined reader, who is not particularly interested in the formal aspects of relativistic quantum field theory, can skip Chapters 1, 2, 3, and 6 on quantization and renormalization. After an introductory study of group theory and the quark model in Chapters 4, 5, and 7 she or he should proceed directly to the parts of Chapters 8, 10, 11, 12, 14, etc. where a general introduction to and applications of gauge theory can be found.

The sections on references and bibliography at the end of the book represent some of the commonly cited references that we ourselves are familiar with. They are not a comprehensive listing. We apologize to our colleagues who have been inadequately referenced. Our hope is that we have provided a sufficient set so that an interested reader can use it to go on to find further reviews and research articles.

It is a pleasure to acknowledge the aid we have received from our colleagues and students; many have made helpful comments about the preliminary version of the book. We are very grateful to Professor Mahiko Suzuki who undertook a critical reading of the manuscript, and also to Professors James Bjorken, Sidney Drell, Jonathan Rosner, and Lincoln Wolfenstein for having encouraged us to begin the conversion of the lecture notes into a book. One of us (T.P.C.) would like to thank the National Science Foundation, UMSL Summer Research Fellowship Committee, and the Weldon Spring Endowment for support. During various stages of working on this project he has enjoyed the hospitality of the theoretical physics groups at the Lawrence Berkeley Laboratory, the Stanford Linear Accelerator Center and the University of Minnesota. L.F.L. would like to thank the Institute for Theoretical Physics at the University of California—Santa Barbara for hospitality and the Department of Energy and the Alfred P. Sloan Foundation for support. Finally, we also gratefully acknowledge the encouragement and help given by our wives throughout this project. And, we are much indebted to Ms Susan Swyers for the painstaking task of typing this manuscript. Other technical assistance by Ms Tina Ramey and Mr Jerry McClure is also much appreciated.

Note added in proof. As this manuscript was being readied for publication we received the news that the CERN UA1 and UA2 groups have observed events in $p\bar{p}$ collisions which may be interpreted as the production of an intermediate vector boson W with a mass approximately 80 GeV. Also, the Irvine–Michigan–Brookhaven collaboration reported a preliminary result setting a lower bound for the lifetime $\tau(p \rightarrow e^+ \pi^0) > 6.5 \times 10^{31}$ years.

St. Louis and Pittsburgh
September 1982

T.P.C.
L.F.L.

目 录

未标记的小节为本书的基本部分;那些用星号标识的地方含有第一次阅读时可以略去的内容。以剑形符号(dagger)标出的章节是对稍微处在本书主要发展路线之外的高级论题的基本介绍。

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第一部分

1. 场量子化基础

The dynamics of a classical field $\phi(x)$ are determined by the Lagrangian density $\mathcal{L}(\phi, \partial_\mu\phi)$ through the action principle

$$\delta S = 0 \quad (1.1)$$

where S is the action

$$S = \int d^4x \mathcal{L}(\phi, \partial_\mu\phi).$$

This extremization leads to the Euler–Lagrange equation of motion

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu\phi)} - \frac{\delta \mathcal{L}}{\delta\phi} = 0. \quad (1.2)$$

To quantize a system we can adopt either of two equivalent approaches. The canonical formalism involves the identification of the true dynamical variables of the system. They are taken to be operators and are postulated to satisfy the canonical commutation relations. The Hamiltonian of the system is constructed and used to find the time evolution of the system. This allows us to compute the transition amplitude from the state at an initial time to the state at final time. Alternatively, we can use the Feynman path-integral formalism to describe the quantum system. Here the transition amplitude is expressed directly as the sum (a functional integral) over all possible paths between the initial and final states, weighted by the exponential of i times the action (in units of the Planck's constant \hbar) for the particular path. Thus in the classical limit ($\hbar \rightarrow 0$) the integrand oscillates greatly, making a negligible contribution to the integral except along the stationary path selected by the action principle of eqn (1.1).

In this chapter we present an elementary study of field quantization. First we review the more familiar canonical quantization procedure and its perturbative solutions in the form of Feynman rules. Since we will find that gauge field theories are most easily quantized using the path-integral formalism we will present an introduction to this technique (and its connection to Feynman rules) in §1.2. For the most part the simplest case of the self-interacting scalar particle will be used as the illustrative example; path-integral formalism for fermions will be presented in §1.3.

Since the path-integral formalism will not be used until Chapter 9 when we quantize the gauge fields, the reader may wish to postpone the study of §§1.2 and 1.3 until then. It should also be pointed out that even for gauge theories we shall use these two quantization formalisms in an intermixed fashion. By this we mean that we will use whatever language is most convenient for the task at hand, regardless of whether it implies path-integral or canonical

quantization. For example, in the discussion of the short-distance phenomena in Chapter 10, we continue to use the language of ‘operator product expansion’ even though strictly speaking this implies canonical quantization. The reader is also referred to Appendix B at the end of the book where one can find a practical guide to derivation of Feynman rules via path-integral formalism.

1.1 正则量子化形式回顾

We assume familiarity with the transition from a classical nonrelativistic particle system to the corresponding quantum system. The Schrödinger equation is obtained after we replace the canonical variables by operators and the Poisson brackets by commutators. These operators act on the Hilbert space of square integrable functions (the wavefunctions), and they satisfy equations of motion which are formally identical to the classical equations of motion.

A relativistic field may be quantized by a similar procedure. For a system described by the Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$, the field $\phi(x)$ satisfies the classical equation of motion given in eqn (1.2). We obtain the corresponding quantum system by imposing the canonical commutation relations at equal time

$$\begin{aligned} [\pi(\mathbf{x}, t), \phi(\mathbf{x}', t)] &= -i\delta^3(\mathbf{x} - \mathbf{x}') \\ [\pi(\mathbf{x}, t), \pi(\mathbf{x}', t)] &= [\phi(\mathbf{x}, t), \phi(\mathbf{x}', t)] = 0 \end{aligned} \quad (1.3)$$

where the conjugate momentum is defined by

$$\pi(x) = \frac{\delta \mathcal{L}}{\delta(\partial_0 \phi)}. \quad (1.4)$$

The Hamiltonian

$$H = \int d^3x [\pi(x) \partial_0 \phi(x) - \mathcal{L}(x)] \quad (1.5)$$

governs the dynamics of the system

$$\begin{aligned} \partial_0 \phi(\mathbf{x}, t) &= i[H, \phi(\mathbf{x}, t)] \\ \partial_0 \pi(\mathbf{x}, t) &= i[H, \pi(\mathbf{x}, t)]. \end{aligned} \quad (1.6)$$

Example 1.1. Free scalar field. Given the Lagrangian density

$$\mathcal{L} = \frac{1}{2}[(\partial_\lambda \phi)(\partial^\lambda \phi) - \mu^2 \phi^2],$$

eqn (1.2) yields the Klein–Gordon equation

$$(\partial^2 + \mu^2)\phi(x) = 0. \quad (1.7)$$

In quantum field theory the field $\phi(x)$ and its conjugate momentum operators given by eqn (1.4), $\pi(x) = \partial_0 \phi(x)$, satisfy the canonical commutation

relations

$$\begin{aligned} [\partial_0 \phi(\mathbf{x}, t), \phi(\mathbf{x}', t)] &= -i\delta^3(\mathbf{x} - \mathbf{x}') \\ [\partial_0 \phi(\mathbf{x}, t), \partial_0 \phi(\mathbf{x}', t)] &= [\phi(\mathbf{x}, t), \phi(\mathbf{x}', t)] = 0. \end{aligned} \quad (1.8)$$

The Hamiltonian is given by

$$H_0 = \int d^3x \frac{1}{2} [(\partial_0 \phi)^2 + (\nabla \phi)^2 + \mu^2 \phi^2]. \quad (1.9)$$

The time evolution equation (1.6), which is basically Hamilton's equation of motion, can be cast in the form of (1.7). Thus the field operator $\phi(x)$ formally satisfies the Klein-Gordon equation. This simple non-interacting case can be solved and we have

$$\phi(\mathbf{x}, t) = \int \frac{d^3k}{[(2\pi)^3 2\omega_k]^{1/2}} [a(\mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega_k t)} + a^\dagger(\mathbf{k}) e^{-i(\mathbf{k} \cdot \mathbf{x} - \omega_k t)}] \quad (1.10)$$

where $\omega_k = (\mathbf{k}^2 + \mu^2)^{1/2}$. The coefficients of expansion $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ are operators. The canonical commutation relations of eqn (1.8) are transcribed into

$$\begin{aligned} [a(\mathbf{k}), a^\dagger(\mathbf{k}')] &= \delta^3(\mathbf{k} - \mathbf{k}') \\ [a(\mathbf{k}), a(\mathbf{k}')] &= [a^\dagger(\mathbf{k}), a^\dagger(\mathbf{k}')] = 0 \end{aligned} \quad (1.11)$$

and the Hamiltonian of eqn (1.9) can be expressed as

$$H_0 = \int d^3k \omega_k a^\dagger(\mathbf{k}) a(\mathbf{k}) \quad (1.12)$$

where we have discarded an irrelevant constant. Remembering the situation of the harmonic oscillator, we see immediately that $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ can be interpreted as destruction and creation operators. Thus the one-particle state with momentum \mathbf{k} is given by the creation operator acting on the vacuum state

$$|\mathbf{k}\rangle = [(2\pi)^3 2\omega_k]^{1/2} a^\dagger(\mathbf{k}) |0\rangle \quad (1.13)$$

where the normalization is

$$\langle \mathbf{k}' | \mathbf{k} \rangle = (2\pi)^3 2\omega_k \delta^3(\mathbf{k} - \mathbf{k}').$$

The product $a^\dagger a$ has the usual interpretation as a number operator and eqn (1.12) shows that H_0 is the Hamiltonian for a system of non-interacting particles.

Given the solution, (1.10), and (1.11), we can easily calculate the Feynman propagator function, which is the vacuum expectation value for a time-ordered product of two fields,

$$\begin{aligned} i\Delta(x_1 - x_2) &\equiv \langle 0 | T(\phi(x_1)\phi(x_2)) | 0 \rangle \\ &= \theta(t_1 - t_2) \langle 0 | \phi(x_1)\phi(x_2) | 0 \rangle + \theta(t_2 - t_1) \langle 0 | \phi(x_2)\phi(x_1) | 0 \rangle \\ &= \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - \mu^2 + i\epsilon} \exp\{ik \cdot (x_1 - x_2)\}. \end{aligned} \quad (1.14)$$