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Damiano Brigo
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Interest Rate Models – Theory and Practice

With Smile, Inflation and Credit

cond Edition

利率模型理论和实践

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Preface

"Professor Brigo, will there be any new quotes in the second edition?"

"Yes... for example this one!"

A student at a London training course, following a similar question by a Hong Kong student to Massimo Morini, 2003.

"I would have written you a shorter letter, but I didn't have the time"

Benjamin Franklin

MOTIVATION.... five years later.

...I'm sure he's got a perfectly good reason... for taking so long...

Emily, "Corpse Bride", Tim Burton (2005).

Welcome onboard the second edition of this book on interest rate models, to all old and new readers. We immediately say this second edition is actually almost a new book, with four hundred fifty and more new pages on smile modeling, calibration, inflation, credit derivatives and counterparty risk.

As explained in the preface of the first edition, the idea of writing this book on interest-rate modeling crossed our minds in early summer 1999. We both thought of different versions before, but it was in Banca IMI that this challenging project began materially, if not spiritually (more details are given in the trivia Appendix G). At the time we were given the task of studying and developing financial models for the pricing and hedging of a broad range of derivatives, and we were involved in medium/long-term projects.

The first years in Banca IMI saw us writing a lot of reports and material on our activity in the bank, to the point that much of those studies ended up in the first edition of the book, printed in 2001.

In the first edition preface we described motivation, explained what kind of theory and practice we were going to address, illustrated the aim and readership of the book, together with its structure and other considerations. We do so again now, clearly updating what we wrote in 2001.

Why a book on interest rate models, and why this new edition?

"Sorry I took so long to respond, Plastic Man. I'd like to formally declare my return to active duty, my friends... This is J'onn J'onzz activating full telepathic link. Counter offensive has begun". JLA 38, DC Comics (2000).

In years where every month a new book on financial modeling or on mathematical finance comes out, one of the first questions inevitably is: why one more, and why one on interest-rate modeling in particular?

The answer springs directly from our job experience as mathematicians working as quantitative analysts in financial institutions. Indeed, one of the major challenges any financial engineer has to cope with is the practical implementation of mathematical models for pricing derivative securities.

When pricing market financial products, one has to address a number of theoretical and practical issues that are often neglected in the classical, general basic theory: the choice of a satisfactory model, the derivation of specific analytical formulas and approximations, the calibration of the selected model to a set of market data, the implementation of efficient routines for speeding up the whole calibration procedure, and so on. In other words, the general understanding of the theoretical paradigms in which specific models operate does not lead to their complete understanding and immediate implementation and use for concrete pricing. This is an area that is rarely covered by books on mathematical finance.

Undoubtedly, there exist excellent books covering the basic theoretical paradigms, but they do not provide enough instructions and insights for tackling concrete pricing problems. We therefore thought of writing this book in order to cover this gap between theory and practice.

The first version of the book achieved this task in several respects. However, the market is rapidly evolving. New areas such as smile modeling, inflation, hybrid products, counterparty risk and credit derivatives have become fundamental in recent years. New bridges are required to cross the gap between theory and practice in these recent areas.

The Gap between Theory and Practice

But Lo! Siddârtha turned/ Eyes gleaming with divine tears to the sky,/ Eyes lit with heavenly pity to the earth;/ From sky to earth he looked, from earth to sky,/ As if his spirit sought in lonely flight/ Some far-off vision, linking this and that,/ Lost - past - but searchable, but seen, but known.

From "The Light of Asia", Sir Edwin Arnold (1879).

A gap, indeed. And a fundamental one. The interplay between theory and practice has proved to be an extremely fruitful ingredient in the progress of science and modeling in particular. We believe that practice can help to appreciate theory, thus generating a feedback that is one of the most important and intriguing aspects of modeling and more generally of scientific investigation.

If theory becomes deaf to the feedback of practice or vice versa, great opportunities can be missed. It may be a pity to restrict one's interest only to extremely abstract problems that have little relevance for those scientists or quantitative analysts working in "real life".

Now, it is obvious that everyone working in the field owes a lot to the basic fundamental theory from which such extremely abstract problems stem.

It would be foolish to deny the importance of a well developed and consistent theory as a fundamental support for any practical work involving mathematical models. Indeed, practice that is deaf to theory or that employs a sloppy mathematical apparatus is quite dangerous.

However, besides the extremely abstract refinement of the basic paradigms, which are certainly worth studying but that interest mostly an academic audience, there are other fundamental and more specific aspects of the theory that are often neglected in books and in the literature, and that interest a larger audience.

Is This Book about Theory? What kind of Theory?

"Our paper became a monograph. When we had completed the details, we rewrote everything so that no one could tell how we came upon our ideas or why. This is the standard in mathematics."

David Berlinski, "Black Mischief" (1988).

In the book, we are not dealing with the fundamental no-arbitrage paradigms with great detail. We resume and adopt the basic well-established theory of Harrison and Pliska, and avoid the debate on the several possible definitions of no-arbitrage and on their mutual relationships. Indeed, we will raise problems that can be faced in the basic framework above. Insisting on the subtle aspects and developments of no-arbitrage theory more than is necessary would take space from the other theory we need to address in the book and that is more important for our purposes.

Besides, there already exist several books dealing with the most abstract theory of no-arbitrage. On the theory that we deal with, on the contrary, there exist only few books, although in recent years the trend has been improving. What is this theory? For a flavor of it, let us select a few questions at random:

- How can the market interest-rate curves be defined in mathematical terms?
- What kind of interest rates does one select when writing the dynamics? Instantaneous spot rates? Forward rates? Forward swap rates?
- What is a sufficiently general framework for expressing no-arbitrage in interest-rate modeling?
- Are there payoffs that do not require the interest-rate curve dynamics to be valued? If so, what are these payoffs?
- Is there a definition of volatility (and of its term structures) in terms of interest-rate dynamics that is consistent with market practice?
- What kinds of diffusion coefficients in the rate dynamics are compatible with different qualitative evolutions of the term structure of volatilities over time?
- How is "humped volatility shape" translated in mathematical terms and what kind of mathematical models allow for it?

- What is the most convenient probability measure under which one can price a specific product, and how can one derive concretely the related interest-rate dynamics?
- Are different market models of interest-rate dynamics compatible?
- What does it mean to calibrate a model to the market in terms of the chosen mathematical model? Is this always possible? Or is there a degree of approximation involved?
- Does terminal correlation among rates depend on instantaneous volatilities or only on instantaneous correlations? Can we analyze this dependence?
- What is the volatility smile, how can it be expressed in terms of mathematical models and of forward-rate dynamics in particular?
- Is there a diffusion dynamics consistent with the quoting mechanism of the swaptions volatility smile in the market?
- What is the link between dynamics of rates and their distributions?
- What kind of model is more apt to model correlated interest-rate curves of different currencies, and how does one compute the related dynamics under the relevant probability measures?
- When does a model imply the Markov property for the short rate and why is this important?
- What is inflation and what is its link with classical interest-rate modeling?
- How does one calibrate an inflation model?
- Is the time of default of a counterparty predictable or not?
- Is it possible to value payoffs under an equivalent pricing measure in presence of default?
- Why are Poisson and Cox processes so suited to default modeling?
- What are the mathematical analogies between interest-rate models and credit-derivatives models? For what kind of mathematical models do these analogies stand?
- Does counterparty risk render a payoff dynamics-dependent even if without counterparty risk the payoff valuation is model-independent?
- What kind of mathematical models may account for possible jump features in the stochastic processes needed in credit spread modeling?
- Is there a general way to model dependence across default times, and across market variables more generally, going beyond linear correlation? What are the limits of these generalizations, in case?
-

We could go on for a while with questions of this kind. Our point is, however, that the theory dealt with in a book on interest-rate models should consider this kind of question.

We sympathize with anyone who has gone to a bookstore (or perhaps to a library) looking for answers to some of the above questions with little success. We have done the same, several times, and we were able to find only limited material and few reference works, although in the last few years the

situation has improved. We hope the second edition of this book will cement the steps forward taken with the first edition.

We also sympathize with the reader who has just finished his studies or with the academic who is trying a life-change to work in industry or who is considering some close cooperation with market participants. Being used to precise statements and rigorous theory, this person might find answers to the above questions expressed in contradictory or unclear mathematical language. This is something else we too have been through, and we are trying not to disappoint in this respect either.

Is This Book about Practice? What kind of Practice?

If we don't do the work, the words don't mean anything. Reading a book or listening to a talk isn't enough by itself.

Charlotte Joko Beck, "Nothing Special: Living Zen", Harper Collins, 1995.

We try to answer some questions on practice that are again overlooked in most of the existing books in mathematical finance, and on interest-rate models in particular. Again, here are some typical questions selected at random:

- What are accrual conventions and how do they impact on the definition of rates?
- Can you give a few examples of how time is measured in connection with some aspects of contracts? What are "day-count conventions"?
- What is the interpretation of most liquid market contracts such as caps and swaptions? What is their main purpose?
- What kind of data structures are observed in the market? Are all data equally significant?
- How is a specific model calibrated to market data in practice? Is a joint calibration to different market structures always possible or even desirable?
- What are the dangers of calibrating a model to data that are not equally important, or reliable, or updated with poor frequency?
- What are the requirements of a trader as far as a calibration results are concerned?
- How can one handle path-dependent or early-exercise products numerically? And products with both features simultaneously?
- What numerical methods can be used for implementing a model that is not analytically tractable? How are trees built for specific models? Can instantaneous correlation be a problem when building a tree in practice?
- What kind of products are suited to evaluation through Monte Carlo simulation? How can Monte Carlo simulation be applied in practice? Under which probability measure is it convenient to simulate? How can we reduce the variance of the simulation, especially in presence of default indicators?

- Is there a model flexible enough to be calibrated to the market smile for caps?
- How is the swaptions smile quoted? Is it possible to “arbitrage” the swaption smile against the cap smile?
- What typical qualitative shapes of the volatility term structure are observed in the market?
- What is the impact of the parameters of a chosen model on the market volatility structures that are relevant to the trader?
- What is the accuracy of analytical approximations derived for swaptions volatilities and terminal correlations?
- Is it possible to relate CMS convexity adjustments to swaption smiles?
- Does there exist an interest-rate model that can be considered “central” nowadays, in practice? What do traders think about it?
- How can we express mathematically the payoffs of some typical market products?
- How do you handle in practice products depending on more than one interest-rate curve at the same time?
- How do you calibrate an inflation model in practice, and to what quotes?
- What is the importance of stochastic volatility in inflation modeling?
- How can we handle hybrid structures? What are the key aspects to take into account?
- What are typical volatility sizes in the credit market? Are these sizes motivating different models?
- What’s the impact of interest-rate credit-spread correlation on the valuation of credit derivatives?
- Is counterparty risk impacting interest-rate payoffs in a relevant way?
- Are models with jumps easy to calibrate to credit spread data?
- Is there a way to imply correlation across default times of different names from market quotes? What models are more apt at doing so?
-

Again, we could go on for a while, and it is hard to find a single book answering these questions with a rigorous theoretical background. Also, answering some of these questions (and others that are similar in spirit) motivates new theoretical developments, maintaining the fundamental feedback between theory and practice we hinted at above.

AIMS, READERSHIP AND BOOK STRUCTURE

“And these people are sitting up there seriously discussing intelligent stars and trips through time to years that sound like telephone numbers. Why am I here?” Huntress/Helena Bertinelli, DC One Million (1999).

Contrary to what happens in other derivatives areas, interest-rate modeling is a branch of mathematical finance where no general model has been

yet accepted as “standard” for the whole sector, although the LIBOR market model is emerging as a possible candidate for this role. Indeed, there exist market standard models for both main interest-rate derivatives “sub-markets”, namely the caps and swaptions markets. However, such models are theoretically incompatible and cannot be used jointly to price other interest-rate derivatives.

Because of this lack of a standard, the choice of a model for pricing and hedging interest-rate derivatives has to be dealt with carefully. In this book, therefore, we do not just concentrate on a specific model leaving all implementation issues aside. We instead develop several types of models and show how to use them in practice for pricing a number of specific products.

The main models are illustrated in different aspects ranging from theoretical formulation to a possible implementation on a computer, always keeping in mind the concrete questions one has to cope with. We also stress that different models are suited to different situations and products, pointing out that there does not exist a single model that is uniformly better than all the others.

Thus our aim in writing this book is two-fold. First, we would like to help quantitative analysts and advanced traders handle interest-rate derivatives with a sound theoretical apparatus. We try explicitly to explain which models can be used in practice for some major concrete problems. Secondly, we would also like to help academics develop a feeling for the practical problems in the market that can be solved with the use of relatively advanced tools of mathematics and stochastic calculus in particular. Advanced undergraduate students, graduate students and researchers should benefit as well, from seeing how some sophisticated mathematics can be used in concrete financial problems.

The Prerequisites

The prerequisites are some basic knowledge of stochastic calculus and the theory of stochastic differential equations and Poisson processes in particular. The main tools from stochastic calculus are Ito’s formula, Girsanov’s theorem, and a few basic facts on Poisson processes, which are, however, briefly reviewed in Appendix C.

The Book is Structured in Eight Parts

The first part of the book reviews some basic concepts and definitions and briefly explains the fundamental theory of no-arbitrage and its implications as far as pricing derivatives is concerned.

In the second part the first models appear. We review some of the basic short-rate models, both one- and two-dimensional, and then hint at forward-rate models, introducing the so called Heath-Jarrow-Morton framework.

In the third part we introduce the “modern” models, the so-called market models, describing their distributional properties, discussing their analytical tractability and proposing numerical procedures for approximating the interest-rate dynamics and for testing analytical approximations. We will make extensive use of the “change-of-numeraire” technique, which is explained in detail in a initial section. This third part contains a lot of new material with respect to the earlier 2001 edition. In particular, the correlation study and the cascade calibration of the LIBOR market model have been considerably enriched, including the work leading to the Master’s and PhD theses of Massimo Morini.

The fourth part is largely new, and is entirely devoted to smile modeling, with a parade of models that are studied in detail and applied to the caps and swaptions markets.

The fifth part is devoted to concrete applications. We in fact list a series of market financial products that are usually traded over the counter and for which there exists no uniquely consolidated pricing model. We consider some typical interest-rate derivatives dividing them into two classes: i) derivatives depending on a single interest-rate curve; ii) derivatives depending on two interest-rate curves.

Part Six is new and we introduce and study inflation derivatives and related models to price them.

Part Seven is new as well and concerns credit derivatives and counterparty risk, and besides introducing the payoffs and the models we explain the analogies between credit models and interest-rate models.

Appendices

Part Eight regroups our appendices, where we have also moved the “other interest rate models” and the “equity payoffs under stochastic rates” sections, which were separate chapters in the first edition. We updated the appendix on stochastic calculus with Poisson processes and updated the “Talking to the Traders” appendix with conversations on the new parts of the book.

We also added an appendix with trivia and frequently asked questions such as “who’s who of the two authors”, “what does the cover represent”, “what about all these quotes” etc.

It is sometimes said that no one ever reads appendices. This book ends with eight appendices, and the last one is an interview with a quantitative trader, which should be interesting enough to convince the reader to have a look at the appendices, for a change.

FINAL WORD AND ACKNOWLEDGMENTS

Whether our treatment of the theory fulfills the targets we have set ourselves, is for the reader to judge. A disclaimer is necessary though. Assembling a

book in the middle of the “battlefield” that is any trading room, while quite stimulating, leaves little space for planned organization. Indeed, the book is not homogeneous, some topics are more developed than others.

We have tried to follow a logical path in assembling the final manuscript, but we are aware that the book is not optimal in respect of homogeneity and linearity of exposition. Hopefully, the explicit contribution of our work will emerge over these inevitable little misalignments.

Acknowledgments

A book is always the product not only of its authors, but also of their colleagues, of the environment where the authors work, of the encouragements and critique gathered from conferences, referee reports for journal publications, conversations after seminars, university lectures, training courses, summer and winter schools, e-mail correspondence, and many analogous events. While we cannot do justice to all the above, we thank explicitly our recently acquired colleagues Andrea “Fifty levels of backtrack and I’m not from Vulcan” Pallavicini, who joined us in the last year with both analytical and numerical impressive skills, and Roberto “market-and-modeling-super-speed” Torresetti, one of the founders of the financial engineering department, who came back after a tour through Chicago and London, enhancing our activity with market understanding and immediate and eclectic grasp of modeling issues.

Some of the most important contributions, physically included in this book, especially this second edition, come from the “next generation” of quants and PhD students. Here is a roll call:

- Aurélien Alfonsi (PhD in Paris and Banca IMI trainee, Credit Derivatives with Damiano);
- Cristina Capitani (Banca IMI trainee, LIBOR model calibration, with Damiano);
- Laurent Cousot (PhD student in NY and Banca IMI trainee, Credit Derivatives with Damiano);
- Naoufel El-Bachir (PhD student in Reading and Banca IMI trainee, Credit Derivatives with Damiano);
- Eymen Errais (PhD student at Stanford and Banca IMI trainee, Credit Derivatives with Damiano and Smile Modeling with Fabio);
- Jan Liinev (PhD in Ghent, LIBOR / Swap models distance with differential geometric methods, with Damiano);
- Dmitri Lvov (PhD in Reading and Banca IMI trainee, Bermudan Swaption Pricing and Hedging with the LFM, with Fabio);
- Massimo Masetti (PhD in Bergamo, currently working for a major bank in London, Counterparty Risk and Credit Derivatives with Damiano);
- Nicola Moreni (PhD in Paris, and Banca IMI trainee, currently our colleague, Inflation Modeling with Fabio);

- Giulio Sartorelli (PhD in Pisa, Banca IMI trainee and currently our colleague, Short-Rate and Smile Modeling with Fabio);
- Marco Tarengi (Banca IMI trainee and our former colleague, Credit derivatives and counterparty risk with Damiano);

Special mention is due to Massimo Morini, Damiano's PhD student in Milan, who almost learned the first edition by heart. His copy is the most battered and travel-worn we have ever seen; Massimo is virtually a co-author of this second edition, having contributed largely to the new parts of Chapters 6 and 7, and having developed recent and promising results on smile calibration and credit derivatives market models that we have not been in time to include here. Massimo also taught lectures and training courses based on the book all around the world, helping us whenever we were too busy to travel abroad.

As before we are grateful to our colleagues Gianvittorio "Tree and Optimization Master" Mauri and Francesco "Monte Carlo" Rapisarda, for their help and continuous interaction concerning both modeling and concrete implementations on computers. Francesco also helped by proofreading the manuscript of the first edition and by suggesting modifications.

The feedback from the trading desks (interest-rate-derivatives and credit derivatives) has been fundamental, first in the figures of Antonio Castagna and Luca Mengoni (first edition) and then Andrea Curotti (now back in London), Stefano De Nuccio (now with our competitors), Luca Dominici, Roberto Paoletti, and Federico Veronesi. They have stimulated many developments with their objections, requirements and discussions. Their feeling for market behavior has guided us in cases where mere mathematics and textbook finance could not help us that much. Antonio has also helped us with stimulating discussions on inflation modeling and general pricing and hedging issues. As before, this book has been made possible also by the farsightedness of our head Aleardo Adotti, who allowed us to work on the frontiers of mathematical finance inside a bank.

Hundreds of e-mails in the last years have reached us, suggesting improvements, asking questions, and pointing out errors. Again, it would be impossible to thank all single readers who contacted us, so we say here a big collective "Thank-you" to all our past readers. All mistakes that are left are again, needless to say, ours.

It has been tough to remain mentally sane in these last years, especially when completing this almost-one-thousand pages book. So the list of "external" acknowledgments has lengthened since the last time.

Damiano is grateful to the next generation above, in particular to the ones he is working/has worked with most: his modeling colleagues Andrea and Roberto, and then Aurélien, Eymen, Jan and Massimo Masetti (who held the "fortress" all alone in a difficult moment); Marco and Massimo Morini are gratefully mentioned also for the Boston MIT -Miami-Key West-Cape Canaveral "tournee" of late 2004. Umberto Cherubini has been lots

of fun with the “Japanese experiences” of 1999-2004 and many professional suggestions. Gratitude goes also to Suzuki “Freccetta” SV650, to the Lake of Como (Lario) and the Dolomites (Dolomiti), to Venice, Damiano’s birthplace, a dream still going after all these years, to the Venice carnival for the tons of fun with the “Difensori della Terra” costume players, including Fabrizio “Spidey”, Roberto “Cap”, Roberto “Ben” and Graziano “Thor” among many others; to Diego and Bojana, they know why, to Chiara and Marco Salcoacci (and the newly arrived Carlo!), possibly the nicest persons on Earth, to Lucia and Massimo (Hayao Miyazaki is the greatest!), and to the many on-line young friends at ComicUS and DCForum. Damiano’s gratitude goes finally to his young fiancée, who in the best tradition of comic-books and being quite shy asked to maintain a secret identity here, and especially to his whole family past and present, for continued affection, support and encouragement, in particular to Annamaria, Francesco, Paolo, Dina and Mino.

Fabio is grateful to his colleagues Gianvittorio Mauri, Andrea Pallavicini, Francesco Rapisarda and Giulio Sartorelli for their invaluable contribution in the modeling, pricing and hedging of the bank’s derivatives. Their skilled efficiency has allowed (and still allows) him to devote himself also to more speculative matters. Special thanks then go to his friends, and especially to the “ammiragliato” (admiralty) group, for the fun they have planning their missions around a table in “trattorie” near Treviso, to Antonio, Jacopo and Raffaele for the great time they spend together in Milan and travelling all over the world, to Chiara and Eleonora for their precious advices and sincere affection, to his pastoral friends for their spiritual support, and last, but not least, to his family for continued affection and support.

Finally, our ultimate gratitude is towards transcendence and is always impossible to express with words. We just say that we are grateful for the Word of the Gospel and the Silence of Zen.

A Special Final Word for Young Readers and Beginners

It looked insanely complicated, and this was one of the reasons why the snug plastic cover it fitted into had the words “Don’t Panic” printed on it in large friendly letters.
Douglas Adams (1952 - 2001).

We close this long preface with a particular thought and encouragement for young readers. Clearly, if you are a professional or academic experienced in interest-rate modeling, we believe you will not be scared by a first quick look at the table of contents and at the chapters.

However, even at a first glance when flipping through the book, some young readers might feel discouraged by the variety of models, by the difference in approaches, by the book size, and might indeed acquire the impression of a chaotic sequence of models that arose in mathematical finance without a particular order or purpose. Yet, we assure you that this subject is interesting, relevant, and that it can (and should) be fun, however “clichéd” this

XVIII Preface

may sound to you. We have tried at times to be colloquial in the book, in an attempt to avoid writing a book on formal mathematical finance from A to Zzzzzzzzzzz... (where have you heard this one before?).

We are trying to avoid the two apparent extremes of either scaring or boring our readers. Thus you will find at times opinions from market participants, guided tours, intuition and discussion on things as they are seen in the market. We would like you to give it at least a try. So, if you are one of the above young readers, and be you a student or a practitioner, we suggest you take it easy. This book might be able to help you a little in entering this exciting field of research. This is why we close this preface with the by-now classic recommendations...

.. a brief hiss of air as the green plasma seals around him and begins to photosynthesize oxygen, and then the dead silence of space. A silence as big as everything. [...] Cool green plasma flows over his skin, maintaining his temperature, siphoning off sweat, monitoring muscle tone, repelling micro-meteorites. He thinks green thoughts. And his thoughts become things. Working the ring is like giving up cigarettes.

He feels like a "sixty-a-day" man.

Grant Morrison on Green Lantern (Kyle Rayner)'s ring, *JLA*, 1997

"May fear and dread not conquer me". Majjhima Nikaya VIII.6

"Do not let your hearts be troubled and do not be afraid". St. John XIV.27

Martian manhunter: *"...All is lost..."*

Batman: *"I don't believe that for a second. What should I expect to feel?"*

M: *"Despair. Cosmic despair. Telepathic contact with Superman is only possible through the Mageddon mind-field that holds him in thrall. It broadcasts on the lowest psychic frequencies...horror...shame...fear...anger..."*

B: *"Okay, okay. Despair is fine. I can handle despair and so can you."*

Grant Morrison, *JLA: World War Three*, 2000, DC Comics.

"Non abbiate paura!" [Don't be afraid!]. Karol Wojtyla (1920- 2005)

"For a moment I was afraid." "For no reason".

Irma [Kati Outinen] and M [Markku Peltola], *"The Man without a Past"*, Aki Kaurismaki (2002).

Venice and Milan, May 4, 2006

Damiano Brigo and Fabio Mercurio

DESCRIPTION OF CONTENTS BY CHAPTER

We herewith provide a detailed description of the contents of each chapter, highlighting the updates for the new edition.

Part I: BASIC DEFINITIONS AND NO ARBITRAGE

Chapter 1: Definitions and Notation. The chapter is devoted to standard definitions and concepts in the interest-rate world, mainly from a static point of view. We define several interest-rate curves, such as the LIBOR, swap, forward-LIBOR and forward-swap curves, and the zero-coupon curve.

We explain the different possible choices of rates in the market. Some fundamental products, whose evaluation depends only on the initially given curves and not on volatilities, such as bonds and interest-rate swaps, are introduced. A quick and informal account of fundamental derivatives depending on volatility such as caps and swaptions is also presented, mainly for motivating the following developments.

Chapter 2: No-Arbitrage Pricing and Numeraire Change. The chapter introduces the theoretical issues a model should deal with, namely the no-arbitrage condition and the change of numeraire technique. The change of numeraire is reviewed as a general and powerful theoretical tool that can be used in several situations, and indeed will often be used in the book.

We remark how the standard Black models for either the cap or swaption markets, the two main markets of interest-rate derivatives, can be given a rigorous interpretation via suitable numeraires, as we will do later on in Chapter 6.

We finally hint at products involving more than one interest-rate curve at the same time, typically quanto-like products, and illustrate the no-arbitrage condition in this case.

Part II: FROM SHORT RATE MODELS TO HJM

Chapter 3: One-Factor Short-Rate Models. In this chapter, we begin to consider the dynamics of interest rates. The chapter is devoted to the short-rate world. In this context, one models the instantaneous spot interest rate via a possibly multi-dimensional driving diffusion process depending on some parameters. The whole yield-curve evolution is then characterized by the driving diffusion.

If the diffusion is one-dimensional, with this approach one is directly modeling the short rate, and the model is said to be “one-factor”. In this chapter, we focus on such models, leaving the development of the multi-dimensional (two-dimensional in particular) case to the next chapter.

As far as the dynamics of one-factor models is concerned, we observe the following. Since the short rate represents at each instant the initial point

of the yield curve, one-factor short-rate models assume the evolution of the whole yield curve to be completely determined by the evolution of its initial point. This is clearly a dangerous assumption, especially when pricing products depending on the correlation between different rates of the yield curve at a certain time (this limitation is explicitly pointed out in the guided tour of the subsequent chapter).

We then illustrate the no-arbitrage condition for one-factor models and the fundamental notion of market price of risk connecting the objective world, where rates are observed, and the risk-neutral world, where expectations leading to prices occur. We also show how choosing particular forms for the market price of risk can lead to models to which one can apply both econometric techniques (in the objective world) and calibration to market prices (risk-neutral world). We briefly hint at this kind of approach and subsequently leave the econometric part, focusing on the market calibration.

A short-rate model is usually calibrated to some initial structures in the market, typically the initial yield curve, the caps volatility surface, the swaptions volatility surface, and possibly other products, thus determining the model parameters. We introduce the historical one-factor time-homogeneous models of Vasicek, Cox Ingersoll Ross (CIR), Dothan, and the Exponential Vasicek (EV) model. We hint at the fact that such models used to be calibrated only to the initial yield curve, without taking into account market volatility structures, and that the calibration can be very poor in many situations.

We then move to extensions of the above one-factor models to models including “time-varying coefficients”, or described by inhomogeneous diffusions. In such a case, calibration to the initial yield curve can be made perfect, and the remaining model parameters can be used to calibrate the volatility structures. We examine classic one-factor extensions of this kind such as Hull and White’s extended Vasicek (HW) model, classic extensions of the CIR model, Black and Karasinski’s (BK) extended EV model and a few more.

We discuss the volatility structures that are relevant in the market and explain how they are related to short-rate models. We discuss the issue of a humped volatility structure for short-rate models and give the relevant definitions. We also present the Mercurio-Moraleda short-rate model, which allows for a parametric humped-volatility structure while exactly calibrating the initial yield curve, and briefly hint at the Moraleda-Vorst model.

We then present a method of ours for extending pre-existing time-homogeneous models to models that perfectly calibrate the initial yield curve while keeping free parameters for calibrating volatility structures. Our method preserves the possible analytical tractability of the basic model. Our extension is shown to be equivalent to HW for the Vasicek model, whereas it is original in case of the CIR model. We call CIR++ the CIR model being extended through our procedure. This model will play an important role in the final part of the book devoted to credit derivatives, in the light of the