

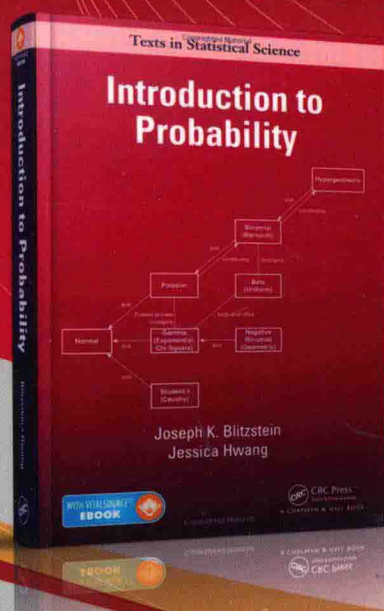
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概率论导论

(英文影印 中文导读版)

Introduction to Probability

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本书源自著名的哈佛大学统计学讲座，介绍了帮助读者理解统计方法、随机性和不确定性的基本语言和工具，并列举了多种多样的应用实例，内容涉及偶然性、概率悖论、谷歌 PageRank 及马尔可夫链蒙特卡罗方法 (MCMC) 等。本书还探讨了概率论在诸如基因学、药学、计算机科学和信息科学等领域的应用。

全书共分 13 章，分别介绍了概率与计数、条件概率、随机变量及其分布、数学期望、连续型随机变量、矩、联合分布、变换、条件期望、不等式与极限定理、马尔可夫链、马尔可夫链蒙特卡罗方法、泊松过程等内容。用容易理解的方式来呈现内容，用实例来揭示统计学中基本分布之间的联系，并通过条件化将复杂的问题归约为易于解决的若干小问题。书中还包含了很多直观的解释、图示和实践问题。每一章的结尾部分都给出了如何利用 R 软件来完成相关仿真和计算的方法。

本书可作为高等院校本科生概率论课程的教材，也可作为相关科研人员的参考书。

前言 (译)

本书通过现代的观点来介绍概率论，为理解统计方法、随机性和不确定性奠定了基础。书中包含了丰富的应用实例，从基本的抛硬币问题和偶然性的研究到谷歌PageRank算法以及马尔可夫链蒙特卡罗方法等。由于概率论是一门经常被认为是反直觉的学科，所以书中给出了很多可以凭直觉得到的解释、图示和实例以证明这个观点的偏颇。每章的结尾部分还结合R软件来更详细地探讨这一章的思想（R软件是一种用于统计计算和仿真的免费软件）。

本书取材于哈佛大学的视频公开课Stat110（从2006年起，这门课程每年均由Joseph讲授），课程视频可在stat110.net网站上免费获取。其他附加的补充材料，诸如R代码及标记了⑤的练习题的解答均可在该网站获取。

掌握微积分是学习本书的一个前提，而对统计学的基础则没有要求。数学方面的主要挑战不在于完成微积分求解，而在于能够在抽象的概念和具体的例子之间转换。

本书的主要特征概括如下：

1. 实例。书中的定义、定理和证明都是通过实例来呈现的，这种呈现既保留了数学的精确性又概括性地对现实世界的一些现象做出了解释。通过那些让概率分布广泛地在统计建模中使用的实例来探究概率分布。我们尽可能避免冗长乏味的推导，取而代之的是致力于给出解释和直觉判断来说明为什么那些主要结论是正确的。事实证明，通过深刻理解来替代死记硬背的方法可以提高学生对内容的长期记忆力。

2. 图。由于图本身就能表达很多内容，所以我们通过图来补充定义，使得那些主要概念与让人印象深刻的图相联系。在很多领域中，一名初学者与一名专家的差距常被描述如下：初学者总是努力去记住大量看似不相关的事实和公式，而专家则会领悟出一个统一的结构，在这个结构中仅通过少量的原理和思想就可将那些事实连贯地联系在一起。为了帮助学生领会概率论的结构，我们特别强调了思想间的联系（同时从语言上和视觉效果上加以巩固），并在大多数章节的结尾部分给出了概念与分布的循环、扩展图。

3. 概念和策略的双重教学。我们的目的在于让学生在读本书时不仅能够学习概率论的概念，同时还能够掌握广泛适用于概率论之外的一系列解决问题的策略。对于书中的例子，由于经常对相同的问题会给出多种不同的解答。我们对求解的每一步都进行了解释，同时也对如何思考并选择采用的方法进行了评述。

我们对诸如对称性和模式识别这样的重要策略进行了明确的标记和命名，并且通过给出了标有⚠（生物危害标识）的内容来消除常见误解。

4. 实践问题。本书包含大约600道不同难度的练习题。目的是为了让学生加强对内容的理解，同时强化他们解决问题的能力。这些练习题中有些是策略实践问题，根据主题进行了分组以促进对特定主题的实践，而有些则是混合型实践问题，在这些实践问题中需要综合一些前面章节中的内容。大约250道练习题已有详细的在线解答以

供线下实践及自学使用。

5. 仿真、蒙特卡罗方法和 R 软件。很多概率问题都因计算太难而不能精确求解，并且在任何情况下，对所给答案进行核查都是很重要的。我们介绍了通过仿真来研究概率论的方法，并证明了借助简短的几行 R 软件代码就足以对一个看似复杂的问题进行仿真。

6. 聚焦真实世界的关联性和统计思维。书中所有的例子和练习题都有明确的现实背景，都聚焦于如何为进一步学习统计推断和统计建模打下坚实的理论基础。我们简要介绍了重要的统计思想，例如抽样、仿真、贝叶斯推断和马尔可夫链蒙特卡罗方法，及其应用领域，包括基因学、药学、计算机科学和信息科学等。对例子和练习题的选择都是为了突出概率思维的力量、适用性及其美之所在。

致谢

感谢我们的同事、Stat110 的教学助理和数千位 Stat110 的学生所给出的与这门课程和这本书相关的评论及想法。特别要感谢 Alvin Siu、Angela Fan、Anji Tang、Carolyn Stein、David Jones、David Rosengarten、David Watson、Johannes Ruf、Kari Lock、Keli Liu、Kevin Bartz、Lazhi Wang、Martin Lysy、Michele Zempenyi、Peng Ding、Rob Phillips、Sam Fisher、Sebastian Chiu、Sofia Hou、Theresa Gebert、Valeria Espinosa、Viktoria Liublinska、Viviana Garcia、William Chen 和 Xander Marcus 对本书的反馈。尤其感谢 Bo Jiang、Raj Bhuptani、Shira Mitchell 和那些匿名的审稿人针对本书草稿所给出的详细评论，及 Andrew Gelman、Carl Morris、Persi Diaconis、Stephen Blyth、Susan Holmes 和 Xiao-Li Meng 关于概率的无数次有深刻见解的讨论。

CRC 出版社的 John Kimmel 在本书的写作过程中提供了极好的编辑上的专家意见，对他的支持深表感激。

最后，对我们的家人致以最深的谢意，感谢他们对我们的爱和鼓励。

Joe Blitzstein 和 Jessica Hwang
于 Cambridge, MA and Stanford, CA
2014 年 5 月

原书前言

This book provides a modern introduction to probability and develops a foundation for understanding statistics, randomness, and uncertainty. A variety of applications and examples are explored, from basic coin-tossing and the study of coincidences to Google PageRank and Markov chain Monte Carlo. As probability is often considered to be a counterintuitive subject, many intuitive explanations, diagrams, and practice problems are given. Each chapter ends with a section showing how to explore the ideas of that chapter in R, a free software environment for statistical calculations and simulations.

Lecture videos from Stat 110 at Harvard, the course which gave rise to this book (and which has been taught by Joe every year since 2006), are freely available at stat110.net. Additional supplementary materials, such as R code and solutions to exercises marked with (S), are also available at this site.

Calculus is a prerequisite for this book; there is no statistics prerequisite. The main mathematical challenge lies not in performing technical calculus derivations, but in translating between abstract concepts and concrete examples. Some major themes and features are listed below.

1. *Stories.* Throughout this book, definitions, theorems, and proofs are presented through stories: real-world interpretations that preserve mathematical precision and generality. We explore probability distributions using the generative stories that make them widely used in statistical modeling. When possible, we refrain from tedious derivations and instead aim to give interpretations and intuitions for why key results are true. Our experience is that this approach promotes long-term retention of the material by providing insight instead of demanding rote memorization.
2. *Pictures.* Since pictures are thousand-word stories, we supplement definitions with illustrations so that key concepts are associated with memorable diagrams. In many fields, the difference between a novice and an expert has been described as follows: the novice struggles to memorize a large number of seemingly disconnected facts and formulas, whereas the expert sees a unified structure in which a few principles and ideas connect these facts coherently. To help students see the structure of probability, we emphasize the connections between ideas (both verbally and visually), and at the end of most chapters we present recurring, ever-expanding maps of concepts and distributions.

3. *Dual teaching of concepts and strategies.* Our intent is that in reading this book, students will learn not only the concepts of probability, but also a set of problem-solving strategies that are widely applicable outside of probability. In the worked examples, we explain each step of the solution but also comment on how we knew to take the approach we did. Often we present multiple solutions to the same problem.

We explicitly identify and name important strategies such as symmetry and pattern recognition, and we proactively dispel common misunderstandings, which are marked with the ☣ (biohazard) symbol.

4. *Practice problems.* The book contains about 600 exercises of varying difficulty. The exercises are intended to reinforce understanding of the material and strengthen problem-solving skills instead of requiring repetitive calculations. Some are *strategic practice problems*, grouped by theme to facilitate practice of a particular topic, while others are *mixed practice*, in which several earlier topics may need to be synthesized. About 250 exercises have detailed online solutions for practice and self-study.
5. *Simulation, Monte Carlo, and R.* Many probability problems are too difficult to solve exactly, and in any case it is important to be able to check one's answer. We introduce techniques for exploring probability via simulation, and show that often a few lines of R code suffice to create a simulation for a seemingly complicated problem.
6. *Focus on real-world relevance and statistical thinking.* Examples and exercises in this book have a clear real-world motivation, with a particular focus on building a strong foundation for further study of statistical inference and modeling. We preview important statistical ideas such as sampling, simulation, Bayesian inference, and Markov chain Monte Carlo; other application areas include genetics, medicine, computer science, and information theory. Our choice of examples and exercises is intended to highlight the power, applicability, and beauty of probabilistic thinking.

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Joe Blitzstein and Jessica Hwang
Cambridge, MA and Stanford, CA
May 2014

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