

经 典 原 版 书 库

机器人学导论

[美] 约翰 J. 克雷格 (John J. Craig) 著

(英文版·第4版)

Introduction to

ROBOTICS

MECHANICS AND CONTROL

Fourth Edition

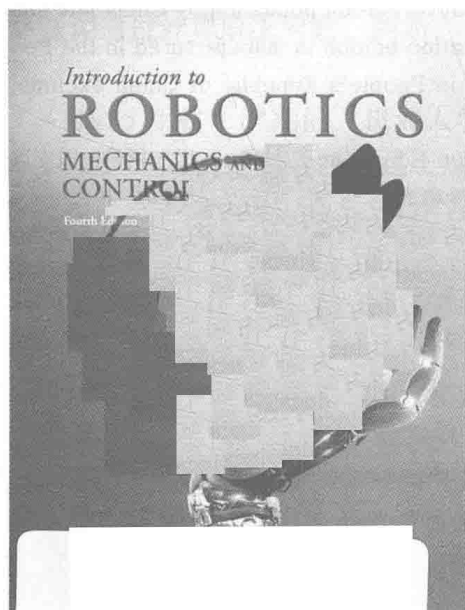


经 典 原 版 书 库

机器人学导论

(英文版·第4版)

Introduction to Robotics
Mechanics and Control (Fourth Edition)



[美] 约翰 J. 克雷格 (John J. Craig) 著



机械工业出版社
China Machine Press

图书在版编目 (CIP) 数据

机器人学导论 (英文版·第4版) / (美) 约翰 J. 克雷格 (John J. Craig) 著. —北京: 机械工业出版社, 2018.1

(经典原版书库)

书名原文: Introduction to Robotics: Mechanics and Control, Fourth Edition

ISBN 978-7-111-58672-2

I. 机… II. 约… III. 机器人学—高等学校—教材—英文 IV. TP24

中国版本图书馆 CIP 数据核字 (2017) 第 305525 号

本书版权登记号: 图字 01-2017-6461

Authorized Adaptation from the English Language edition, entitled *Introduction to Robotics: Mechanics and Control*, Fourth Edition, ISBN 978-0-13-348979-8 by John J. Craig, Copyright © 2018, 2005 by Pearson Education, Inc.

All rights reserved. No part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage retrieval system, without permission from Pearson Education, Inc.

English language adaptation edition published by China Machine Press Copyright © 2018.

English language adaptation edition is manufactured in the People's Republic of China and is authorized for sale only in People's Republic of China excluding Taiwan, Hong Kong SAR and Macau SAR.

本书英文影印版由 Pearson Education Asia Ltd. 授权机械工业出版社独家出版。未经出版者书面许可, 不得以任何方式复制或抄袭本书内容。

仅限于中华人民共和国境内 (不包括香港、澳门特别行政区及台湾地区) 销售发行。

本书封面贴有 Pearson Education (培生教育出版集团) 激光防伪标签, 无标签者不得销售。

出版发行: 机械工业出版社 (北京市西城区百万庄大街 22 号 邮政编码: 100037)

责任编辑: 吴晋瑜

责任校对: 殷虹

印刷: 中国电影出版社印刷厂

版次: 2018 年 1 月第 1 版第 1 次印刷

开本: 186mm × 240mm 1/16

印张: 27

书号: ISBN 978-7-111-58672-2

定价: 99.00 元

凡购本书, 如有缺页、倒页、脱页, 由本社发行部调换

客服热线: (010) 88378991 88361066

投稿热线: (010) 88379604

购书热线: (010) 68326294 88379649 68995259

读者信箱: hzjsj@hzbook.com

版权所有·侵权必究

封底无防伪标均为盗版

本书法律顾问: 北京大成律师事务所 韩光 / 邹晓东

前 言

科学家们常会感到自己通过研究工作能够不断地认识自我。物理学家认识到了这一点，同样，心理学家和化学家也认识到了这一点。在机器人学的研究中，研究领域和研究者自身之间的关系尤为明显。与仅追求分析的自然科学不同，当前机器人学所追求的是倾向于综合的工程学科。也许正是这个原因，这个领域才让我们当中的许多人为之着迷。

机器人学研究的是怎样综合运用机械、传感器、驱动器和计算机来实现人类某些方面的功能。显然，这是一项庞大的任务，它必然需要运用各种“传统”领域的研究思想。

现今，机器人学诸方面的研究工作都是由不同领域的专家们进行的。通常没有一个人能够完全掌握机器人领域的所有知识。因此，自然有必要对这个研究领域进行划分。在更高的层次上，可把机器人学划分为四个主要领域：机械操作、移动、计算机视觉和人工智能。

本书介绍机械操作的理论和工程知识。这是机器人学的分支学科，它是建立在几个传统学科基础之上的。主要的相关学科有力学、控制理论和计算机科学。在本书中，第1~8章介绍机械工程和数学的主题，第9~11章为控制理论的题材，第12章和第13章属于计算机科学的内容。另外，本书始终强调通过计算解决问题，例如，与力学密切相关的每一章都有一节简要介绍计算方面的问题。

本书源于斯坦福大学1983~1985年秋季学期的“机器人学导论”课程的讲稿。前3版在1986~2016年期间为许多大学所采用。第4版得益于这些教材的广泛应用，并且根据多方面的反馈意见做了修正和改进。在此，向对本书作者提出修正意见的所有人表示感谢。

本书适用于高年级本科生或者低年级研究生课程。选修此课程的学生如果学过静力学和动力学这两门基础课程，同时学习过线性代数，并且能够使用计算机高级语言编程，将有助于他们的学习。此外，虽然不必先修控制理论方面的入门课程，但学过这门课程也是有益的。本书的目标之一是以简单、直观的方式介绍机器人学的知识。特别需要指出的是，虽然本书很多内容选自机械工程领域，但并不要求本书的读者一定得是机械工程师。在斯坦福大学，很多电气工程师、计算机科学家、数学家都认为本书具有很强的可读性。

虽然本书直接由机器人系统的研发工程师使用，但是任何将要从事机器人研究工作的人，应将本书内容看作重要的背景资料。同样，至少从事过某种硬件的软件开发人员以及不直接参与机器人的机械和控制的研究人员，应当具备一些本书提供的背景知识。

与第3版一样，第4版分为13章。本书的材料适合于用一学期的时间来讲授，如果要在半学期内讲授，教师需要略去一些章节。即便如此，仍然无法深入讲解所有专题。本书在编写时从某些方面考虑了这一点，例如，多数章节只采用一种方法去解决常见的问题。编写本书的主要问题之一就是尽量在限定的教学时间内为每个主题合理地分配时间。为此，我的

办法是只考虑那些直接影响机器人机械操作学习的材料。

在每章的最后都有一组习题。每道习题题号后的方括号中给出了习题的难度系数。难度系数在 [00] 和 [50] 之间。[00] 是最简单的题目，[50] 是尚未解决的研究性问题[⊖]。当然，一个人认为困难的问题在另一个人看来可能会觉得容易，因此，一些读者会发现那些难度系数在某些情况下会引起误解。不过，我们尽力评价了这些习题的难度。

每章的末尾都有一个编程作业，学生可以把相应章节的知识应用到一个简单的三关节平面操作臂中。这个简单的操作臂足以用来证明大多数一般操作臂的所有原理，而不必使学生陷入过于复杂的问题中。每个编程作业都建立在前一个作业的基础上，到课程结束时，学生就会得到一个完整的操作臂软件程序库。

第 1 ~ 9 章共有 12 道使用 MATLAB 的习题。这些习题由俄亥俄大学的 Robert L. Williams II 教授编写，我对他所作的贡献深表感谢。这些习题可以配合澳大利亚 CSIRO 首席研究科学家 Peter Corke 编写的 MATLAB Robotics Toolbox[⊖]使用。

第 1 章是机器人学的概述，介绍一些背景资料、基本思想和本书所使用的符号，并预览后面各章的内容。

第 2 章包括描述三维空间中位置与方法的数学知识。这是极为重要的内容：通过定义机械操作本身与周围空间的移动物体（工件、工具、机器人自身）联系起来。我们需要用一种易于理解并且尽可能直观的方式来描述这些动作。

第 3 章和第 4 章讨论机械操作臂的几何问题。介绍机械工程学科中的运动学分支，这个分支研究运动但不考虑引起这种运动的力。在这两章里，我们讨论操作臂运动学，但把研究范围限定在静态定位问题上。

第 5 章将运动学的研究范围扩展到速度和静力方面。

第 6 章开始研究引起操作臂运动的力和力矩。这就是操作臂动力学问题。

第 7 章描述操作臂在空间的运动轨迹。

第 8 章涉及许多与操作臂机械设计有关的问题。例如，设计多少关节是适宜的、关节的类型应是什么以及如何对它们进行布局。

第 9 章和第 10 章研究操作臂的控制方法（通常利用计算机），以准确地跟踪预先设定的空间轨迹。第 9 章研究线性控制方法，第 10 章将研究拓展到非线性领域。

第 11 章讨论操作臂的主动动力控制。也就是研究如何控制操作臂的运用，这种控制模式在操作臂接触周围环境的情况下非常重要，比如操作臂用海绵擦窗户。

第 12 章概述机器人编程方法，特别是机器人编程系统中所需的基本成分以及与工业机器人编程相关的特殊问题。

第 13 章介绍离线仿真和编程系统，描述人 - 机接口的最新进展。

⊖ 我采用了与 D. Knuth 所著《The Art of Computer Programming》(Addison-Wesley 出版) 同样的难度等级。

⊖ 关于 MATLAB 机器人工具箱，请访问：<http://www.ict.csiro.au/robotics/ToolBox7.htm>。

第 4 版新增内容

- 每章增加了若干习题
- 8.9 节的光学编码器
- 10.9 节的自适应控制
- 由于技术的发展进步，更新了文献材料和参考文献
- 更新或新增了几幅图片
- 改正了 100 多个打印错误和其他小错误

非常感谢牺牲宝贵时间协助我完成本书的朋友们！首先感谢斯坦福大学 1983 秋季到 1985 秋季 ME219 班的同学们，他们学习了本书的初稿，发现了不少错误，并提出了许多建议。Bernard Roth 教授在多方面给予了帮助，不仅对草稿提出了建设性的意见，还为我完成第 1 版提供了环境。在 SILMA 公司，我得到了很好的仿真环境和资源，这些帮助我完成了第 2 版。Jeff Kerr 博士写了第 8 章的初稿，Robert L. Williams II 教授设计了每章最后的 MATLAB 习题。Peter Corke 扩充了他的 Robotics Toolbox (机器人工具箱)，以支持本书采用的 Denavit-Hartenberg 符号体系。在此，我也深深地感谢我在机器人学方面的导师 Marc Raibert、Carl Ruoff、Tom Binford 和 Bernard Roth。

我还要感谢来自斯坦福大学、SILMA 公司、Adept 公司和其他地方的许多人，他们以各种方式对我提供过帮助，他们是 John Mark Agosta、Mike Ali、Lynn Balling、Al Barr、Stephen Boyd、Chuck Bunkley、Joel Burdick、Jim Callan、Brian Carlisle、Monique Craig、Subas Desa、Tri Dai Do、Karl Garcia、Ashitava Ghosal、Chris Goad、Ron Goldman、Bill Hamilton、Steve Holland、Peter Jackcon、Eric Jacobs、Johann Jäger、Paul James、Jeff Kerr、Oussama Khatib、Jim Kramer、Dave Lowe、Jim Maples、Dave Marimont、Dave Meer、Kent Ohlund、Madhusudan Raghavan、Richard Roy、Ken Salisbury、Bruce Shimano、Donalda Speight、Bob Tilove、Sandy Wells 和 Dave Williams。

我还要感谢 Pearson 公司的 Tom Robbins 为我编写本书的前两版提供了指导。

斯坦福大学的 Roth 教授在给 2002 级的学生们讲授机器人课程时使用了本书第 2 版，并指出了许多错误，这些错误在第 3 版中得到了修订。

最后，我还要感谢那些帮助我完成第 4 版的人们：Matt Marshall 为每章新增习题和收集有用的反馈做出了贡献；还有 Pearson 公司的 Julie Bai 和 Michelle Bayman。

约翰 J. 克雷格

目 录

1 概述	1
2 空间描述和变换	21
3 操作臂运动学	67
4 操作臂逆运动学	109
5 雅克比：速度和静力	145
6 操作臂动力学	177
7 轨迹生成	215
8 操作臂的机构设计	245
9 操作臂的线性控制	285
10 操作臂的非线性控制	315
11 操作臂的力控制	351
12 机器人编程语言及编程系统	375
13 离线编程系统	389
附录 A 三角恒等式	409
附录 B 24 种角度组合	411
附录 C 运动学公式	415
精选习题答案	417

Contents

1	Introduction	1
2	Spatial Descriptions and Transformations	21
3	Manipulator Kinematics	67
4	Inverse Manipulator Kinematics	109
5	Jacobians: Velocities and Static Forces	145
6	Manipulator Dynamics	177
7	Trajectory Generation	215
8	Manipulator-Mechanism Design	245
9	Linear Control of Manipulators	285
10	Nonlinear Control of Manipulators	315
11	Force Control of Manipulators	351
12	Robot Programming Languages and Systems	375
13	Off-Line Programming Systems	389
A	Trigonometric Identities	409
B	The 24 Angle-Set Conventions	411
C	Some Inverse-Kinematic Formulas	415
	Solutions to Selected Exercises	417

Introduction

1.1 BACKGROUND

1.2 THE MECHANICS AND CONTROL OF MECHANICAL MANIPULATORS

1.3 NOTATION

1.1 BACKGROUND

The history of industrial automation is characterized by periods of rapid change in popular methods. Either as a cause or, perhaps, an effect, such periods of change in automation techniques seem closely tied to world economics. Use of the **industrial robot**, which became identifiable as a unique device in the 1960s [1], along with computer-aided design (CAD) systems and computer-aided manufacturing (CAM) systems, characterizes the latest trends in the automation of the manufacturing process. These technologies are leading industrial automation through another transition, the scope of which is still unknown [2].

In North America, there was much adoption of robotic equipment in the early 1980s, followed by a brief pull-back in the late 1980s. Since that time, the market has been growing (see Fig. 1.1), although it is subject to economic swings, as are all markets.

Figure 1.2 shows the number of robots being installed per year worldwide. A major reason for the growth in the use of industrial robots is their declining cost and increasing abilities. By 2025 it is estimated that the average manufacturing employer will save 16% on labor by replacing human workers with robots. In some countries or regions, it is even more favorable to employ robots (see Fig. 1.3). As robots become more cost effective at their jobs, and as human labor continues to become more expensive, more and more industrial jobs become candidates for robotic automation. This is the single most important trend propelling growth of the industrial robot market. A secondary trend is that, economics aside, as robots become more capable, they become *able* to do more and more tasks that might be dangerous or impossible for human workers to perform.

This book focuses on the mechanics and control of the most important form of the industrial robot, the **mechanical manipulator**. Exactly what constitutes an industrial robot is sometimes debated. Devices such as that shown in Fig. 1.4 are always included, while numerically controlled (NC) milling machines usually are not. The distinction lies somewhere in the sophistication of the programmability of the device; if a mechanical device can be programmed to perform a wide variety of applications, it is probably an industrial robot. Machines which are for the most part limited to one class of task are considered **fixed automation**. For the purposes of this

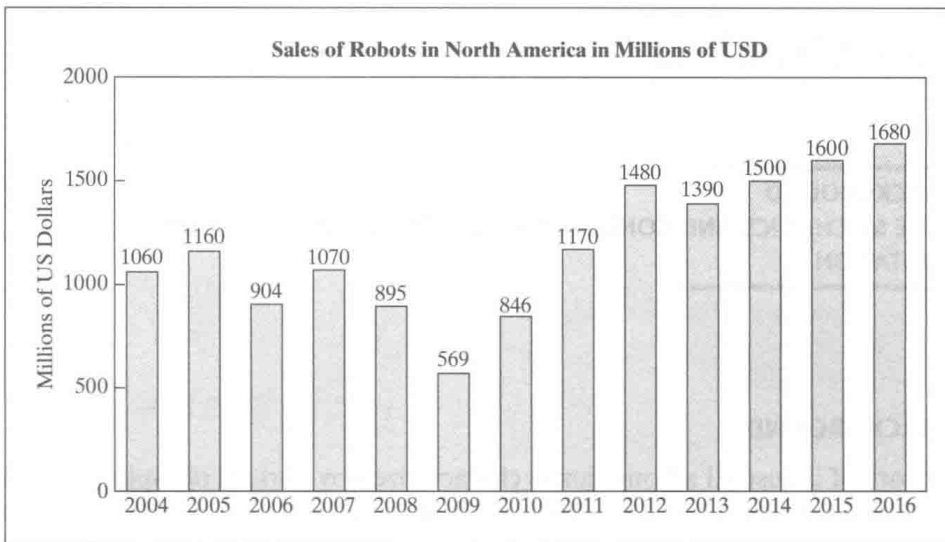


FIGURE 1.1: Sales of industrial robots in North America in millions of U.S. dollars. *Source:* Robotic Industries Association.

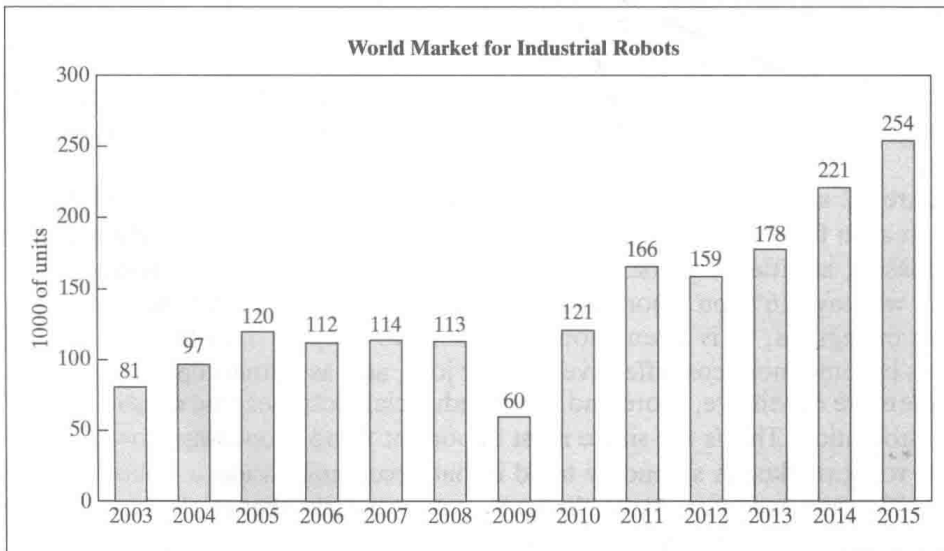


FIGURE 1.2: Yearly installations of multipurpose industrial robots. *Source:* World Robotics 2016.

text, the distinctions need not be debated; most material is of a basic nature that applies to a wide variety of programmable machines.

By and large, the study of the mechanics and control of manipulators is not a new science, but merely a collection of topics taken from “classical” fields. Mechanical engineering contributes methodologies for the study of machines in static and dynamic situations. Mathematics supplies tools for describing spatial motions and other attributes of manipulators. Control theory provides tools for designing and evaluating algorithms to realize desired motions or force applications.

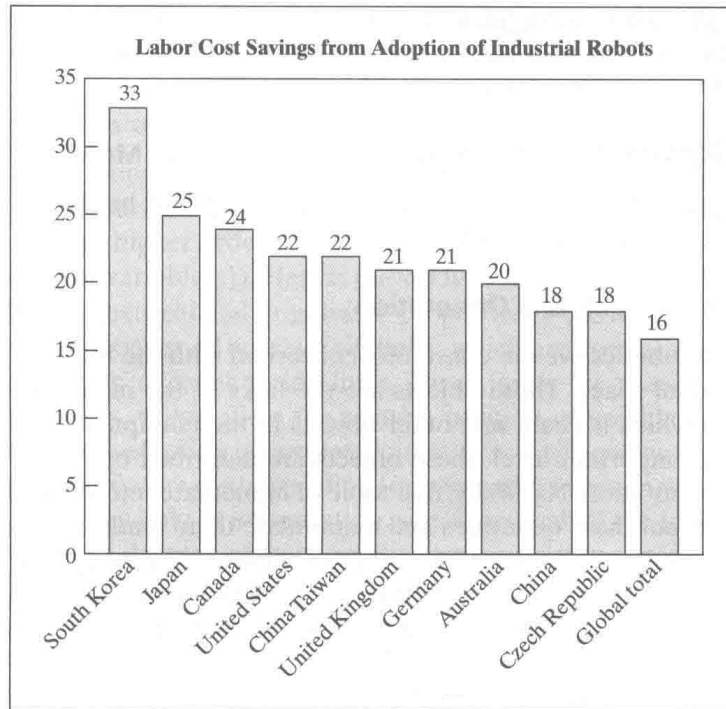


FIGURE 1.3: Labor cost savings from adoption of industrial robots. Estimated as a percentage in 2025. *Source:* The Boston Consulting Group.

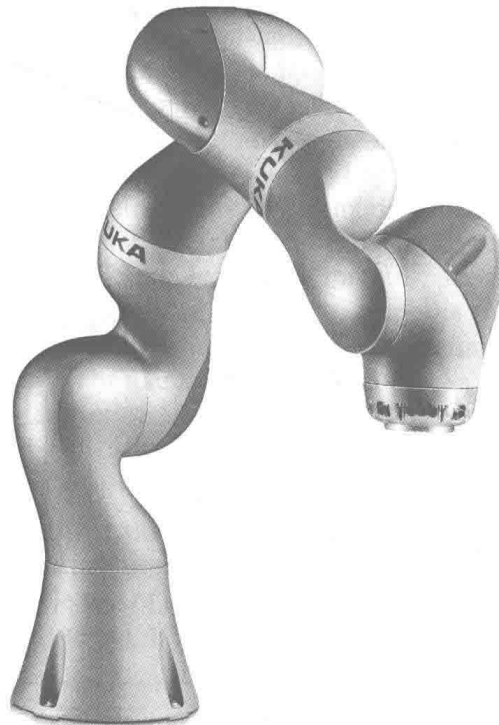


FIGURE 1.4: A modern 7 degree-of-freedom robot. Image courtesy KUKA Roboter GmbH.

Electrical-engineering techniques are brought to bear in the design of sensors and interfaces for industrial robots, and computer science contributes a basis for programming these devices to perform a desired task.

1.2 THE MECHANICS AND CONTROL OF MECHANICAL MANIPULATORS

The following sections will introduce some terminology, and briefly preview each of the topics that will be covered in the text.

Description of Position and Orientation

In the study of robotics, we are constantly concerned with the location of objects in three-dimensional space. These objects are the links of the manipulator, the parts and tools with which it deals, and other objects in the manipulator's environment. At a crude but important level, these objects are described by just two attributes: position and orientation. Naturally, one topic of immediate interest is the manner in which we represent these quantities and manipulate them mathematically.

In order to describe the position and orientation of a body in space, we will always attach a coordinate system, or **frame**, rigidly to the object. We will then proceed to describe the position and orientation of this frame with respect to some reference coordinate system (see Fig. 1.5).

Any frame can serve as a reference system within which to express the position and orientation of a body, so we often think of *transforming* or *changing the description of* these attributes of a body from one frame to another. Chapter 2 will discuss conventions and methodologies for dealing with the description of position and orientation, and the mathematics of manipulating these quantities with respect to various coordinate systems.

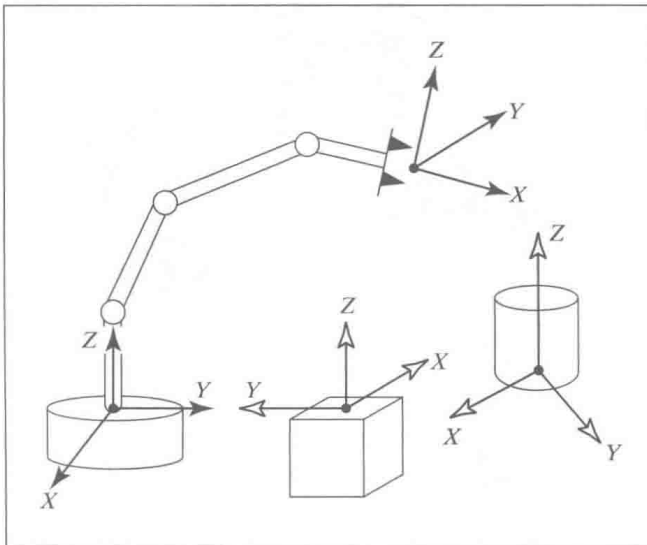


FIGURE 1.5: Coordinate systems or “frames” are attached to the manipulator and to objects in the environment.

Developing good skills concerning the description of position and rotation of rigid bodies is highly useful even in fields outside of robotics.

Forward Kinematics of Manipulators

Kinematics is the science of motion that treats motion without regard to the forces which cause it. Within the science of kinematics, one studies position, velocity, acceleration, and all higher order derivatives of the position variables (with respect to time or any other variable(s)). Hence, the study of the kinematics of manipulators refers to all the geometrical and time-based properties of the motion.

Manipulators consist of nearly rigid **links**, which are connected by **joints** that allow relative motion of neighboring links. These joints are usually instrumented with position sensors, which allow the relative position of neighboring links to be measured. In the case of rotary or **revolute** joints, these displacements are called **joint angles**. Some manipulators contain sliding (or **prismatic**) joints, in which the relative displacement between links is a translation, sometimes called the **joint offset**.

The number of **degrees of freedom** that a manipulator possesses is the number of independent position variables that would have to be specified in order to locate all parts of the mechanism. This is a general term used for any mechanism. For example, a four-bar linkage has only one degree of freedom (even though there are three moving members). In the case of typical industrial robots, because a manipulator is usually an open kinematic chain, and because each joint position is usually defined with a single variable, the number of joints equals the number of degrees of freedom.

At the free end of the chain of links that make up the manipulator is the **end-effector**. Depending on the intended application of the robot, the end-effector could be a gripper, a welding torch, an electromagnet, or another device. We generally describe the position of the manipulator by giving a description of the **tool frame**, which is attached to the end-effector, relative to the **base frame**, which is attached to the nonmoving base of the manipulator (see Fig. 1.6).

A very basic problem in the study of mechanical manipulation is called **forward kinematics**. This is the static geometrical problem of computing the position and orientation of the end-effector of the manipulator. Specifically, given a set of joint angles, the forward kinematic problem is to compute the position and orientation of the tool frame relative to the base frame. Sometimes, we think of this as changing the representation of manipulator position from a **joint space** description into a **Cartesian space** description.¹ This problem will be explored in Chapter 3.

Inverse Kinematics of Manipulators

In Chapter 4, we will consider the problem of **inverse kinematics**. This problem is posed as follows: Given the position and orientation of the end-effector of the manipulator, calculate all possible sets of joint angles that could be used to attain this given position and orientation (see Fig. 1.7). This is a fundamental problem in the practical use of manipulators.

¹By *Cartesian space*, we mean the space in which the position of a point is given with three numbers, and in which the orientation of a body is given with three numbers. It is sometimes called *task space* or *operational space*.

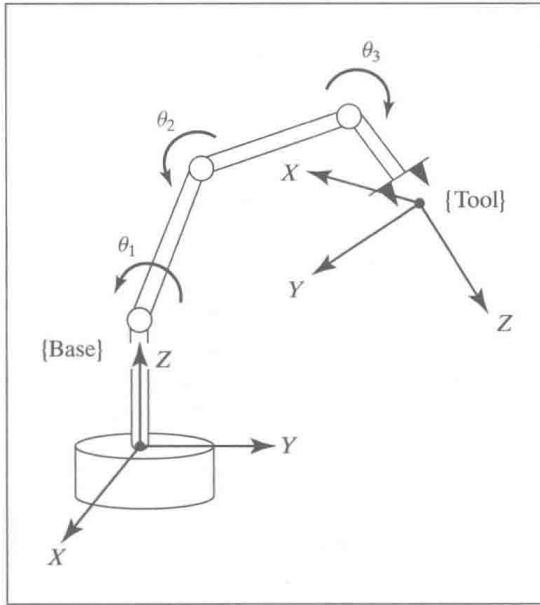


FIGURE 1.6: Kinematic equations describe the tool frame relative to the base frame as a function of the joint variables.

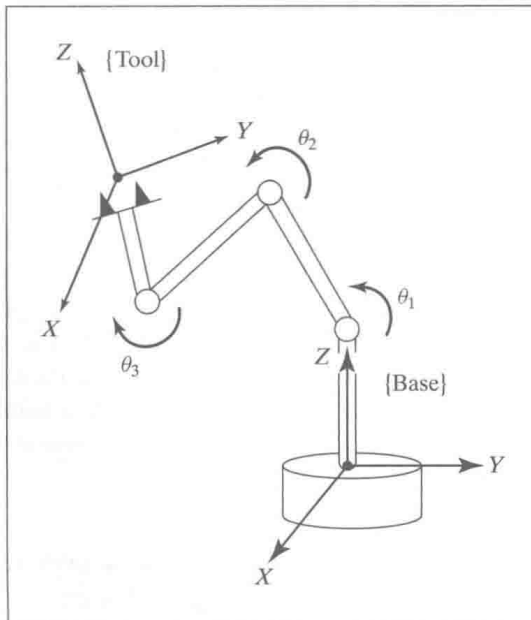


FIGURE 1.7: For a given position and orientation of the tool frame, values for the joint variables can be calculated via the inverse kinematics.

This is a rather complicated geometrical problem that is routinely solved thousands of times daily in human and other biological systems. In the case of an artificial system like a robot, we will need to create an algorithm in the control computer that can make this calculation. In some ways, solution of this problem is the most important element in a manipulator system.

We can think of this problem as a *mapping* of “locations” in 3-D Cartesian space to “locations” in the robot’s internal joint space. This need naturally arises anytime a goal is specified in external 3-D space coordinates. Some early robots lacked this algorithm—they were simply moved (sometimes by hand) to desired locations, which were then recorded as a set of joint values (i.e., as a location in joint space) for later playback. Obviously, if the robot is used purely in the mode of recording and playback of joint locations and motions, no algorithm relating joint space to Cartesian space is needed. These days, however, it is rare to find an industrial robot that lacks this basic inverse kinematic algorithm.

The inverse kinematics problem is not as simple as the forward kinematics one. Because the kinematic equations are nonlinear, their solution is not always easy (or even possible) in a closed form. Also, questions about the existence of a solution and about multiple solutions arise.

Study of these issues gives one an appreciation for what the human mind and nervous system are accomplishing when we, seemingly without conscious thought, move and manipulate objects with our arms and hands.

The existence or nonexistence of a kinematic solution defines the **workspace** of a given manipulator. The lack of a solution means that the manipulator cannot attain the desired position and orientation, because it lies outside of the manipulator’s workspace.

Velocities, Static Forces, Singularities

In addition to dealing with static positioning problems, we may wish to analyze manipulators in motion. Often, in performing velocity analysis of a mechanism, it is convenient to define a matrix quantity called the **Jacobian** of the manipulator. The Jacobian specifies a **mapping** from velocities in joint space to velocities in Cartesian space (see Fig. 1.8). The nature of this mapping changes as the configuration of the manipulator varies. At certain points, called **singularities**, this mapping is not invertible. An understanding of the phenomenon is important to designers and users of manipulators.

Consider the rear gunner in a World War I—vintage biplane fighter plane (illustrated in Fig. 1.9). While the pilot flies the plane from the front cockpit, the rear gunner’s job is to shoot at enemy aircraft. To perform this task, his gun is mounted in a mechanism that rotates about two axes, the motions being called azimuth and elevation. Using these two motions (two degrees of freedom), the gunner can direct his stream of bullets in any direction he desires in the upper hemisphere.

An enemy plane is spotted at azimuth one o’clock and elevation 25 degrees! The gunner trains his stream of bullets on the enemy plane and tracks its motion so as to hit it with a continuous stream of bullets for as long as possible. He succeeds and thereby downs the enemy aircraft.

A second enemy plane is seen at azimuth one o’clock and elevation 70 degrees! The gunner orients his gun and begins firing. The enemy plane is moving so as to

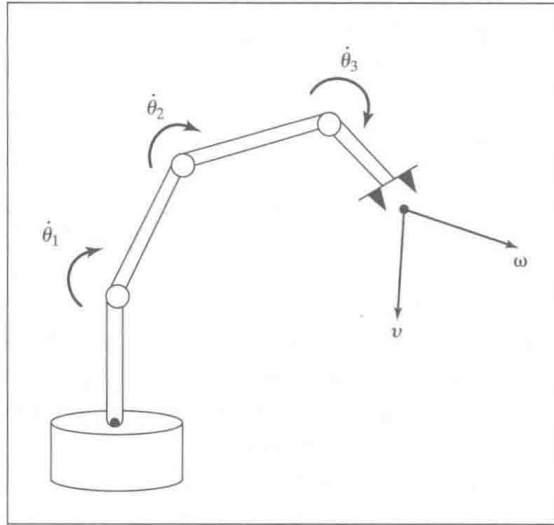


FIGURE 1.8: The geometrical relationship between joint rates and velocity of the end-effector can be described in a matrix called the Jacobian.

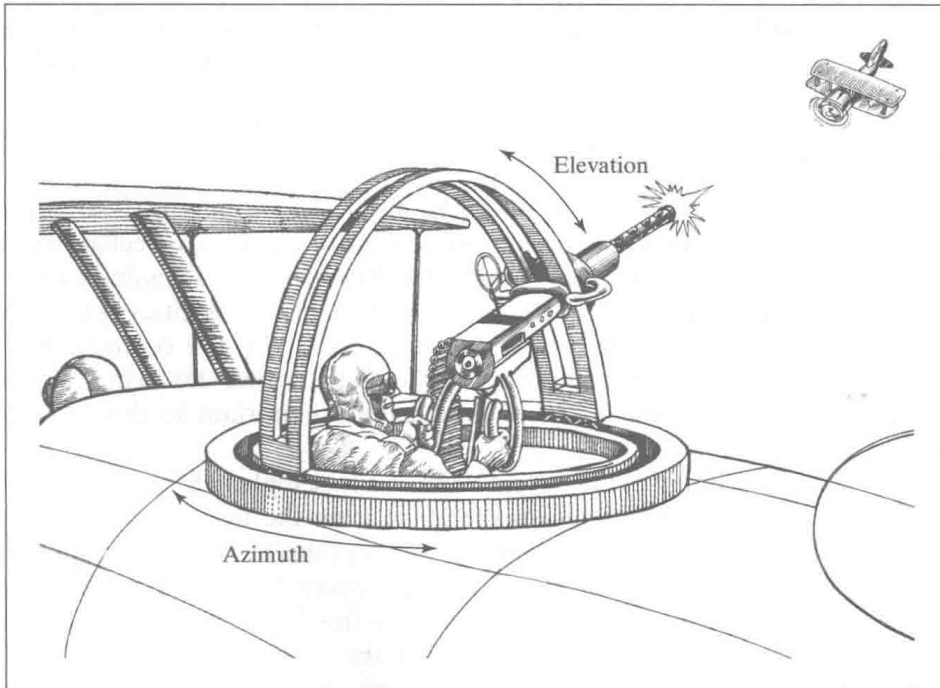


FIGURE 1.9: A World War I biplane with a pilot and a rear gunner. The rear-gunner mechanism is subject to the problem of singular positions.

obtain a higher and higher elevation relative to the gunner's plane. Soon the enemy plane is passing nearly overhead. What's this? The gunner is no longer able to keep his stream of bullets trained on the enemy plane! He found that, as the enemy plane flew overhead, he was required to change his azimuth at a very high rate. He was not able to swing his gun in azimuth quickly enough, and the enemy plane escaped!

In the latter scenario, the lucky enemy pilot was saved by a *singularity*! The gun's orienting mechanism, while working well over most of its operating range, becomes less than ideal when the gun is directed straight upwards or nearly so. To track targets that pass through the position directly overhead, a very fast motion around the azimuth axis is required. The closer the target passes to the point directly overhead, the faster the gunner must turn the azimuth axis to track the target. If the target flies directly over the gunner's head, he would have to spin the gun on its azimuth axis at infinite speed!

Should the gunner complain to the mechanism designer about this problem? Could a better mechanism be designed to avoid this problem? It turns out that you really can't avoid the problem very easily. In fact, any two-degree-of-freedom orienting mechanism that has exactly two rotational joints cannot avoid having this problem. In the case of this mechanism, with the stream of bullets directed straight up, their direction aligns with the axis of rotation of the azimuth rotation. This means that, at exactly this point, the azimuth rotation does not cause a change in the direction of the stream of bullets. We know we need two degrees of freedom to orient the stream of bullets, but, at this point, we have lost the effective use of one of the joints. Our mechanism has become **locally degenerate** at this location, and behaves as if it only has one degree of freedom (the elevation direction).

This kind of phenomenon is caused by what is called a **singularity of the mechanism**. All mechanisms are prone to these difficulties, including robots. Just as with the rear gunner's mechanism, these singularity conditions do not prevent a robot arm from positioning anywhere within its workspace. However, they can cause problems with *motions* of the arm in their neighborhood.

Manipulators do not always move through space; sometimes they are also required to touch a workpiece or work surface and apply a static force. In this case, the problem arises: Given a desired contact force and moment, what set of **joint torques** is required to generate them? Once again, the Jacobian matrix of the manipulator arises quite naturally in the solution of this problem.

Dynamics

Dynamics is a huge field of study devoted to studying the forces required to cause motion. In order to accelerate a manipulator from rest, glide at a constant end-effector velocity, and finally decelerate to a stop, a complex set of torque functions must be applied by the joint actuators.² The exact form of the required functions of actuator torque depend on the spatial and temporal attributes of the path taken by the end-effector and on the mass properties of the links and payload, friction in the joints, and so on. One method of controlling a manipulator to follow a desired path involves calculating these actuator torque functions by using the dynamic equations of motion of the manipulator.

²We use *joint actuators* as the generic term for devices that power a manipulator—for example, electric motors, hydraulic and pneumatic actuators, and muscles.