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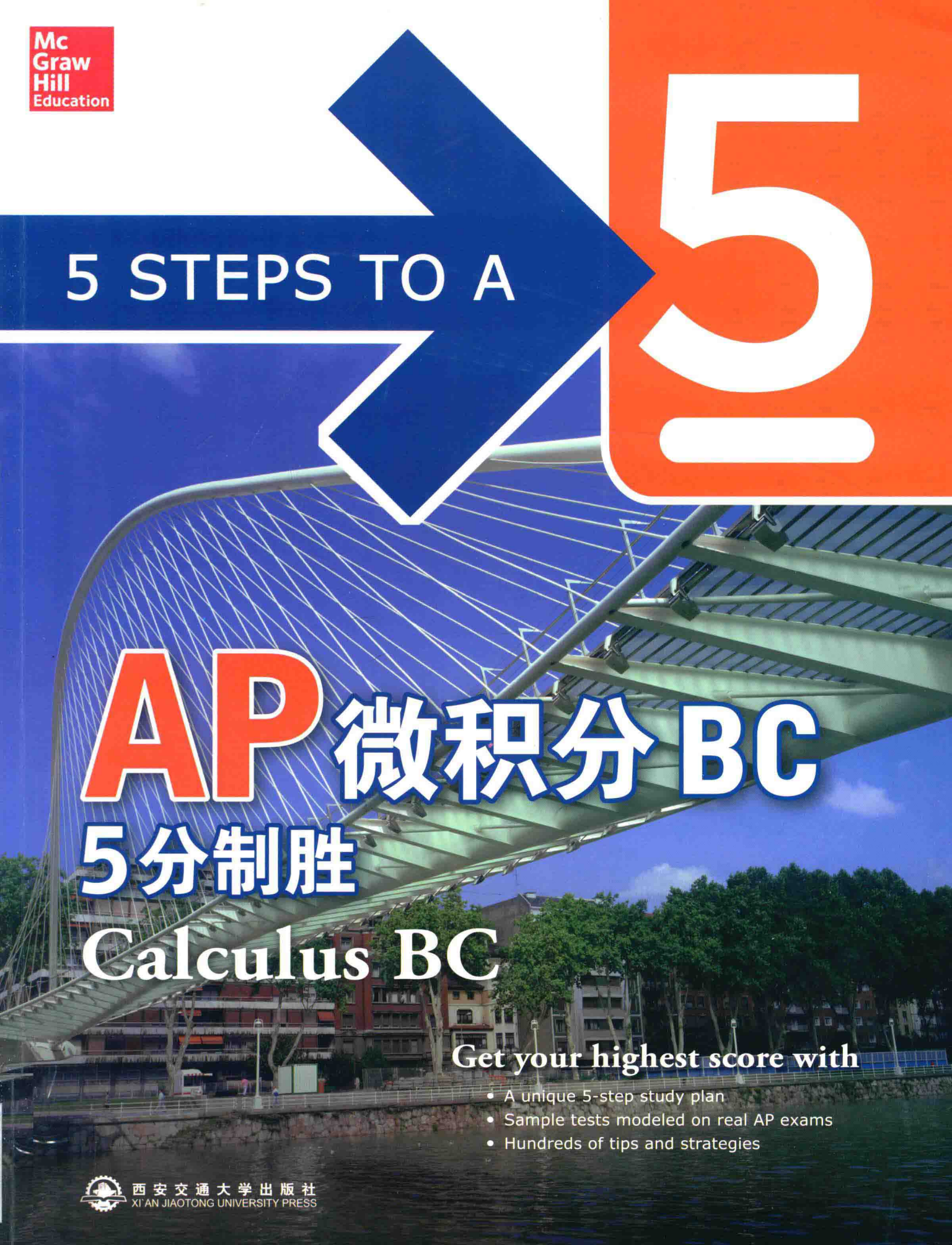
AP 微积分 BC 5分制胜

Calculus BC



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AP微积分 BC

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Calculus BC

(美) William Ma 编著

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前　言

AP项目（Advanced Placement Program）始于1955年，由美国大学理事会（the College Board）主持，是在高中阶段开设的具有大学水平的课程，即大学预修课程。AP项目目前设有34门课程和考试，它可以使有余力、有能力、成绩优秀的高中生有机会先修部分美国大学基础课程以获得大学学分，因此吸引了很多成绩优秀的学生选修。目前，已有60多个国家的几千所大学把AP学分作为其入学参考标准，其中包括哈佛大学、耶鲁大学、牛津大学、剑桥大学等世界知名大学。

美国每年约有200万高中毕业生，他们都要参加美国高考SAT和AP课程的考试。美国高中生会在11年级时完成SAT考试，在12年级（高中最后一年）完成两件大事：第一，根据SAT的考试成绩申请大学和奖学金；第二，选修AP课程，并进行备考。在高中选修AP课程和通过AP考试不仅是对学生能力和学业水平的证明，还可以使学生：1. 在申请大学时具有很大的优势。美国大学把学生在AP考试中的表现作为衡量其是否能够胜任大学学习的依据。从美国大学录取顾问委员会公布的影响大学录取因素的比较分析可以看出，AP成绩以80.3%的影响力位居第一，因为它向学校充分展示了学生的才智、专长及学习能力。2. 进入大学后，可以获得大学学分，免修同类课程，提早选修更高级的课程或跳级。3. 提前毕业。4. 节省大学学费。在美国，初等教育是免费的，但高等教育是收费的。选修的AP课程越多，免修的大学课程也就越多，节省的学费也就越多。另外，对中国学生而言，除了可以获得美国大学学分、省时省钱外，还可以在国内提前适应美国大学课程。

AP考试成绩的评定为5分制，满分5分表示极为优秀，4分为优秀，3分相当于合格，即可为大多数学校所接受，2分为可能有资格，1分则不予推荐。AP考试在每年5月份举行一次，为期两周。每门课程的考试时间约为2～3个小时，考试费用为每科1100元人民币或者1790港币左右。

更多信息可查询以下网站：

AP考试官网：http://www.collegeboard.com

AP国内报名网站：http://apchina.net.cn

香港考务局报名网址：https://www2.hkeaa.edu.hk

为满足国内考生对AP考试资料日益增长的需求，我们从美国知名教育出版公司McGraw-Hill Education引进了本系列AP考试丛书，共包括7本，分别为《AP微观经济学5分制胜》、《AP宏观

经济学5分制胜》、《AP微积分AB 5分制胜》、《AP微积分BC 5分制胜》、《AP美国历史5分制胜》、《AP化学5分制胜》和《AP生物5分制胜》。AP各学科分册由AP考试相关领域专家编写，精准把握考试命题特点，设计“五步”高效学习方案，总结与考试相关的学科内容和要点，精选针对性练习以及全真模拟试题，并配以答案和准确详尽的解析。本系列丛书适用于备考AP的所有考生，便于考生巩固所学，紧抓重点，取得高分。

本书为其中的《AP微积分BC 5分制胜》。

恭喜你！成为了一名AP微积分考生。众所周知，AP微积分是高中最具有挑战性的科目之一，你学习的数学概念曾经改变了世界。不久之前，微积分课程还只对大学生开放，而现在和你一样年轻聪明的考生在高中就开始了微积分学习。如果你在AP微积分BC考试中取得3分或以上的成绩，大多数大学都会给予学分。

怎样才能在AP微积分考试中取得优异成绩呢？怎样才能得到满分5分呢？其实你已经开始第一步了，因为你正在阅读这本书。你需要做的下一步就是确保读懂书中内容并做完全部习题。近年来，AP微积分考试发生了许多改变。例如，现在的试题不再强调冗长枯燥的代数运算，而是期望考生具备解决各种问题的能力，包括以图表或文字的形式出现的题目。对于很多题目，你都可以使用计算器来得出答案。

我们拥有多年AP微积分的教学经验，并与考生和其他教师有过很多交流，十分了解考生在AP微积分考试中会遇到的一些困难。例如，有些考生抱怨不能读懂题目的含义，也有考生说即使给出了解题方法，他们还是无法理解解题步骤。遇到这样的情况，谁能不感觉到沮丧呢？本书将为考生解决这些问题。书中的题目能附图表的都附有图表，解题方法会逐一呈现。本书从关于极限与连续的章节开始。这部分内容通常在初级微积分课程中教授。如果你对这些概念已经十分熟悉，那就可以略过这些内容，从第六章开始学习。

那么，怎样才能在AP微积分BC考试中取得满分5分呢？

第一步：从本书第二章中给出的三个学习计划中挑选一个适合你自己的。

第二步：利用第三章的诊断测试（Diagnostic Exam）来检测你的备考程度。

第三步：通过学习第四章提供的应试技巧，研究考试成功的策略。

第四步：通过学习第五章至第十四章的全部内容来巩固获取高分所必需的知识点。

第五步：利用本书提供的模拟测试（Practice Exams）来树立考试信心。

正如大部分AP考试专家所说的那样，“首先必须理解，然后练习。”

祝你好运！

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INTRODUCTION: THE FIVE-STEP PROGRAM

How is this Book Organized?

This book begins with an introduction to the Five-Step Program followed by 14 chapters reflecting the 5 steps.

- Step 1 provides an overview of the AP Calculus BC Exam, and offers three study plans for preparing for the Exam.
- Step 2 contains a diagnostic test with answers and explanations.
- Step 3 offers test-taking strategies for answering both multiple-choice and free-response questions, and for using a graphing calculator.
- Step 4 consists of 10 chapters providing a comprehensive review of all topics covered on the AP Calculus BC Exam. At the end of each chapter (beginning with Chapter 5), you will find a set of practice problems with solutions, a set of cumulative review problems with solutions, and a Rapid Review section giving you the highlights of the chapter.
- Step 5 provides three full practice AP Calculus BC Exams with answers, explanations, and worksheets to compute your score.

The book concludes with a summary of math formulas and theorems needed for the AP Calculus BC Exam. *(Please note that the exercises in this book are done with the TI-89 Graphing Calculator.)*

Introducing the Five-Step Preparation Program

This book is organized as a five-step program to prepare you to succeed in the AP Calculus BC Exam. These steps are designed to provide you with vital skills, strategies, and the practice that can lead you to that perfect 5. Here are the 5 steps.

Step 1: Set Up Your Study Plan

In this step you will read an overview of the AP Calculus BC Exam, including a summary of topics covered in the exam and a description of the format of the exam. You will also follow a process to help determine which of the following preparation programs is right for you:

- Full school year: September through May.
- One semester: January through May.
- Six weeks: Basic training for the exam.

Step 2: Determine Your Test Readiness

In this step you will take a diagnostic multiple-choice exam in calculus. This pre-test should give you an idea of how prepared you are to take the real exam before beginning to study for the actual AP Calculus BC Exam.

Step 3: Develop Strategies for Success

In this step you will learn strategies that will help you do your best on the exam. These strategies cover both the multiple-choice and free-response sections of the exam.

- Learn to read multiple-choice questions.
- Learn how to answer multiple-choice questions.
- Learn how to plan and write answers to the free-response questions.

Step 4: Review the Knowledge You Need to Score High

In this step you will learn or review the material you need to know for the test. This review section takes up the bulk of this book. It contains:

- A comprehensive review of AP Calculus BC.
- A set of practice problems.
- A set of cumulative review problems beginning with Chapter 5.
- A rapid review summarizing the highlights of the chapter.

Step 5: Build Your Test-Taking Confidence

In this step you will complete your preparation by testing yourself on practice exams. We have provided you with three complete practice exams in AP Calculus BC with solutions and scoring guides. Although these practice exams are not reproduced questions from the actual AP calculus exam, they mirror both the material tested by AP and the way in which it is tested.

Finally, at the back of this book you will find additional resources to aid your preparation. These include:

- A brief bibliography.
- A list of websites related to the AP Calculus BC exam.
- A summary of formulas and theorems related to the AP Calculus BC exam.

Introduction to the Graphics Used in this Book

To emphasize particular skills and strategies, we use several icons throughout this book. An icon in the margin will alert you that you should pay particular attention to the accompanying text. We use these icons:

This icon points out a very important concept or fact that you should not pass over.

This icon calls your attention to a strategy that you may want to try.

This icon indicates a tip that you might find useful.

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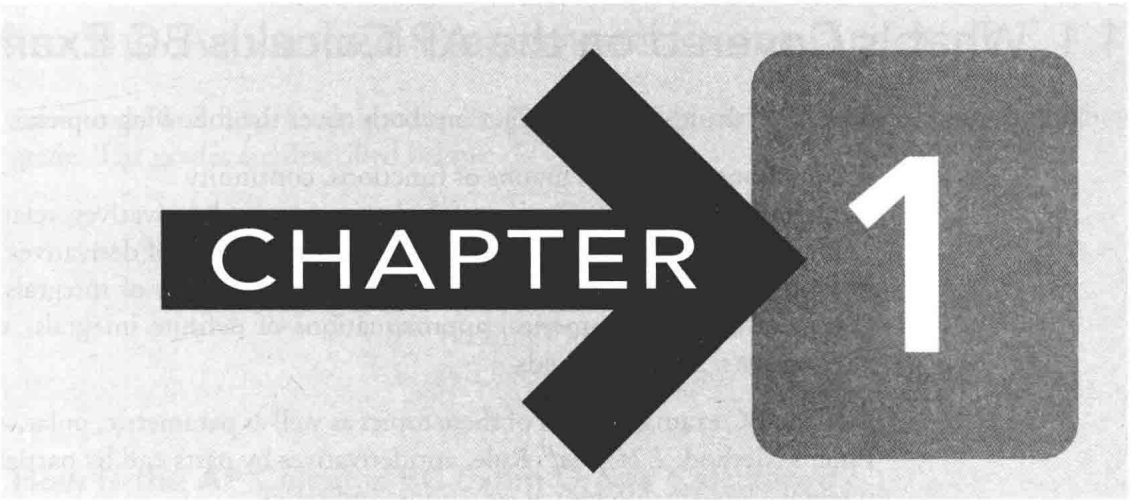
Set Up Your Study Plan

CHAPTER 1 What You Need to Know About the AP Calculus BC Exam

CHAPTER 2 How to Plan Your Time



What You Need to Know About the AP Calculus BC Exam

IN THIS CHAPTER

Summary: Learn what topics are tested in the exam, what the format is, which calculators are allowed, and how the exam is graded.

Key Ideas

- The AP Calculus BC exam covers all of the topics in the AB exam as well as additional topics including Euler's Method, logistic differential equations, series, and more.
- The AP Calculus BC exam has 45 multiple-choice questions and 6 free-response questions. Each of the two types of questions makes up 50% of the grade.
- Many graphing calculators are permitted on the exam, including the TI-98.
- You may bring up to two approved calculators for the exam.
- You may store programs in your calculator and you are not required to clear the memories in your calculator for the exam.

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