

(英文版)

应用数值方法 使用MATLAB和C语言

Applied Numerical Methods
for Engineers Using MATLAB and C

(美) 罗伯特 J.奇林 (Robert J.Schilling) 著
桑德拉 L.哈里斯 (Sandra L.Harris)



时代教育·国外高校优秀教材精选

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随着我国加入 WTO，国际间的竞争越来越激烈，而国际间的竞争实际上也就是人才的竞争、教育的竞争。为了加快培养具有国际竞争力的高水平技术人才，加快我国教育改革的步伐，国家教育部近来出台了一系列倡导高校开展双语教学、引进原版教材的政策。以此为契机，机械工业出版社陆续推出了一系列国外影印版教材，其内容涉及高等学校公共基础课，以及机、电、信息领域的专业基础课和专业课。

引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

为了做好教材的引进工作，机械工业出版社特别成立了由著名专家组成的国外优秀教材审定委员会。这些专家对高校实施双语教学做了深入细致的调查研究，对引进原版教材提出了许多建设性意见，并慎重地对每一本将要引进的原版教材一审再审，精选再精选，确认教材本身的质量水平，以及权威性和先进性，以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中，审定委员会还结合我国高校教学课程体系的设置和要求，对原版教材的教学思想和方法的先进性、科学性进行了严格把关。同时，尽量考虑了原版教材的系统性和经济性。

这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

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序

这是一本从实际例子的角度全面概括数值计算方法的工科研究生教材，写作风格非常简洁优美。总的说来，本书具有以下特点：

特点一是该书重点介绍了解决工程实践中的各种有用的数值方法，而不是数学的定理和证明，所述问题涉及到土木、机电、力学、化工等学科，书中给出的许多事例都有很新的应用背景，这是国内同类教材所缺乏的。

该书的另一特点是求解这些问题的数学软件(MATLAB 和 C 语言)的计算机程序和结果，便于学生上机实算，可以加深学生对数值计算方法的特点的直观认识。

作为教材该书的编排很有心意，每章的第一节都设为“动机和目标”，从实际例子切入将要讨论的问题；每章后的练习题都分成了分析题和计算题两部分；附录中还安排了对向量和矩阵的有益的回顾。

该书内容包括数值计算，线性代数系，特征根和特征向量，曲线拟合，根的求解，最优化方法，微分和积分，常微分方程，偏微分方程，数字信号处理等。它涵盖的内容多于国内同类教材，如最优化方法(第 6 章)和数字信号处理(第 10 章)。最优化方法部分国内计算方法课程一般不讲，但随着这一学科的发展，这部分内容对学生掌握新的数值方法是需要的。

最后，书中有些例题反复出现在不同章节中，通过比照可以学习到处理问题的不同方法。

鉴于以上特点和优点，本书非常适合作为工科研究生教材，或者理工科本科生的教学参考，也可作为工程技术人员的参考书。

北京航空航天大学

丁丽娟

Preface

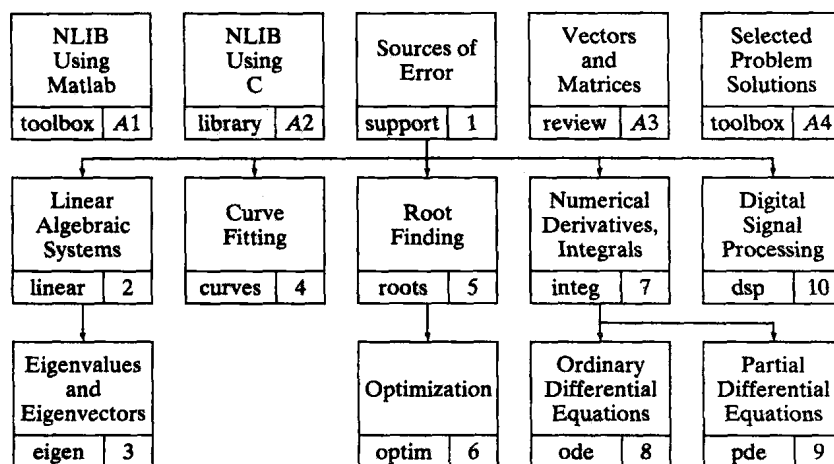
Numerical computing is a powerful tool for solving practical mathematical problems that occur throughout engineering. In this book, we focus on the application of numerical methods to solve both analysis and design problems. In today's computing environment, inexpensive hardware and software are available to solve realistic numerical problems very quickly and with modest effort.

TOPICAL COVERAGE

The topics included in this book are summarized in the block diagram shown in Figure 1. The number in the lower right corner of each block indicates the chapter or appendix where the topic is discussed, while the text in the lower left corner indicates the software module associated with the block.

In Chapter 1, we examine potential sources of error in numerical computations. Two of the most common mistakes that casual users of numerical software make are to accept the validity of numerical output at face value and to confuse precision for accuracy. In addition to examining the effects of round-off error and formula truncation error, Chapter 1 also focuses on techniques for efficiently generating random numbers with desired statistical properties. We included random number generation in Chapter 1 because, like round-off error, it can be tied directly to the internal representation of numbers within the computer. In addition, random numbers prove useful for testing

FIGURE 1
Topics*



*In each box, the lower left corner lists the appropriate software module; the lower right corner lists the chapter or appendix number.

virtually all the numerical methods that follow. Vectors and matrices are used in engineering applications because they provide an elegant, concise way to describe the essential relationships between problem variables. They are used extensively both in this book and in the MATLAB and C software packages that accompany it. We highly recommend that students whose mathematical background includes little or no exposure to the use of vectors and matrices read the brief review of the topic in Appendix 3 before proceeding beyond Chapter 1. It is *not* necessary for students to have taken a course in linear algebra to use this book. However, it is important that students at least be comfortable with the *notation* of vectors and matrices and be generally familiar with basic operations and fundamental definitions.

Chapter 2 focuses on the problem of solving systems of linear algebraic equations using both direct methods based on elementary row operations and iterative methods that are attractive for large sparse systems. In Chapter 3, we examine the problem of finding the eigenvalues and eigenvectors of a square matrix. Knowing the locations of the eigenvalues allows one to draw conclusions about the stability of linear dynamic systems, and this knowledge is also helpful in analyzing the convergence properties of linear iterative methods.

Chapter 4 addresses the problem of fitting curves and surfaces to experimental data. In this chapter, we have included techniques for interpolation, extrapolation, least-squares curve fitting, and cubic splines. Several of these techniques surface again in later chapters as parts of other algorithms. Chapter 5 is devoted to the problem of solving nonlinear algebraic equations, also called root finding. Iterative techniques are developed and compared on the basis of speed and ease of implementation. We also consider in detail the important special case of finding the roots of polynomials. Chapter 6 focuses on the more general problem of minimizing an objective function subject to equality and inequality constraints. Optimal design problems in engineering can often be formulated as constrained minimizations. In this chapter, we examine both local and global search methods.

Chapter 7 investigates the problem of performing numerical differentiation and integration of functions whose values are available only at discrete points. We examine the sensitivity of numerical differentiation to noise and develop techniques for accurately estimating the value of single and multidimensional integrals. Chapter 8 examines an important generalization of numerical integration, namely, the solution of systems of first-order ordinary differential equations (ODEs). In addition, we explore initial value problems and boundary value problems. We also consider implicit techniques applicable to stiff systems of differential equations. Chapter 9 generalizes the problem still further by introducing additional independent variables in the form of partial differential equations (PDEs). We also present finite difference techniques for solving Poisson's equation, the heat equation, and the wave equation, in one and two dimensions.

Chapter 10 examines a relatively recent topic, the extraction of information from discrete samples of continuous-time signals. Digital signal processing (DSP) techniques include the highly efficient fast Fourier transform, digital filter design, correlation, and convolution. They also include the identification of linear discrete-time systems from input and output measurements using least-squares and adaptive methods.

STYLE OF PRESENTATION

This book is written in an informal style in order to ease the student gradually into each new topic and to smooth the transition between topics. The book contains numerous algorithms and examples, but there are no formal explicit theorems, definitions, or proofs. Terms defined or emphasized within the text are *italicized*. The usage of mathematical notation is summarized in Appendix 3.

Each chapter follows the template shown in Figure 2. Chapters start with a brief introduction to the problem or class of problems to be solved. This is followed by a section on motivation (“Why solve this problem?”) and chapter objectives (“What will you learn?”). The core of each chapter is the development of a sequence of increasingly sophisticated numerical methods for solving the problem. We introduce the simplest and most specialized techniques first; then we present the more general ones. Each method is demonstrated with at least one example. For all the computational examples, corresponding example programs are available on the CD that accompanies the text. The example programs follow the naming convention *xyz.m* (for MATLAB) and *xyz.c* (for C), where *x* is the chapter number, *y* is the section within the chapter, and *z* is the example within the section.

The algorithm development sections are followed by an applications section that includes several case study-type examples from different fields of engineering (chemical, civil, electrical, and mechanical). The case study problems are developed in detail, and complete computational solutions are provided in MATLAB and C. Several case study examples reappear throughout subsequent chapters as we investigate different aspects of the problem and apply different mathematical techniques. The applications section is followed by a chapter summary that compares the different methods in terms of speed, accuracy, and applicability. At the end of each chapter is a section with homework problems, including subsections on analysis and computation. Problem solutions are available both in an Instructor’s Manual and on an accompanying Solution Disk. In addition, solutions to selected problems are provided in Appendix 4. We encourage students to use these problems, marked with an (S), to check their understanding of the material.

MATHEMATICAL BACKGROUND

An increasingly common problem faced by college and university instructors is the large variation in the mathematical backgrounds of students taking a course such as

FIGURE 2
Chapter
Format

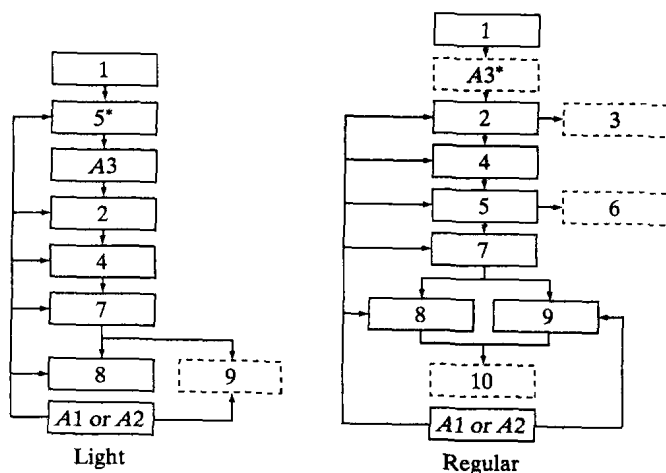
Chapter Problem
Motivation and Objectives
Algorithm Development with Examples
Applications
Chapter Summary
Homework Problems

applied numerical methods. This book is targeted for use by undergraduates in all fields of engineering. It is assumed that the students have taken the standard sequence of courses in calculus and differential equations and are in at least the spring semester or quarter of their sophomore year.

There is enough material in the book, and enough flexibility in the order in which topics can be covered, to provide for two distinct ways the book can be used. The choice depends on the academic maturity of the class (sophomore, junior, senior) and the length of time available for the course (one quarter, one semester, two quarters). For students who have a less sophisticated mathematical background, the *light* chapter sequence shown in Figure 3 is probably most appropriate. In the *light* sequence, the sources-of-error material in Chapter 1 is followed directly by root-finding techniques in Chapter 5, skipping the section on systems of nonlinear equations at the end of the chapter. This way, students can see important problems and solution techniques using only *scalar* mathematics. The notation, basic operations, and fundamental definitions associated with the use of vectors and matrices are then covered in Appendix 3. This is necessary because vectors and matrices are used extensively in both the algorithm development and the accompanying software. Once students are comfortable with problem formulations that use vectors and matrices, they can go on to the fundamental topic of solving linear systems of algebraic equations, covered in Chapter 2. Eigenvalues and eigenvectors (Chapter 3) can be skipped without interrupting the flow, which then leads to curve fitting techniques in Chapter 4. Optimization (covered in Chapter 6) is a more advanced topic that can be skipped without loss of continuity, so students can proceed directly to Chapter 7, in which we discuss numerical differentiation and integration. This leads naturally to the solution of ordinary differential equations in Chapter 8. As an alternative, and with time permitting (one semester), the topic of partial differential equations can be considered after either Chapter 7 or Chapter 8, for courses for which PDEs are important.

For classes that are at least junior level and for courses of longer duration (that is, one semester or two quarters), some variation of the more complete treatment

FIGURE 3
Typical
Chapter
Sequences



shown in the *regular* sequence in Figure 3 can be used. The main difference here is that the chapters are covered more or less in the order they are presented in the book, with the optional chapters on eigenvalues and eigenvectors (Chapter 3), optimization (Chapter 6), and digital signal processing (Chapter 10) included, depending on the students' interests, the students' backgrounds, and the time available. It should be emphasized that even though inclusion of Appendix 3 is optional in the regular course sequence, most students should read at least the first part of Appendix 3, which reviews vector and matrix notation and basic operations.

SOFTWARE BACKGROUND

The key to the successful application of numerical methods is effective software. The software included assumes that the student is familiar with the fundamentals of the MATLAB programming environment or the fundamentals of the programming language C. For MATLAB users, a numerical toolbox of MATLAB functions (NLIB) is available. The NLIB toolbox is described in detail in Appendix 1. It includes functions that implement the algorithms developed throughout the text. Also included is a set of main-program support functions, which are low-level utility functions designed to ease user interaction and the display of numerical results. At the start of each Applications section, the student should read the accompanying section of the software appendix to see how the algorithms are implemented.

For C users, there is a corresponding library of functions described in detail in Appendix 2. The NLIB library follows the ANSI C standard so as to maximize *portability* between programming platforms. Although the MATLAB toolbox is somewhat easier to use and is therefore recommended, the advantage of the C library is that the corresponding functions execute faster. This is particularly noticeable for computationally intensive applications, such as optimization and solution of partial differential equations. The complete source code for the NLIB library is provided on the distribution CD. In addition, precompiled versions of the library are available for the Microsoft Visual C++ and the Borland C++ compilers.

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This project has been several years in the making, and many individuals have contributed to its completion. We would like to thank our colleagues at Clarkson University for early reviews of the draft (Rangaswami Mukundan) and for suggesting some of the case study examples (Goodarz Ahmadi and Hayley Shen).

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Robert J. Schilling
Sandra L. Harris

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