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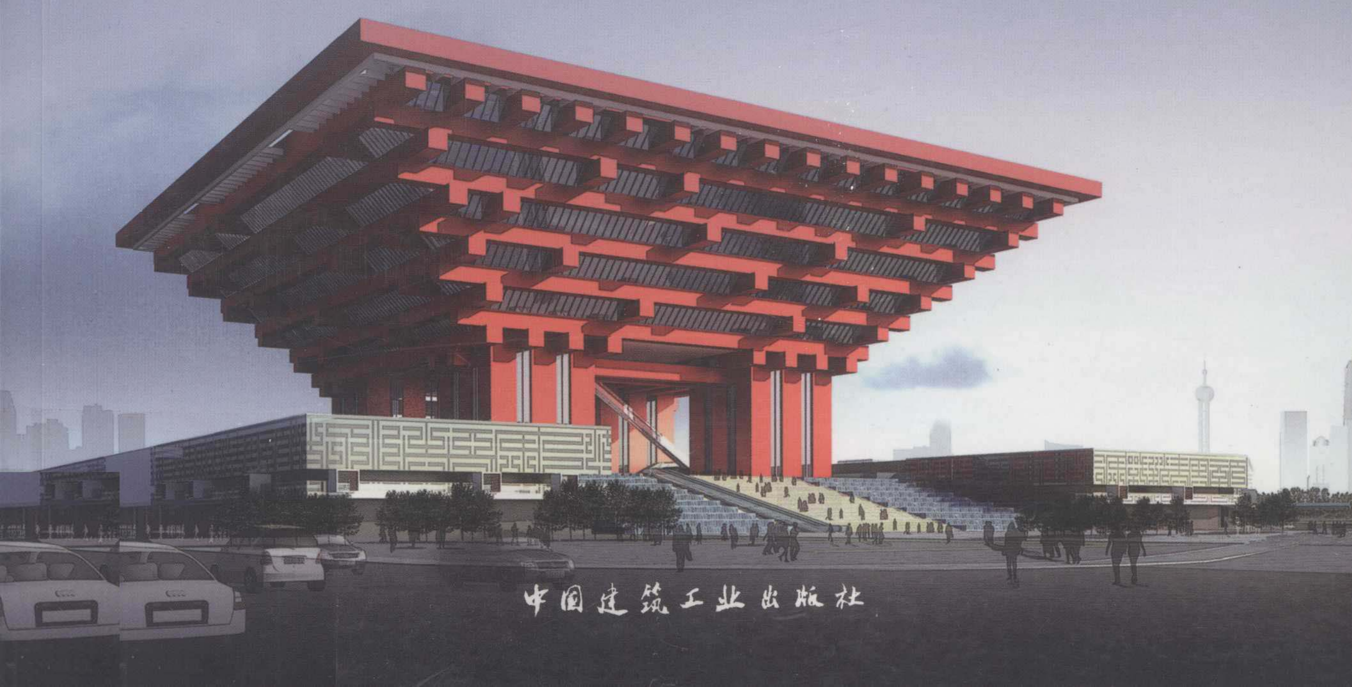
第三届结构工程新进展国际论坛论文集

Proceedings of the Third International Forum on Advances in Structural Engineering

2009 中国·上海

李国强 陆 烨 李元齐 主编

Editors in Chief: LI Guoqiang ,LU Ye & LI Yuanqi



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I 钢结构计算理论与计算方法

Direct Strength Method Design of Welded I-Section Beam-columns with Slender Web

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Abstract: The Direct Strength Method (DSM) of design, which is an alternative to the Effective Width Method (EWM) used for cold-formed steel structural design, has recently been adopted by some design codes (NAS, 2004 Supplement; AS/NZS4600, 2003) for the design of cold- formed steel structural members. The DSM as used in the NAS and AS/NZS 4600 has been limited to cold-formed beams and columns. Till now the method has not been applied to hot-rolled and welded steel structural members. Furthermore, even for beam-columns of cold-formed sections there exist no DSM formulas either for in-plane stability or for out-of-plane stability. Welded I-section beam-columns with slender web are designed by the EWD to the current Chinese Standard GB 50017—2003 (Code for Design of Steel Structures) . The DSM is much simpler to apply than the EWM as it uses gross section properties rather than effective section properties as commonly used in EWM. Due to this simplicity to use, the question naturally arises: if the DSM can also be used to such members. In this paper this possibility is preliminarily explored for such sections both for in-plane and out-of-plane stability and it is concluded that based on the GB 50017—2003, the DSM was found to give rather good design accuracy compared with the current EWM for most cases discussed and it is much easier to implement, therefore a set of design proposals based on the GB 50017—2003 is put forward.

Keywords: Direct Strength Method, Effective Width Method, beam-column, slender web

1 Introduction

The Direct Strength Method (DSM) of design, which is an alternative to the Effective Width Method (EWM) used for over 50 years for cold-formed steel structural design, has recently been adopted by the North American Specification (NAS) (2004 Supplement) (AISI, 2004) for the Design of Cold-Formed Steel Structural Members and the draft Australian/New Zealand Standard for Cold-Formed Steel Structures AS/NZS4600 (2003)

(Hancock et al., 2005). The DSM as used in the NAS and AS/NZS 4600 has been limited at this stage to cold-formed beams and columns. The method has not been applied to hot-rolled sections and those designed to the hot-rolled standards. The DSM is much simpler to apply than the EWM as it uses gross section properties rather than effective section properties as commonly used in EWM. Due to this simplicity to use, the question naturally arises if the DSM can also be used to hot-rolled sections and similar sections design to the hot-rolled standards. Hancock et al. (2005) explored this possibility preliminarily and concluded that based on the AS 4100: 1998, the DSM was found to give better design accuracy than the current EWM for most cases discussed.

Welded I-section beam-columns with slender web are designed with the EWD to the current Chinese Standards GB 50017—2003 (Code for Design of Steel Structures) (2003) and CECS102: 2002 (Technical Specification for Steel Structure of light-weight Buildings with gabled Frames) (2003). In this paper, the DSM is applied to such members and corresponding design proposals are put forward. As prescribed in the Chinese Standard GB 50017—2003, the stability design of beam-columns composes of two aspects, namely, in-plane and out-of-plane stability, in this paper both aspects are discussed.

2 DSM for In-plane Stability

Based on the DSM in AISI (2004) and the design formula for checking the in-plane stability of beam-columns in GB 50017—2003 (2003) the following DSM procedure is proposed for the in-plane stability check of welded beam-columns with slender web:

$$\frac{N}{P_{nl}}+\frac{\beta_{mx}M_x}{\alpha_e\gamma_x W_x f_y\left(1-0.8\frac{N}{N'_{Ex}}\right)}\leqslant 1.0 \tag{1}$$

P_{nl} is calculated as following (AISI, 2004):

if $\lambda_{nl}\leqslant 0.776$, then

$$P_{nl}=P_{ne} \tag{2a}$$

if $\lambda_{nl}>0.776$, then

$$P_{nl}=\left[1-0.15\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}P_{ne} \tag{2b}$$

with

$$\lambda_{nl}=\sqrt{P_{ne}/P_{crl}} \tag{3}$$

$$P_{ne}=f_y\varphi_x A \tag{4}$$

$$P_{crl}=\sigma_{crl}A \tag{5}$$

Here σ_{crl} is the local buckling stress of the section under concentrated compression, obtained with the finite strip software CUFSM 2.6b developed by Schafer (2003); α_e is a reduction coefficient to the section modulus accounting for the local buckling of the web under pure bending, calculated as following (GB 50017—2003, 2003):

$$\alpha_e = 1 - \frac{(1-\rho)h_c^3 t_w}{2I_x} \tag{6}$$

in which, I_x is the second moment of area of the section about axis x (the strong axis of the section) ; h_c the height of the compression zone of the web; t_w the thickness of the web; ρ is the effective section coefficient for the web plate under pure bending, calculated as:

$$\begin{aligned} &\rho = 1.0 && (\lambda_b \leqslant 0.85) \\ &\rho = 1 - 0.82(\lambda_b - 0.85) && (0.85 < \lambda_b \leqslant 1.25) \\ &\rho = (1 - 0.2/\lambda_b)/\lambda_b && (\lambda_b > 1.25) \end{aligned} \tag{7}$$

with

$$\lambda_b = \frac{h_w/t_w}{153}\sqrt{\frac{f_y}{235}} \tag{8}$$

A, W_x are the section area and section modulus, respectively; N is the concentrated compression and M_x the maximum bending moment on the column, respectively. β_{mx} is the in-plane equivalent moment coefficient, γ_x is the plastic adaptation coefficient, φ_x is in-plane stability coefficient for columns under concentrated compression, f_y is the yielding strength of steel, N'_{Ex} is modified Euler buckling load of the column defined as:

$$N'_{Ex} = \pi^2 EA/(1.1\lambda_x^2) \tag{9}$$

with λ_x being the in-plane slenderness of the column, E being the Young' s modulus of steel.

3 DSM for Out-of-plane Stability

Similarly, based on the DSM in AISI (2004) and the design formula for checking the out-of-plane stability of beam-columns in GB 50017—2003 (2003), the following DSM procedure is proposed for the out-of-plane stability check of welded beam-columns with slender web:

$$\frac{N}{P_{nl}} + \frac{\beta_{tx}M_x}{M_{nl}} \leqslant 1.0 \tag{10}$$

P_{nl} is calculated as in Sec. 2, but equation (4) is now about the weak axis y:

$$P_{ne} = f_y \varphi_y A \tag{11}$$

M_{nl} is calculated as following (AISI, 2004):

if $\lambda_{nl} \leqslant 0.776$, then

$$M_{nl} = M_{ne} \tag{12a}$$

if $\lambda_{nl} > 0.776$, then

$$M_{nl} = \left[1 - 0.15\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} M_{ne} \tag{12b}$$

with

$$\lambda_{ml} = \sqrt{M_{ne} / M_{crl}} \tag{13}$$

$$M_{ne} = f_y \varphi_b W_x \tag{14}$$

$$M_{crl} = \sigma_{mcrl} W_x \tag{15}$$

Here σ_{mcrl} is the local buckling stress of the section under pure bending, as in Sec. 2 obtained with the finite strip software CUFSM 2. 6b developed by Schafer (2003); A, W_x are the section area and section modulus, respectively; β_{tx} is the out-of-plane equivalent moment coefficient, φ_y is out-of-plane stability coefficient for columns under concentrated compression, φ_b is stability coefficient for beams under pure bending.

4 Comparisons with Tests

4.1 In-plane Stability

Gu and Chen (1997) conducted a series of nine specimens of welded I-section beam-columns with slender web to investigate their in-plane stability behavior. The test data of the specimens are given in table 1. In the table h_w, t_w, b, t represent the web depth, web thickness, flange width and flange thickness respectively, they are all in millimeter; v_0, w_0 denote the maximum initial in-plane member imperfections and maximum initial out-of-flatness of the web plate, respectively, all measured in millimeter; $\varepsilon = eA/W_x$ is the relative loading eccentricity, through which bending moment is applied; P_u is the tested ultimate in-plane load bearing capacity. The specimens are made of Q235 steel, with tested web plate yield point of 238. 6MPa and flange yield point of 274. 5MPa.

Test data of Gu and Chen (1997) **Table 1**

Specimen	h_w	t_w	b	t	λ_x	ε	v_0	w_0	P_u/ kN
1-1	180	2. 5	120	5	70. 7	3. 07	4. 0	1. 50	96. 04
1-2	180	2. 5	120	5	70. 5	1. 23	0. 0	3. 07	165. 62
2-1	210	2. 5	120	5	61. 4	0. 91	−1. 0	1. 90	207. 76
2-2	210	2. 5	120	5	61. 3	2. 48	1. 0	2. 45	121. 03
3-1	240	2. 5	120	5	54. 9	0. 97	3. 0	1. 80	203. 84
3-2	240	2. 5	120	5	54. 3	2. 35	4. 0	3. 12	121. 52
4-1	270	2. 5	120	5	49. 2	2. 28	-7. 5	2. 87	129. 36
4-2	270	2. 5	120	5	49. 1	0. 76	3. 5	2. 20	231. 28
5-1	300	2. 5	120	5	44. 7	2. 07	2. 0	3. 07	147. 00

Given in table 2 are the ultimate in-plane loads bearing capacities in kilo-Newton calculated with two current Chinese codes and with the proposed method above. P_{20tw} denotes the calculated ultimate load by the method in GB 50017—2003, P_e is the calculated value by the method in CECS102: 2002, both methods are EWM. P_d is the calculated ultimate load by the above-proposed DSM. From comparisons in table 2 it is evident that the proposed DSM can rather accurately predict the in-plane stability capacity of I-section beam-columns with slender web.

Comparison with test results of Gu and Chen (1997) **Table 2**

Specimen	P_u	P_e	P_{20tw}	P_d	P_u/P_e	P_u/P_{20tw}	P_u/P_d
1-1	96.04	97.79	90.35	102.66	0.99	1.07	0.94
1-2	165.62	157.83	145.85	170.41	1.05	1.14	0.97
2-1	207.76	186.95	178.76	193.15	1.11	1.16	1.08
2-2	121.03	114.66	110.49	120.00	1.06	1.10	1.01
3-1	203.84	195.02	185.45	189.77	1.04	1.10	1.07
3-2	121.52	125.63	120.68	126.06	0.97	1.01	0.96
4-1	129.36	134.5	127.93	129.68	0.96	1.01	1.00
4-2	231.28	225.46	212.21	204.19	1.03	1.09	1.13
5-1	147.00	149.16	140.39	137.90	0.99	1.05	1.07

4.2 Out-of-plane Stability

Yang and Chen (1998) conducted a series of eight specimens of welded I-section beam-columns with slender web to investigate their out-of-plane stability behavior. The test data of the specimens are shown in table 3. In the table h_w, t_w, b, t are the same as in table 1; u_0, w_0 denote the maximum initial out-of-plane member imperfections and maximum initial out-of-flatness of the web plate, respectively, all measured in millimeter; e is the loading eccentricity in mm, through which bending moment is applied; P_u is the tested ultimate out-of-plane load bearing capacity. The specimens are made of Q235 steel, with tested web plate yield point of 179.4MPa and flange yield point of 292.6MPa.

Test data of Yang and Chen (1998) **Table 3**

Specimen	h_w	t_w	b	t	λ_x	λ_y	e	u_0	w_0	P_u/ kN
LC70A	140	2	100	4	21.4	50.2	26.4	0.0	0.20	201.6
LC80A	160	2	100	4	23.3	63.3	27.0	0.0	0.25	195.0
LC90A	180	2	100	4	24.7	76.8	70.6	1.0	0.28	112.9
LC90C	180	2	100	4	24.7	76.8	49.6	1.0	0.50	172.5
LC100A	200	2	100	4	25.8	90.7	65.5	1.0	1.08	92.8
LC100C	200	2	100	4	25.8	90.7	65.0	1.5	1.33	122.3
LC120A1	240	2	100	4	24.8	107.0	109.4	0.0	2.35	75.5
LC120A2	240	2	100	4	26.7	115.8	149.6	2.0	2.30	70.0

Shown in table 4 are the ultimate out-of-plane loads bearing capacities in kilo-Newton calculated with two current Chinese codes and with the proposed method above. P_{20tw}, P_e denote similar loads as in table 2. P_d is the calculated ultimate load by the above-proposed DSM. From comparisons in table 4 it is also evident that the proposed DSM can rather accurately predict the out-of-plane stability capacity of I-section beam-columns with slender web.

Comparison with test results of Yang and Chen (1998) **Table 4**

Specimen	P_u	P_d	P_e	P_{20tw}	P_u/P_d	P_u/P_e	P_u/P_{20tw}
LC70A	201.6	162.6	177.1	168.3	1.24	1.14	1.20
LC80A	195.0	149.4	165.0	157.1	1.31	1.18	1.24
LC90A	112.9	106.6	118.7	111.1	1.06	0.95	1.02
LC90C	172.5	131.2	133.0	137.5	1.31	1.30	1.25
LC100A	92.8	100.7	110.9	104.5	0.92	0.84	0.89
LC100C	122.3	112.3	112.3	116.6	1.09	1.09	1.05
LC120A1	75.5	78.5	91.0	81.7	0.96	0.83	0.92
LC120A2	70.0	65.2	73.6	67.4	1.07	0.95	1.04

5 Conclusions

A Direct Strength Method (DSM) is proposed for the stability design of welded I-section beam-columns with slender web, which are currently treated with the traditional Effective Width Method (EWM). Both in-plane and out-of-plane stability are dealt with. The column and beam design curves are those in the Chinese Steel Structures Standard GB 50017—2003, while the local buckling loads of the complete section under pure compression and bending are calculated using a finite strip buckling analysis software. From the comparisons with test results and the methods in the two Chinese codes GB 50017—2003 and CECS102: 2002 it is found that the proposed DSM can fairly well predict the ultimate stability load bearing capacity of such members. However, before practical application more studies to confirm the proposals are needed and practical methods for computing the local buckling loads of the whole section also need be put forward.

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薄壁构件弯扭屈曲总势能方程的若干问题分析*

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摘　要：介绍了推导薄壁构件弯扭屈曲总势能方程的“传统理论”和“新型理论”，指出了“传统理论”中的应变能实则为线性正应变能与线性剪应变能之和，而外荷载势能的负值则为非线性正应变能，在“传统理论”中同时存在应变能与外荷载势能是不严密的。回顾了采用“新型理论”推导薄壁构件弯扭屈曲总势能方程存在的若干争议，分析了 Власов В. З. 教授提出的薄壁构件“刚周边假定”和“中面剪应变为零假定”的含义，并指出部分学者在错误应用这两条假定推导薄壁构件弯扭屈曲总势能方程时出现的若干问题，为建立较为严密的薄壁构件弯扭屈曲总势能方程的推导过程提供参考。

关键词：薄壁构件；总势能方程；线性应变能；非线性应变能
中图分类号：TU391

Analysis on some problems in total potential energy equation of flexural-torsional buckling of thin-walled members

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Abstract：Two different theories in conducting total potential energy equation of flexural-torsional buckling of thin-walled members are introduced，the strain energy of “traditional

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theory" is the summation of linear strain energy and linear shear strain energy and the negative potential energy of force is non-linear strain energy is pointed out, it is not closely that both strain energy and potential energy of force are included in "traditional theory". Some controversies of "new theory" in conducting total potential energy equation of flexural-torsional buckling of thin-walled member are recalled, the meanings of two widely accepted hypotheses that the shape of the cross-section remains unchanged and the shear strain can be omitted on mid surface of the cross-section when the member undergoes its flexural-torsional deformations that first proposed by Власов В. З. is analyzed, and some wrong applications of the two hypotheses in conducting total potential energy equation are pointed out, it is benefit for conducting a closely total potential energy equation of flexural-torsional buckling of thin-walled members.

Keywords: thin-walled members, total potential energy equation, linear strain energy, non-linear strain energy

1 引言

总势能驻值原理是解算薄壁构件弯扭屈曲荷载的有效方法之一，其关键是推导准确的薄壁构件弯扭屈曲总势能方程。Tong Gengshu（童根树）等[1]指出目前存在两种具有代表性的建立薄壁构件弯扭屈曲总势能方程的理论："传统理论"和"新型理论"。"传统理论"自 F. Bleich[2]于 1952 年建立后，已被广大科研人员所采用[3~5]，但由于其未考虑非线性剪切应变能，存在着难以克服的理论缺陷[6]。"新型理论"由吕烈武等[7]于 1982 年提出，该理论采用非线性力学的观点，由于推导过程较前者较为严密，因此越来越多地被人们所接受[1]。Gengshu Tong（童根树）等[8]在"新型理论"的基础上于 2003 年通过引入横向正应力给出了考虑横向正应力的薄壁构件弯扭屈曲总势能方程，但方山峰[9]从弹性力学、材料力学和薄壁杆件的基本概念出发，认为童根树提出的横向正应力的非线性应变能实则为横向外荷载和等效横向外荷载的势能，并给出了证明。

鉴于"传统理论"和"新型理论"在推导薄壁构件弯扭屈曲总势能方程上存在着较大差异，本文试图对两种理论分别进行分析，指出每种理论中存在的若干问题，为进一步建立准确的薄壁构件弯扭屈曲总势能方程提供参考。

2 两种基本理论

2.1 传统理论

"传统理论"认为薄壁构件弯扭屈曲的总势能[2]为：

$$\Pi_1 = U + V \tag{1}$$

式中，Π_1 为薄壁构件弯扭屈曲的总势能，U 为薄壁构件中的应变能，V 为外荷载势能。

文［2～5］认为薄壁构件中的应变能 U 由平面内的弯曲应变能 U_1、侧向弯曲应变能

U_2、纯扭转应变能 U_3 和翘曲应变能 U_4 四部分组成，分别求出 U_1、U_2、U_3、U_4 后相加即得应变能 U，即：

$$U = U_1 + U_2 + U_3 + U_4 \tag{2}$$

文［2～5］在求外荷载势能 V 时，认为外荷载势能等于外力功的负值，即：

$$V = -W \tag{3}$$

求解过程为先求出外荷载功 W，然后利用式（3）求出外荷载势能 V。

由"传统理论"推导薄壁构件中的应变能 U 的过程可以看出，该理论没有严密的数学过程；而外荷载势能 V 的求解是以先求外荷载功 W 开始的。对于所研究的对象薄壁构件，其中的应变能 U 的产生来自于外荷载，因此应变能 U 也是外荷载对薄壁构件做功的结果，而外荷载功 W 最终还是以薄壁构件应变能的形式存在于薄壁构件之中。对于"传统理论"，把除应变能 U 之外的另一部分应变能当作外荷载势能 V，进而又通过求外荷载功 W 来求出外荷载势能 V，由此可看出"传统理论"的理论体系不具备严密性。

此外，文［10］从屈曲问题的变分原理的角度分析了"传统理论"的总势能表达式中不应包含"外荷载功"，同时指出在"传统理论"中对薄壁构件横截面的转角 φ 简化不一致也是该理论的缺陷之一。

需要指出的是，"传统理论"对应变能 U 没有指出是线性还是非线性，所以文［6］指出"传统理论"的缺陷在于未考虑非线性剪切应变能即使正确，但有牵强之意。

那么对本文阐述的"传统理论"中外荷载势能 V 是除应变能 U 之外的另一部分应变能的观点是否正确，"新型理论"给出了证明。

2.2 新型理论

"新型理论"[7]把薄壁构件的正应变和剪应变分为线性部分和非线性部分，即 $\varepsilon=\varepsilon^{L}+\varepsilon^{N}$，$\gamma=\gamma^{L}+\gamma^{N}$，同时也把正应力和剪应力分为线性部分和非线性部分，即 $\sigma=\sigma^{L}+\sigma^{N}$，$\tau=\tau^{L}+\tau^{N}$，利用应力和应变的乘积：

$$\sigma\varepsilon = (\sigma^{L} + \sigma^{N})(\varepsilon^{L} + \varepsilon^{N}) = \sigma^{L}\varepsilon^{L} + \sigma^{L}\varepsilon^{N} + \sigma^{N}\varepsilon^{L} + \sigma^{N}\varepsilon^{N} \tag{4}$$

$$\tau\gamma = (\tau^{L} + \tau^{N})(\gamma^{L} + \gamma^{N}) = \tau^{L}\gamma^{L} + \tau^{L}\gamma^{N} + \tau^{N}\gamma^{L} + \tau^{N}\gamma^{N} \tag{5}$$

同时利用 $\sigma^{L}\varepsilon^{N}=\sigma^{N}\varepsilon^{L}$ 和 $\tau^{L}\gamma^{N}=\tau^{N}\gamma^{L}$ 并忽略非线性应力和非线性应变的乘积 $\sigma^{N}\varepsilon^{N}$ 和 $\tau^{N}\gamma^{N}$，得到应变能即薄壁构件弯扭屈曲的总势能为：

$$\begin{aligned}\Pi_2 =& \frac{1}{2}\int_l\int_s(\sigma^{L}\varepsilon^{L})t\mathrm{d}s\mathrm{d}t + \frac{1}{2}\int_l\int_s(\tau^{L}\gamma^{L})t\mathrm{d}s\mathrm{d}t + \frac{1}{2}\int_l\int_s(2\sigma^{L}\varepsilon^{N})t\mathrm{d}s\mathrm{d}t \\ &+ \frac{1}{2}\int_l\int_s(2\tau^{L}\gamma^{N})t\mathrm{d}s\mathrm{d}t\end{aligned} \tag{6}$$

按先后顺序记式（6）中的 4 项分别为线性正应变能 $U_{\sigma}{}^{L}$、线性剪应变能 $U_{\tau}{}^{L}$、非线性正应变能 $U_{\sigma}{}^{N}$、非线性剪应变能 $U_{\tau}{}^{N}$。对比文［2］与文［10］可知下列关系式成立：

$$U_{\sigma}{}^{L} = U_1 + U_2 + U_4 \tag{7}$$

$$U_{\tau}{}^{L} = U_3 \tag{8}$$

$$U_{\sigma}{}^{N} = -V \tag{9}$$