Letters of Euler on Different Subjects in Natural Philosophy: Addressed to a German Princess

欧拉致德国公主的书信: 关于自然哲学的不同学科

第1卷

LEONHARD EULER



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Volume I

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This book reproduces the text of the original edition. The content and language reflect the beliefs, practices and terminology of their time, and have not been updated.

#### 内容简介

18世纪60年代,著名数学家欧拉应邀为德国公主函授哲学、物理学、宇宙学、化学、音乐等多个 科学和艺术分支的知识。为此他写下了一系列文笔优美的文章,它们充分体现了欧拉渊博的知识、极 高的文学修养和哲学修养。这些通信被整理成书并翻译成英文,分两卷出版,本卷收录了115篇文章。

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Leonhard Euler (1707—1783)

欧拉 (Leonhard Euler, 1707—1783), 瑞士数学家, 自然科学家. 1707 年 4 月 15 日生于瑞士巴塞尔, 1783 年 9 月 18 日去世于俄国圣彼得堡. 15 岁在巴塞尔大学获学士学位, 翌年获硕士学位. 父亲希望他学神学, 而他最感兴趣的是数学, 并受到约翰第一·伯努利的指导. 18 岁时, 彻底放弃当牧师的念头而专攻数学, 并开始发表文章. 1727 年, 欧拉应圣彼得堡科学院的邀请到俄国. 1731 年接替丹尼尔第一·伯努利成为物理教授. 他以旺盛的精力投入研究, 在俄国的 14 年中, 他在分析学、数论和力学方面做了大量出色的工作. 他还应俄国政府的要求, 解决不少诸如地图学、造船业中的实际问题. 大量的写作带来的眼疾使他在 1735 年右眼失明. 1741 年受普鲁士腓特烈大帝的邀请到柏林科学院工作, 达 25 年之久. 在柏林期间他的研究内容更加广泛, 涉及行星运动、刚体运动、热力学、弹道学、人口学, 这些工作和他的数学研究相互推动. 欧拉这个时期在微分方程、曲面微分几何以及其他数学领域的研究都是开创性的. 1766 年他又回到了圣彼得堡. 一场重病使他的左眼于 1771 年也完全失明. 然而由于他惊人的记忆力和心算技巧使他的创造力继续得到发挥. 他通过与助手们讨论以及直接口授等方式又完成了大量科学著作, 直至生命的最后一刻.

欧拉是 18 世纪数学界最杰出的人物之一,他不但在数学上做出伟大贡献,而且把数学用到了几乎整个物理领域.他又是一个无与伦比的多产作者,他写了大量的力学、分析学、几何学、变分法的教科书,《无穷小分析引论》、《微分学原理》、《积分学原理》都成为数学中的经典著作.除了教科书外,他在工作的时期几乎以每年 800 页的速度写出创造性论文,他的全集将近 74 卷.

欧拉的最大功绩是扩展了微积分的领域,为分析学的一些重要分支 (如无穷级数、微分方程) 与微分几何的产生和发展奠定了基础. 除了分析之外,欧拉在数学上的发现还有很多,在代数学上,他发现了每个实系数多项式必能分解为一次或二次因子之积,因此根必是 a+bi 的形式. 数论作为数学中一个独立分支的基础是由欧拉的一系列成果所奠定的. 他还给出了费马小定理的三个证明,并引入了数论中重要的欧拉函数  $\varphi(n)$ ; 他发现了二次互反律;他利用连分数给出佩尔方程  $x^2-ay^2=1$  的最小解;他用解析方法讨论数论问题,发现了  $\zeta$  函数所满足的函数方程,引入了欧拉乘积. 他还解决了著名的组合问题: 哥尼斯堡七桥问题. 在数学的许多分支中都常常见到以他的名字命名的重要常数、公式和定理.

## LETTERS OF EULER

ON DIFFERENT SUBJECTS IN

# NATURAL PHILOSOPHY,

ADDRESSED TO

A GERMAN PRINCESS.

WITH NOTES, AND A LIFE OF EULER,

BY DAVID BREWSTER, LL.D., F.R.S. LOND. AND ED.

CONTAINING A GLOSSARY OF SCIENTIFIC TERMS.

WITH ADDITIONAL NOTES,
BY JOHN GRISCOM, LL.D.

IN TWO VOLUMES.

VOL. I.

NEW YORK:

HAMPER & BROTHERS, PUBLISHERS, 329 & 331 PEARL STREET,

FRANKLIN SQUARE.

1858.

The title page of the original version

THE work now presented for the first time to the American public is the production of one of the most learned and highly-gifted men whose names adorn the annals of sound philosophy, and whose labours gained for him a reputation inferior only to that of Newton. They will be read with great advantage by the young student, on account of the remarkable clearness with which the subjects are treated, the copiousness of the style, and its adaptation to the purpose of easy and familiar instruction. The author, in addition to his profound knowledge, had the advantage of being, both by profession and by taste, a teacher of youth; and the volumes before us bear evidence of the tact which an experienced instructer acquires in discovering the points which stand most in need of varied and reiterated explanation.

These Letters, therefore, designed as they were for the benefit of a lady, will predispose the ingenuous mind to the love of scientific truth. Nor is the evidence of piety he has brought into connexion with some of the highest contemplations of philosophy one of the least interesting features of these agreeable volumes.

AMERICAN EDITOR.

### PREFACE.

IT has long been a reproach against English literature, that the composition of elementary and popular works has been left almost exclusively in the hands of inferior writers, who possess only a general and superficial knowledge of the subjects of which they treat. The influence of this practice upon the diffusion of general and correct knowledge has been deeply felt by those who are desirous of introducing a system of education which embraces a wider range than the ordinary routine of classical instruction. The popular writings of those who acquire a knowledge of science for the purpose of teaching it differ in the most essential manner from those of a philosopher who devotes himself to the task of perspicuous illustration. However correct may be the principles, and however copious the details of a compiled work, it must always be defective in the selection of its topics, in the clearness of its reasoning, in the generality of its views, and in the suitableness of its illustrations. A mind like that of EULER, which from its infancy has been devoted to the study of Nature, selects at once the prominent features of the science which it is proposed to explain; excludes all perplexing and extraneous facts, and combines under general views the important truths which it is the object of the pupil to seize and retain. The justness of this remark cannot fail to be admitted by those who read the following Letters, which may be justly characterized as the most popular work that ever was written, and as the production of the profoundest philosopher that ever wrote.

EULER'S Letters to a German Princess were first made known in Europe by an edition published by the Marquis CONDORCET and M. LACROIX, who enriched it with a variety of notes, and whose opinion of the work for the purposes of public instruction may be acceptable to the English reader.

"The Letters of Euler to a German Princess," says M. Condorcet, "have acquired over all Europe a celebrity in which the reputation of the Author, the choice and importance of the several subjects, and the clearness of elucidation justly entitle them. They have deservedly been considered as a treasury of science, adapted to the purposes of every common seminary of learning. They may be studied to advantage without much previous elementary knowledge; they convey accurate ideas respecting a variety of objects, highly interesting in themselves, or calculated to excite a laudable curiosity; they inspire a proper taste for the sciences, and for that sound philosophy which, supported by science, and never losing sight of her cautious, steady, methodical advances, runs no risk of perplexing

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or misleading the attentive student."

The English reader is indebted to the late Rev. Dr. Henry Hunter for the following translation, which has gone through two editions. In this third edition the translation has received very essential improvements. The plates have been reengraved and much improved, and a life of the Author has been added, together with various notes, which, the editor trusts, will be both interesting and useful to the reader.

Independently of the great popularity of this work, it possesses a particular interest at the present time, in consequence of its containing a popular view of the doctrine, that light consists in the undulations of an ethereal medium, which is now generally adopted, in consequence of recent discoveries in optics.

EDINBURGH, July, 22d 1823.

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## THE LIFE OF EULER.

LEONARD EULER, one of the most distinguished mathematicians of the 18th century, was the son of Paul Euler, and Margaret Brucker, and was born at Bâsle on the 15th of April, 1707.

His father, who had been instructed in mathematics by the celebrated James Bernoulli, became pastor of the village of Riechen, near Bâsle, in the year 1708; and as soon as his son had arrived at the proper age, he instilled into him a fondness for mathematical learning, although he had destined him for the study of theology. He was afterward sent to the University of Bâsle, where he was found worthy to receive lessons from John Bernoulli, who was at that time regarded as the first mathematician in Europe. The assiduity and amiable disposition of Euler soon gained him the particular esteem of that great master, and the friendship of his two sons, Daniel and Nicolas Bernoulli, who had already become the disciples and the rivals of their father. John Bernoulli even condescended to give him once every week a particular lesson, for the purpose of explaining the difficulties which he encountered in the course of his studies. Euler had not the good fortune to enjoy long this inestimable advantage. In 1723, he received the degree of Master of Arts; and on this occasion he obtained great applause by the Latin discourse which he delivered, containing a comparison between the Newtonian and Cartesian philosophy. At the request of his father, he now began the study of theology; but his attachment to the mathematics was so strong, that his father at last consented to allow him to follow the bent of his own genius.

Nicolas and Daniel Bernoulli having, in 1725, accepted the invitation of Catherine I. to become members of the Academy of Sciences at St. Petersburg, promised at their departure to employ their influence to procure for Euler an appointment in that city. In the following year they announced that they had a situation in view for him, and strongly advised him to apply his mathematical knowledge to physiology. Euler immediately attended the lectures of the most eminent medical professors at Bâsle, and made rapid progress in the study of medicine. His attention, however, was still directed to his favorite pursuits, and he found leisure to compose a dissertation on the Nature and Propagation of Sound, and another on the Masting of Ships, which was written for the prize proposed by the Academy of Sciences in 1727. As this subject was actually suggested by several members of the academy, with the view of bringing into notice the talents of M. Bouguer, who had paid particular attention to the subject, and who was then professor

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of hydrography in the seaport town of Croisic, it was not likely that Euler, who was destitute of all practical knowledge of naval affairs, should have succeeded in the competition. Bouguer, of course, carried off the first prize; but Euler obtained what is called the *accessit*, or second prize, an honor of no trivial magnitude, when we consider that he was then only twenty years of age. About this time Euler was a candidate for the vacant professorship of natural philosophy in the University of Bâsle; but he had not the good fortune to be elected.

Daniel and Nicolas Bernoulli used all their influence to procure an appointment for their young friend; and having at last succeeded, they requested him to repair immediately to St. Petersburg. Euler lost no time in obeying this welcome summons; but after he had begun his journey, he had the mortification to learn that Nicolas Bernoulli had fallen a victim to the severity of the climate; and the very day upon which he entered the Russian territory was that of the death of the Empress Catherine I.; an event which at first threatened the dissolution of the academy, of which she had laid the foundation. Having reached St. Petersburg at this unfortunate period, Euler resolved to enter into the Russian navy, and had actually received the promise of a lieutenancy, and rapid promotion, from Admiral Sievers; but fortunately for geometry, a change took place in the aspect of public affairs in 1730, and Euler obtained the situation of Professor of Natural Philosophy. In 1733 he succeeded Daniel Bernoulli, when that illustrious mathematician retired into the country; and in the same year he married Mademoiselle Gsell, a Swiss lady, and the daughter of a painter whom Peter the Great had carried into Russia upon his return from his first tour. In 1735, a very intricate problem having been proposed by the Academy of St. Petersburg, Euler completed the solution of it in three days; but the exertion of his mind had been so violent, that it threw him into a fever which endangered his life, and deprived him of the use of one of his eyes. In 1738, the Academy of Sciences at Paris crowned his memoir, entitled Sur la Nature et les Proprietés du Feu; and in 1740, he divided with Daniel Bernoulli, and our countryman Colin Maclaurin, the prize given by the same academy for the best dissertation on the flux and reflux of the sea. Daniel Bernoulli had treated the subject with a sagacity and method which characterized all his labours. The dissertation of Maclaurin contained his celebrated theorem on the equilibrium of elliptical spheroids; and that of Euler was marked with an improvement on the integral calculus, which seemed to resolve the fundamental equation of almost all the great problems on the motions of the heavenly bodies.

In consequence of an invitation from the King of Prussia, through his minister the Count de Mardefield, Euler quitted St. Petersburg and went to Berlin in the month of June, 1741. Upon his arrival, he was honored with a letter from the king of Prussia, written from his camp at Reichenbach, and he was soon after presented to the queenmother, a princess who took great pleasure in the conversation of

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illustrious men. She treated Euler with the utmost familiarity; but never being able to draw him into any conversation but that of monosyllables, she one day asked him why he did not wish to speak to her? "Madam," replied Euler, "it is because I have just come from a country where every person who speaks is hanged."

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The memoirs and works with which Euler enriched mathematics and physics are so extremely numerous that it would occupy many pages to give even the briefest account of them. In many of his physical memoirs, Euler has been justly reproached for having applied the calculus to the most unfounded physical hypotheses, or to metaphysical principles which had not been sufficiently examined; and on this account several of his memoirs have no value whatever, except in so far as they exhibit fine specimens of the resources of analysis. His Dissertations on Windmills, on Achromatic Telescopes, on Naval Architecture, and on Gunnery, are among the number of those which are liable to this criticism.

When Euler was at Berlin, the Princess of An halt Dessau, the niece of the King of Prussia, was desirous to receive from him some instruction in the different branches of Natural Philosophy; and for her use he drew up the present work, which was translated into most of the languages of Europe, and which has always been much esteemed, particularly for the singular perspicuity with which its author has explained some of the most profound truths in physics. The King of Prussia often employed Euler in calculations relative to the mint, and other objects of finance, — in the conducting of the waters of Sans Souci, and in the examination of canals, and other public works.

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In 1744, Euler was appointed Director of the Mathematical Class of the Academy, and in the same year he obtained the prize offered by the Academy of Sciences at Paris for the best work on the theory of magnetism.

About this time Robin's Treatise on Gunnery had appeared in England, and though our countryman had treated Euler with great severity, this act of injustice did not prevent him from recommending it to the King of Prussia as the best book on the subject. He even translated it, and in the additions which he made, he gave a complete theory of the motion of projectiles. M. Turgot ordered this work to be translated into French, and introduced into the schools of artillery; and about the same time there appeared a splendid edition of it in England.

In 1746, he published his new Theory of Light and Colours; and in 1759, his memoir *Sur les effets du Roulis et du Tangage*, gained the prize offered by the French Academy of Sciences.

In 1750, Euler went to Frankfort to receive his mother, who was then a widow, and to conduct her to Berlin, where she remained till the time of her death, in 1761; having enjoyed for eleven years the assiduous attention of a favourite son, and the high pleasure of seeing him universally esteemed and ad mired.

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When Euler remained at Berlin, he formed an intimate acquaintance with M. De Maupertuis, the learned President of the Prussian Academy of Sciences, and he defended Maupertuis's celebrated and favourite principle of the least action, by resolving, by means of it, some of the most difficult problems in mechanics. In the dispute into which he was thus led with Kœnig, who had attacked Maupertuis in 1751, he lost for a while his usual serenity, and became one of the enemies of that unfortunate individual.

Although the number of foreign associates in the French Academy of Sciences was limited to eight, yet Euler was appointed to the ninth place in 1755, on the condition that no appointment should take place at the first vacancy.

In the year 1760, the Russian army under General Tottleben penetrated into the March of Brandenburg, and pillaged a farm which Euler possessed near Charlottenberg. As soon as the Russian general was informed of the event, he immediately repaired the loss by a very large sum; and upon giving notice of the circumstance to the Empress Elizabeth, she added to this indemnity a present of four thousand florins. This act of generosity, no doubt, had a powerful effect in attaching Euler to the Russian government, which, in spite of his absence, had always paid him the pension which it granted him in 1742. Having received an invitation from the Empress Catherine, he obtained permission from the King of Prussia to return to St. Petersburg to spend the remainder of his days; but his eldest son was not allowed to accompany him. When Euler was on the eve of his departure, Prince Czartorisky invited him, in the name of the King of Poland, to take the road of Warsaw, where, loaded with kindness, he spent ten days with Stanislaus, who afterward honoured him with his correspondence.

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Shortly after his arrival in St. Petersburg, on the 17th July, 1766, he lost the sight of his other eye, having been for a considerable time obliged to perform his calculations with large characters, traced with chalk upon a slate. His pupils and his children copied his calculations, and wrote all his memoirs, while Euler dictated to them. To one of his servants, who was quite ignorant of mathematical knowledge, he dictated his Elements of Algebra, a work of very great merit, which has been translated into English and many other languages. Euler now acquired the rare faculty of carrying on in his mind the most complicated analytical and arithmetical calculations; and M. d'Alembert, when he saw him at Berlin, was astonished at some examples of this kind which occurred in their conversation. With the design of instructing his grandchildren in the extraction of roots, he formed a table of the six first powers of all numbers, from 1 to 100, and he recollected them with the utmost accuracy. Two of his pupils having computed to the 17th term, a complicated converging series, their results differed one unit in the fiftieth cypher; and an appeal being made to Euler, he went over the calculation in his mind, and his decision was found correct.

His principal amusement, after he lost his sight, was to make artificial loadstones, and to give lessons on mathematics to one of his grandchildren, who seemed to evince a taste for the science.

In 1771, a dreadful fire broke out in St. Petersburg, and reached the house of Euler. Peter Grimm, a native of Bâsle, having learned the danger in which his illustrious countryman was placed, threw himself among the flames, and, reaching Euler's apartment, brought him off on his shoulders, at the risk of his life. His library, however, and his furniture were consumed; but, by the activity of Count Orloff, his MSS. were saved.

Having revised the lunar theory with the aid of his son, and his colleagues Krafft and Lexell, he constructed a set of new lunar tables, which appeared in 1772. These tables were, at the suggestion of Turgot, rewarded by the Board of Longitude in France; and when the more perfect tables of Mayer obtained the great premium of three thousand pounds offered by the British parliament, the sum of three hundred pounds was given to Euler for having furnished the theorems made use of by Mayer in his theory.

In the year 1773, Euler published, at St. Petersburg, his great work on the construction and management of vessels. A new edition soon afterward appeared at Paris, and at the desire of the French king it was introduced into the schools of marine, and a reward of 1000 rubles transmitted to the author, accompanied by a handsome letter from the celebrated Turgot. About the same time an Italian, an English, and a Russian translation of it appeared, and the Russian government presented Euler with a gift of 2000 rubles.

Three of Euler's memoirs on the Inequalities in the Motions of the Planets, were crowned by the French Academy of Sciences; and he also gained the prizes of 1770 and 1772, by his perfection of the lunar theory.

Having lost his first wife, by whom he had thirteen children, eight of whom died in early life, he was married a second time, in 1776, to Mademoiselle Gsell, the aunt of his first wife.

Euler underwent the operation of couching, which was attended with the happy result of restoring his sight; but whether from the negligence of his surgeon, or from his being too eager to avail himself of his new organs, he again lost his sight, and suffered much severe pain from the relapse. His love for science, however, continued unabated, and in the course of seven years he transmitted seventy memoirs to the Academy of St. Petersburg. On the 7th of September, 1783, after having amused himself with calculating upon a slate the laws of the ascensional motion of balloons, which at that time occupied the attention of philosophers, he dined with his relation M. Lexell, and spoke of the planet Herschel, and of the calculations by which its orbit was determined. A short time afterward, he was amusing himself with one of his grandchildren, when, on a sudden, his pipe fell from his hand, and

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he expired of an apoplectic stroke, in the 79th year of his age.

Euler left behind him three sons, having lost his two daughters in the latter years of his life. Twenty-six out of thirty of his grandchildren were alive at the time of his death.

After a long life, so successfully devoted to the sciences, Euler's reputation was very widely extended. Besides being a foreign member of the Academy of Sciences at Paris, he was a Fellow of the Royal Society of London, and he had received from most of the princes of the north, with whom he was well acquainted, the most flattering marks of their esteem. When the Prince-royal of Prussia visited St. Petersburg, he anticipated the visit of Euler, and passed several hours at the bedside of this great man, holding him all the time by the hand, and having, at the same time, upon his knee one of Euler's grandchildren, who had displayed a premature attachment to geometry. The death of Euler was considered as a public loss even in the country where he lived; and the Academy of St. Petersburg decreed to him, at their own expense, a marble bust, which was placed in their public hall. In an allegorical picture which the academy had put up during his life, Geometry was represented as placed upon a basement covered with calculations. These calculations were the formulæ of Euler's Theory of the Lunar Motions.

Euler's knowledge was not limited to mathematics and the physical sciences. He had carefully studied anatomy, chemistry, and botany, and he was also deeply versed in ancient literature. He could repeat the Æneid from the beginning to the end, and he could even tell the first and last lines in every page of the edition which he used. In one of his works there is a learned memoir on a question in mechanics, of which, as he himself informs us, a verse of the Æneid gave him the first idea.

Euler possessed naturally a strong constitution; and when we consider the nature of his studies, and the assiduity with which he pursued them, we cannot fail to be surprised at the great degree of health which he enjoyed. In all his habits he was sober and temperate, — in his manners unaffected and pleasing, — and in his temper lively and cheerful. In his moral and religious character there is much to admire. The high fame which he acquired, and the interruptions which he must have experienced, both at Berlin and St. Petersburg, never induced him to abandon the religious duties to which he had been educated. As long as he preserved his sight, he assembled the whole of his family every evening, and read a chapter of the Bible, which he accompanied with an exhortation. Theology was one of his favourite studies, and the doctrines which he held were the most rigid doctrines of Calvinism.

The following is a list of the principal works which Euler published in a separate form. His papers, which appeared in the Memoirs of the Academies of Berlin and St. Petersburg, are extremely numerous; and he left behind him no fewer than two

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