海外优秀数学类教材系列丛书

Mc Graw Hill Education

影印版

Calculus (Second Edition)

微积分（第2版）

（上册）

Robert T. Smith
Roland B. Minton

高等教育出版社
Higher Education Press

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Robert T. Smith
Millersville University of Pennsylvania

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出版者的话

在我国已经加入 WTO、经济全球化的今天，为适应当前我国高校各类创新人才培养的需要，大力推进教育部倡导的双语教学，配合教育部实施的“高等学校教学质量与教学改革工程”和“精品课程”建设的需要，高等教育出版社有计划、大规模地开展了海外优秀数学类系列教材的引进工作。

高等教育出版社和 Pearson Education，John Wiley & Sons，McGraw-Hill，Thomson Learning 等国外出版公司进行了广泛接触，经国外出版公司的推荐并在国内专家的协助下，提交引进版权总数 100 余种。收到样书后，我们聘请了国内高校一线教师、专家、学者参与这些原版教材的评介工作，并参考国内相关专业的课程设置和教学实际情况，从中遴选出了这套优秀教材组织出版。

这批教材普遍具有以下特点：（1）基本上是近 3 年出版的，在国际上被广泛使用，在同类教材中具有相当的权威性；（2）高版次，历经多年教学实践检验，内容翔实准确、反映时代要求；（3）各种教学资源配套整齐，为师生提供了极大的便利；（4）插图精美、丰富，图文并茂，与正文相辅相成；（5）语言简练、流畅、可读性强，比较适合非英语国家的学生阅读。

本系列丛书中，有 Finney、Weir 等编的《托马斯微积分》（第 10 版，Pearson），其特色可用“呈传统特色、富革新精神”概括，本书自 20 世纪 50 年代第 1 版以来，平均每四五年就有一个新版面世，长在 50 余年始终盛行于西方教坛，作者既有相当高的学术水平，又热爱教学，长期工作在教学第一线，其中，年近 90 的 G.B.Thomas 教授长年在 MIT 工作，具有丰富的教学经验；Finney 教授也在 MIT 工作达 10 年；Weir 是美国数学建模竞赛委员会主任。Stewart 编的立体化教材《微积分》（第 5 版，Thomson Learning）配备了丰富的教学资源，是国际上最畅销的微积分原版教材，2003 年全球销量约 40 余万册，在美国，占据了约 50%~60%的微积分教材市场，其用户包括耶鲁等名牌院校及众多一般院校 600 余所。本系列丛书还包括 Anton 编的经典教材《线性代数及其应用》（第 8 版，Wiley）；Jay L. Devore 编的优秀教材《概率论与数理统计》（第 5 版，Thomson Learning）等。在努力降低引进教材售价方面，高等教育出版社做了大量和细致的工作，这套引进的教材体现了一定的权威性、系统性、先进性和经济性等特点。

通过影印、翻译、编译这批优秀教材，我们一方面要不断地分析、学习、消化吸收国外优秀教材的长处，吸取国外出版公司的制作经验，提升我们自编教材的立体化配套标准，使我国高校教材建设水平上一个新的台阶；与此同时，我们还将尝试组织海外作者和国内作者合编外文版基础课数学教材，并约请国内专家改编部分国外优秀教材，以适应我国实际教学环境。

这套教材出版后，我们将结合各高校的双语教学计划，开展大规模的宣传、培训工作，及时地将本套丛书推荐给高校使用。在使用过程中，我们衷心希望广大高校教师和同学提出宝贵的意见和建议，如有好的教材值得引进，请与高等教育出版社高等理科分社联系。

联系电话：010-58581384，E-mail: xuke@hep. com.cn。

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ABOUT THE AUTHORS

Robert T. Smith is Professor of Mathematics and Chair of the Department of Mathematics at Millersville University of Pennsylvania, where he has taught since 1987. Prior to that, he was on the faculty at Virginia Tech. He earned his Ph.D. in mathematics from the University of Delaware in 1982.

Dr. Smith's mathematical interests are in the application of mathematics to problems in engineering and the physical sciences. He has published a number of research articles on the applications of partial differential equations as well as on computational problems in x-ray tomography. He is a member of the American Mathematical Society, the Mathematical Association of America and the Society for Industrial and Applied Mathematics.

Professor Smith lives in Lancaster, Pennsylvania, with his wife Pam, his daughter Katie and his son Michael. When time permits, he enjoys playing volleyball, tennis and softball. In his spare time, he also coaches youth league soccer. His present extracurricular goal is to learn the game of golf well enough to not come in last in his annual mathematicians/statisticians tournament.

Roland Minton is Professor of Mathematics at Roanoke College, where he has taught since 1986. Prior to that, he was on the faculty at Virginia Tech. He earned his Ph.D. from Clemson University in 1982. He is the recipient of the 1998 Roanoke College Exemplary Teaching Award.

Dr. Minton has supervised numerous student research projects in such topics as sports science, complexity theory and fractals. He has published several articles on the use of technology and sports examples in mathematics, in addition to a technical monograph on control theory. He has received grants for teacher training from the State Council for Higher Education in Virginia. He is a member of the Mathematical Association of America, American Mathematical Society and other mathematical societies.

Professor Minton lives in Salem, Virginia, with his wife Jan, daughter Kelly and son Greg. He enjoys playing golf when time permits and watching any sport on TV (even if time doesn't permit). Jan also teaches mathematics at Roanoke College and is very active in mathematics education. A busy family schedule includes band performances and lacrosse and soccer matches. Favorite entertainers include the Marx Brothers, guitarist Danny Gatton and mystery novelist Kinky Friedman.

In addition to the Premiere Edition of *Calculus*, Professors Smith and Minton have together published three earlier books with McGraw-Hill: *Discovering Calculus with the HP-28 and the HP-48*, *Discovering Calculus with the TI-81 and the TI-85* and *Discovering Calculus with the Casio fx-7700 and the Casio fx-8700*. The Premiere edition of *Calculus* has also been translated into Spanish.

PREFACE

The wide-ranging debate brought about by the calculus reform movement has had a significant impact on the calculus textbook market. In response to many of the questions and concerns surrounding this debate, we have written a modern calculus textbook, intended for students majoring in mathematics, physics, chemistry, engineering and related fields.

This text is intended for the average student, that is, one who does not already know the subject, whose background is somewhat weak in spots and who requires significant motivation to study the subject. Our intention is that students should be able to read our book, rather than merely use it as an encyclopedia filled with the facts of calculus. The book has been written in a conversational style that reviewers have compared to listening to a good lecture. Our sense of what works well with students has been honed by teaching mathematics for more than 20 years at a variety of colleges and universities, both public and private, ranging from a small liberal arts college to large engineering schools.

In an effort to ensure that this textbook successfully addresses our concerns about the effective teaching of calculus, as well as others' concerns, we have continually asked instructors around North America for their opinions on the calculus curriculum, the strengths and weaknesses of current textbooks and the strengths and weaknesses of our own text. In preparing this second edition, as with the Premiere Edition, we enjoyed the benefit of countless insightful comments from a talented panel of reviewers that was carefully selected to help us with this project. Their detailed reviews of our materials and their opinions about the teaching of calculus were invaluable to us during our development of the Premiere Edition and the preparation of this edition. We are deeply indebted to them for their time and effort.

OUR PHILOSOPHY

We agree with many of the ideas that have come out of the calculus reform movement. In particular, we believe in the **Rule of Four:** that concepts should be presented **graphically, numerically, algebraically** and **verbally,** whenever these are appropriate. In fact, we would add **physically** to this list, since the modeling of physical problems is an important skill that students need to develop. We also believe that, while the calculus curriculum has been in need of reform, we should not throw out those things that already work well. Our book thus represents an updated approach to the traditional topics of calculus. We follow an essentially traditional order of presentation, while integrating technology and thought-provoking exercises throughout.

One of the thrusts of the calculus reform movement has been to place greater emphasis on problem solving and to present students with more realistic applications, as well as open-ended problems. We have incorporated meaningful writing exercises and extended, open-ended problems into **every** problem set. You will also find a much wider range of applications than in most traditional texts. We make frequent use of applications from within students' experience to both motivate the development of new topics and to illustrate concepts we have already presented. In particular, we have included numerous examples from the physics of sports to give students a familiar context in which to think of various concepts.

We believe that a conceptual development of the calculus must motivate the text. Although we have integrated technology throughout, we have not allowed the technology to drive the book. We have also not given in to the temptation to show off what technology can do, except where this has a direct bearing on learning the calculus. Our goal is to use the available technology to help students reach a conceptual understanding of the calculus as it is used today.

Perhaps the most difficult task when preparing a new calculus text is the actual *writing* of it. We have endeavored to write this text in a manner that combines an appropriate level of informality with an honest discussion regarding the difficulties that students commonly face in their study of calculus. In addition to the concepts and applications of calculus, we have also included many frank discussions about what is practical and impractical, and what is difficult and not so difficult to students of calculus. We have attempted to provide clarity of presentation in the creation of every example, application and exercise.

The book that we have written represents substantive change. By virtue of integrating technology throughout, utilizing a lively presentation style and incorporating a wider variety of problems, we believe that we meet many of the objectives of the calculus reform movement. At the same time, our relatively traditional outline retains the central strengths of mainstream calculus, enabling instructors to teach from a familiar body of material while integrating technology and modern applications.

DEVELOPMENT OF CONCEPTS

We have endeavored to carefully reconsider the best way in which to present each traditional calculus concept. Our primary objective is to keep students focused on the central concepts and motivated in their learning.

An example of the results of our analysis is the incorporation of an early introduction to logarithms, exponentials and the trigonometric functions. All of our students have seen these functions before they ever set foot in a calculus classroom. It only makes sense to take advantage of their familiarity. We introduce the calculus of these functions in Chapter 2, along with the other rules of differentiation. While we are not able to give complete derivations of the derivatives of the natural logarithm and exponentials at this point, we give very strong numerical evidence and nearly complete algebraic proofs. We have found that this early introduction allows for more varied and interesting examples in the applications of differentiation (including graphing), integration and applications of integration. We follow up on this in Chapter 6 with the definition of the natural logarithm as an integral and complete derivations of the derivative formulas. At this point, we have tied up all of the loose ends and set the calculus of the logarithmic and exponential functions on a firm theoretical footing. Thus, in the end, we have sacrificed no mathematical rigor, but students will have gained much insight from the use of a richer set of functions throughout the introductory chapters.

Again serving this end purpose, we have augmented a simple algebraic presentation of selected ideas with numerical methods. For instance, when we introduce the notion of area, we emphasize the computation of area as a limit of a Riemann sum, but use regular partitions exclusively. We do not introduce the notion of the norm of a partition until Chapter 13, when we develop multiple integrals. By that point, students should already be comfortable with the concept of the definite integral as a limit of a sum and this refinement should only enhance their understanding. We are careful to point out that (without the Fundamental Theorem of Calculus) the limit of Riemann sums can be computed directly only for a very small number of functions. In addition, we allow students to explore the same ideas numerically. We are not restricted to polynomials of low degree and students can observe numerical values of Riemann sums approaching a limit. With this approach, students get to see the same problem from several different viewpoints, thus improving the likelihood that they will grasp the underlying concept. Additionally, students are given a useful tool (numerical integration) that they can bring to bear on a wide variety of problems.

In our view, techniques of integration remain of great importance. Our emphasis is on helping students develop the ability to carefully distinguish among similar looking integrals and identify the appropriate technique of integration to apply to each integral. The attention to detail and mathematical sophistication required by this process are invaluable skills. We do not attempt to be encyclopedic about techniques of integration, especially given the widespread use of computer algebra systems. Accordingly, in section 7.5, we include a discussion of integration tables and the use of computer algebra systems for performing symbolic integration.

In addition to a focus on the central concepts of calculus, we have included several sections that are not typically found in other calculus texts, as well as expanded coverage of specific topics. This provides instructors with the flexibility to tailor their courses to the interests and abilities of each class. For instance, in section 1.6, we explore loss-of-significance errors. Here, we discuss how computers and calculators perform arithmetic operations and how these can cause errors, in the context of numerical approximation of limits. In section 3.8, we present a diverse group of applications of differentiation, including chemical reaction rates and heart rates. Separable differential equations and logistic growth are discussed in section 6.5, followed by direction fields and Euler's method for first-order ordinary differential equations in section 6.6. In Chapter 8, we follow our discussion of power series and Taylor's Theorem with a section on Fourier series. In sections 9.1–9.3 we provide expanded coverage of parametric equations, and in section 10.4 we include a discussion of Magnus force.

■ CALCULUS AND TECHNOLOGY

We do not view technology as a gimmick to be artificially appended to the same old calculus curriculum. Nor do we believe that a calculus course should be a course in how to use technology. We believe that technology can and should be introduced as a natural part of a coherent development of calculus. In our presentation, technology has been seamlessly integrated throughout, but only where appropriate. Users are expected to have access to a graphing calculator or a computer algebra system and to use it routinely. We employ a generic use of technology, using those features that are shared by virtually all graphing calculators and computer algebra systems.

A common concern regarding the substantial use of technology is that our students will become mindless button-pushers. We guard against this by making the technology secondary to understanding and by pointing out errors that can be made by an overreliance on technology. We use technology so that students can focus on the difficult and sometimes subtle connections among the different concepts of the calculus. A student who has mastered these connections will be a much more effective user of the calculus than will a student who is proficient at algebraic methods alone. By engaging students on several different levels, using different approaches, we hope to improve their understanding and empower them to try new problems on their own.

We have chosen not to separate out "technology" exercises in the exercise sets found at the end of each section and chapter. This decision was made quite carefully, so that students would learn when to use the technology. We feel that placing an icon next to technology-based exercises allows students to avoid making this decision on their own. Throughout the text, we provide advice and guidance on the proper use of technology and provide tools to help students determine when the use of technology is most appropriate. These sections are called out by a technology icon. We do, however, make recommendations for technology use in the homework sets in the *Instructor's Resource Manual,* to help instructors plan their homework assignments accordingly.

We assume that students have access to calculator- or computer-generated graphs, allowing us to routinely use graphs as the first step in solving a problem or as a check on the reasonableness of an answer. Being able to visualize a problem is an invaluable aid to students, and we try to take full advantage of this. One benefit of readily available graphics is the ability to solve more realistic application problems. Functions associated with realistic problems are often not mathematically simple, but we can approximate zeros or extrema graphically and numerically. Further, concepts such as the convergence of Taylor series are more meaningful when graphs are used to illustrate this convergence. This same graphical approach benefits our presentation of Fourier series, which is an important tool for understanding much of our digitally enhanced world.

CONTENTS WITH COMMENTARY

The vast majority of the topics found in our book are part of the standard calculus curriculum that has defined the mainstream for the last 30 years or so. We believe that this curriculum still has validity in terms of both mathematical precision and student learning. Nevertheless, we have made a small number of significant changes in the table of contents. In the following, you will find brief explanations of each chapter and its focus.

Chapter 0: Preliminaries

Chapter 0 consists mostly of a review of background material. Depending on the focus of a specific course, instructors may decide to cover all, some or none of this material. This chapter focuses attention on those aspects of algebra and trigonometry that will be most useful to students as they progress through calculus. In particular, Chapter 0 contains a review of the basic properties of exponential, logarithmic and trigonometric functions. Sections 0.3 and 0.4 can be used to familiarize students with the use of a graphing calculator or computer algebra system. Section 0.5 reviews the trigonometric functions, while a detailed presentation of the inverse trigonometric functions is presented in section 6.7. If desired, section 6.2 on inverse functions and section 6.7 may be used without modification along with the Chapter 0 material. Section 0.6 reviews the exponential and logarithmic functions, which are used extensively throughout the book. Section 0.8 can be used as a one-day bridge to calculus, illustrating the ideas that distinguish calculus from precalculus.

Chapter 1: Limits and Continuity

Chapter 1 introduces the central concepts of limit and continuity. Limits are introduced graphically and numerically in section 1.1, with computational rules developed in section 1.2. Complete coverage of the formal definition of the limit is provided in section 1.5, although this material is not required for the remainder of the text. Section 1.6 provides important insight into the computational and graphical accuracy of computers. This section may also be covered independently of other sections.

Chapter 2: Differentiation: Algebraic, Trigonometric, Exponential and Logarithmic Functions

Chapter 2 introduces the derivative and presents the basic rules of differentiation, including the product, quotient and chain rules. The derivatives of algebraic, exponential, logarithmic and trigonometric functions are developed, providing the opportunity to present a rich set of examples of chain rules, product rules, quotient rules and applications.

Chapter 3: Applications of Differentiation

Chapter 3 presents applications of the derivative, beginning with a discussion of linear approximations and an introduction to l'Hôpitals' Rule. A further treatment of l'Hôpital's Rule can be found in Chapter 7. Numerical methods are introduced by means of Newton's method. A thorough development of graphical interpretations of the derivative is followed by sections on optimization and a variety of rates of change.

Chapter 4: Integration

Chapter 4 provides an introduction to integration, starting with the process of antidifferentiation. Technology is used extensively in Chapter 4 to compute Riemann sums and observe their convergence, as well as to develop numerical integration techniques. The computation of distance from velocity provides a unifying theme to the chapter.

Chapter 5: Applications of the Definite Integral

Chapter 5 presents applications of integration, focusing on the development of integral formulas, routinely constructing approximating sums and then passing to the limit to obtain a definite integral. In this chapter we cover the traditional physics applications of work and fluid pressure and force, while adding a discussion of impulse, moments and center of mass. In addition, we include sections on projectile motion and probability. Our focus is on using Riemann sums and integrals to compute the quantities of interest.

Chapter 6: Exponentials, Logarithms and Other Transcendental Functions

Chapter 6 includes a thorough development of the exponential and logarithmic functions. Although these functions have been used throughout the preceding chapters, the complete derivation of the formulas is not provided until Chapter 6. This chapter's discussion of exponential growth and decay leads naturally into an introduction to first-order differential equations. The treatment of differential equations varies widely among current calculus texts. We have found that many students need a sound background in integration to fully appreciate the concept of the solution of a differential equation. On the other hand, simple techniques for solving differential equations are fully accessible to second-semester calculus students and are required of many second-semester engineering students. For these reasons, we discuss separable differential equations in section 6.5, two chapters after the introduction of the integral and in conjunction with exponential growth and decay. In order to maximize the flexibility in using this text, we chose to introduce this material prior to our coverage of techniques of integration. The inverse trigonometric functions and the hyperbolic functions are also discussed in Chapter 6.

Chapter 7: Integration Techniques

Chapter 7 provides a variety of techniques of integration. We believe that students gain understanding and maturity while learning to choose among different techniques, while recognizing that computer algebra systems are routinely used to find antiderivatives. We include the most important techniques of integration and leave room for instructors to cover other topics. L'Hôpital's Rule is revisited in this chapter to develop the theory needed for improper integrals.

Chapter 8: Infinite Series

Chapter 8 presents a thorough coverage of infinite series. Numerous tables of calculations and graphs are included to give students every chance to understand the difficult topic of series. Section 8.8 introduces numerous interesting and accessible applications of Taylor series. Because Fourier series are used widely by engineers and scientists, a section on this important topic has been included in this chapter, in addition to several interesting applications.

Chapter 9: Parametric Equations and Polar Coordinates

Chapter 9 introduces parametric equations and polar coordinates. A large number of parametric graphs and applications are included, made more accessible by computer graphics and equation-solving capabilities. We use many applied examples within this chapter to help the conceptual flow of material, including using the amusement park ride the Scrambler as the foundation of a number of examples.

Chapter 10: Vectors and the Geometry of Space

Chapter 10 introduces students to a third dimension of graphing and calculations. Computer graphics are a valuable aid in this chapter, and are used extensively. A discussion of Magnus force relates vectors to a variety of sports applications, while providing students with practice at thinking in three-dimensional space.

Chapter 11: Vector-Valued Functions

Chapter 11 develops the calculus of vector-valued functions. As the graphs become more complicated, our use of computer graphics increases. To keep students thinking and not simply pushing buttons, several of the examples and exercises involve matching functions and graphs, with students using the properties of functions to identify the graphs. Section 11.5 includes the important derivation of Kepler's laws of planetary motion.

Chapter 12: Functions of Several Variables and Partial Differentiation

Chapter 12 presents the calculus of functions of two or more variables. Given the increasing difficulty of visualizing the mathematics in this chapter, the Rule of Four is particularly useful. We use a variety of graphics options (e.g., wireframe and parametric plots) in this chapter so that students can see the traces, and not lose the details that shaded graphs tend to obscure. Where appropriate, three-dimensional graphs are augmented with contour plots and density plots. Numerically, a steepest ascent (descent) algorithm is presented, requiring some computer assistance, but reinforcing several important concepts of the calculus of functions of several variables.

Chapter 13: Multiple Integrals

Chapter 13 introduces double and triple integrals. Considerable emphasis is placed on helping students develop insight into the proper coordinate system and order of integration to use to simplify a given multiple integral. Applications involving the design of rockets and baseball bats are used to enliven the discussion of moments and centers of mass.

Chapter 14: Vector Calculus

Chapter 14 introduces the vector calculus that is essential to an understanding of fluid mechanics and applications in electricity and magnetism. Numerous graphs of vector fields are included, as well as a thorough discussion of various interpretations of these graphs.

■ CHANGES FOR THE SECOND EDITION

Every section of the text and every pedagogical feature was carefully scrutinized during the development of this Second Edition. In preparing our revisions, we had the benefit of comments from a very large panel of reviewers, including both users and nonusers of the Premiere Edition of the text. Based on our analysis and the reviewers' comments, we have made countless changes for this edition of *Calculus*. These include:

Chapter 3: We have expanded section 3.1 by using linear approximation to provide an introduction to l'Hôpital's Rule for $\frac{0}{0}$ case. A full treatment of indeterminate forms and l'Hôpital's Rule is then found in section 7.6. As a result of this revision, we have moved Newton's method out of section 3.1, to a section of its own, section 3.2.

Chapter 4: We have included a discussion of error bounds for numerical integration rules in section 4.7.

Chapter 8: We have added a discussion of how to use an improper integral to bound the remainder for series to which the Integral Test applies, as well as a new section on applications of Taylor series (section 8.8). We have also added coverage of the Root Test to section 8.5.

Graphs and Tables: All graphs within this edition have been newly rendered to take advantage of the use of color in interpreting figures. This program has been painstakingly poured over to ensure accuracy and consistency throughout.

Examples: Many new examples have been included in this edition, particularly in places where reviewers suggested that additional explanation would be beneficial.

Exercises: Each exercise set was carefully reexamined to ensure that exercises of all levels were included. In particular, we significantly increased the number of exercises of moderate difficulty.

Presentation: All explanations, proofs and derivations have been carefully reconsidered, with countless revisions made to improve clarity.

Historical Notes: Biographical information about prominent mathematicians and their contributions to the development of calculus have been added to this edition. Several dozen mathematicians are profiled in marginal boxes spread throughout the text.

Technology: We have enhanced our discussions of technology in a number of places, including additional tables and functions specified by data.

Design and Color: The text has been completely redesigned to allow for the pedagogical use of color to highlight important results and remarks, as well as for visual emphasis in the graphs and tables.

ACKNOWLEDGMENTS

First and foremost, we want to express our appreciation to our sponsoring editor, Maggie Rogers for the encouragement and guidance to keep us going through this very long and challenging project. We also wish to thank our original editor, Jack Shira, for having the confidence in us to start the project. We must also thank our developmental editors, Michelle Munn and Kris Swanson for their hard work and constant support. We could never have completed this project without Maggie's, Michelle's and Kris' tireless efforts and professionalism. We could truly not think of better people with whom to work. We also wish to thank our publishers, Bill Barter, J. P. Lenney and Denise Schanck, for their strong support over the last six years.

A project of this magnitude requires the collaboration of an incredible number of talented people. Our developmental editor team for the Premiere Edition, Glenn and Meg Turner of Burrston House, ran the project through their demanding schedule of reviews and meetings. Our understanding of the project and our ability to create a quality book improved tremendously through their efforts. Our production team at McGraw-Hill, particularly our project manager, Jill Peter, kept the project on schedule and helped us to produce a well-designed text. The problem solvers at Laurel Technical Services, led by Carrie Mallery, and Elka Block and Frank Purcell of Twin Prime Editorial made numerous suggestions to

improve the exercise sets. The TEX experts at Interactive Composition Corporation produced the current book design from our original TEX files. George Morris and the staff of Scientific Illustrators created each figure from scratch for the Second Edition. Their diligence in following our design requirements and their skill are greatly appreciated.

A project of this scope could not have been completed without the many insightful comments from our reviewer panels, both past and present. During the development of the manuscript, we continually asked our reviewers for advice on the placement and treatment of everything from major concepts to figures. Thanks to their often unerring judgment, we have made numerous improvements to the text.

With many thanks to our previous edition reviewer panel:

David Anderson, *University of Tennessee*
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Robert T. Smith
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Roland B. Minton
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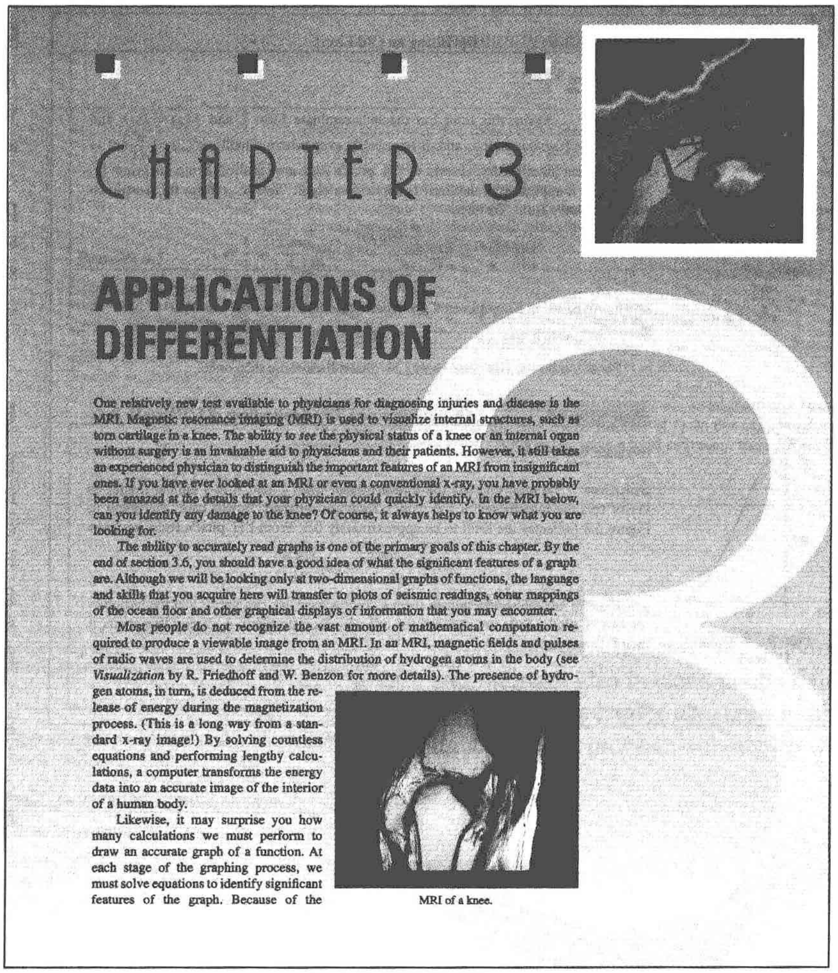
VISUAL GUIDE: EXPANDING UPON THE TEXT

TOOLS FOR LEARNING

Real-World Emphasis

Real-world examples are an important aid to the understanding of calculus. We introduce each chapter with a brief application related to the mathematical concepts being developed in order to put each chapter into a larger problem-solving context. Subsequently, both examples within the text and exercises are used to further demonstrate the importance of calculus within the world.



CHAPTER 3

APPLICATIONS OF DIFFERENTIATION

One relatively new test available to physicians for diagnosing injuries and disease is the MRI. Magnetic resonance imaging (MRI) is used to visualize internal structures, such as torn cartilage in a knee. The ability to *see* the physical status of a knee or an internal organ without surgery is an invaluable aid to physicians and their patients. However, it still takes an experienced physician to distinguish the important features of an MRI from insignificant ones. If you have ever looked at an MRI or even a conventional x-ray, you have probably been amazed at the details that your physician could quickly identify. In the MRI below, can you identify any damage to the knee? Of course, it always helps to know what you are looking for.

The ability to accurately read graphs is one of the primary goals of this chapter. By the end of section 3.6, you should have a good idea of what the significant features of a graph are. Although we will be looking only at two-dimensional graphs of functions, the language and skills that you acquire here will transfer to plots of seismic readings, sonar mappings of the ocean floor and other graphical displays of information that you may encounter.

Most people do not recognize the vast amount of mathematical computation required to produce a viewable image from an MRI. In an MRI, magnetic fields and pulses of radio waves are used to determine the distribution of hydrogen atoms in the body (see *Visualization* by R. Friedhoff and W. Benzon for more details). The presence of hydrogen atoms, in turn, is deduced from the release of energy during the magnetization process. (This is a long way from a standard x-ray image!) By solving countless equations and performing lengthy calculations, a computer transforms the energy data into an accurate image of the interior of a human body.

Likewise, it may surprise you how many calculations we must perform to draw an accurate graph of a function. At each stage of the graphing process, we must solve equations to identify significant features of the graph. Because of the

MRI of a knee.

Definitions, Theorems and Proofs

All formal definitions and theorems are clearly boxed within the text for easy visual reference. Selected proofs are provided for reference. Proofs of some results are found in Appendix A.

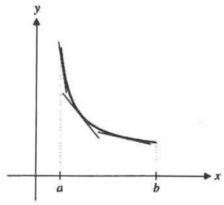


Figure 3.48a
Concave up.

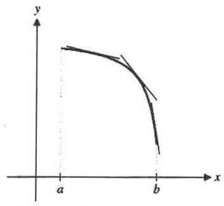


Figure 3.48b
Concave down.

increasing) and the one shown in Figure 3.48b is concave down (slopes of tangent lines decreasing). We have the following definition.

Definition 5.1
For a function f that is differentiable on an interval I, the graph of f is
(i) **concave up** on I, if f' is increasing on I or
(ii) **concave down** on I, if f' is decreasing on I.

How can you tell when f' is increasing or decreasing? The derivative of f' (i.e., f'') yields that information. The following theorem connects this definition with what we already know about increasing and decreasing functions. The proof of the theorem is a straightforward application of Theorem 4.1 to Definition 5.1.

Theorem 5.1
Suppose that f'' exists on an interval I.
(i) If $f''(x) > 0$ on I, then the graph of f is concave up on I.
(ii) If $f''(x) < 0$ on I, then the graph of f is concave down on I.

Example 5.1 **Determining Concavity**

Determine where the graph of $f(x) = 2x^3 + 9x^2 - 24x - 10$ is concave up and concave down and draw a graph showing all significant behavior of the function.

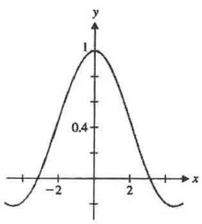


Figure 3.7
$y = \frac{\sin x}{x}$.

Example 1.5 **Revisiting an Old Limit**

Evaluate $\lim_{x\to 0} \frac{\sin x}{x}$.

Solution Again, this limit has the indeterminate form $\frac{0}{0}$ and $f(x) = \sin x$ and $g(x) = x$ are both continuous and differentiable everywhere. Finally, $g'(x) = \frac{d}{dx}(x) = 1 \neq 0$, so that all of the hypotheses of l'H pital's Rule are satisfied. From the graph in Figure 3.7, it appears that the limit is approximately 1. We can confirm this suspicion with l'H pital's Rule. We have

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} = \lim_{x\to 0} \frac{\cos x}{1} = \frac{1}{1} = 1,$$

as we proved using a complicated geometric argument in section 2.5. ■

For some limits, you must apply l'H pital's Rule more than once.

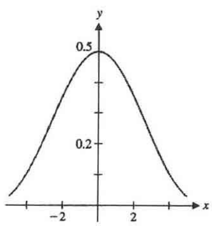


Figure 3.8
$y = \frac{1 - \cos x}{x^2}$.

Example 1.6 **A Limit Requiring Two Applications of L'Hôpital's Rule**

Evaluate $\lim_{x\to 0} \frac{1 - \cos x}{x^2}$.

Solution Again, this has the indeterminate form $\frac{0}{0}$ and it is a simple matter to verify that the hypotheses of l'H pital's Rule are satisfied. In this case, the graph in Figure 3.8 indicates the limit to be approximately 0.5. From l'H pital's Rule, we have

$$\lim_{x\to 0} \frac{1 - \cos x}{x^2} = \lim_{x\to 0} \frac{\sin x}{2x},$$

which again has the indeterminate form $\frac{0}{0}$. In this case, we can verify that the hypotheses of l'H pital's Rule are satisfied for this new limit problem. Applying this again, it then follows that

$$\lim_{x\to 0} \frac{1 - \cos x}{x^2} = \lim_{x\to 0} \frac{\sin x}{2x} = \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}.$$

■

Examples

Each chapter contains a large number of worked examples, ranging from the simple and concrete to more complex and abstract. A thorough understanding of the initial problem presented and the step-by-step solution will greatly enhance your problem-solving capabilities and your further study of the subject.

Use of Graphs and Tables

Being able to visualize a problem is an invaluable aid in understanding the concept presented. To this purpose, we have integrated more than 1500 computer-generated graphs throughout the text. You should use them routinely to aid in solving most problems, even if only as a check on the reasonableness of an answer. Each graph and table has been created very carefully to ensure that the ideas presented are clear and accurate. In many places, we have included multiple graphical perspectives, such as with the contour and density plots found in Chapter 12.