普通高等教育“十三五”规划教材

高等电磁场理论

Advanced Theory of Electromagnetic Fields

徐晓文 ◎ 编著

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PREFACE

The knowledge of electromagnetic field and waves is one of the important fundamental theories in the field of electronic information and related ones. The contents in this book are based on the basic theory of electromagnetic fields for the undergraduate curriculum. Focusing on modeling and solving electromagnetic boundary value problems, this book further discusses some important principles, theorems and related concepts in electromagnetic fields, and some typical boundary value problem solution methods such as the analytical method, numerical method, dyadic Green's function method and high-frequency approximation method. The electromagnetic radiation and scattering models in this book include both conductor and dielectric structures in either the homogeneous or the inhomogeneous medium space.

There are eight topics in this book. The first chapter discusses five important theorems or principles in electromagnetic field theory; Chapters 2, 3 and 4 discuss the modal expansion method (MEM) solving electromagnetic boundary value problems, including the plane wave expansion, the cylindrical wave expansion and the spherical wave expansion; the fifth chapter introduces the basic model of electromagnetic radiation in the stratified media and its solution; the sixth chapter establishes the integral equation (IE) model for the electromagnetic radiation and scattering problems and discusses the basic principle of the method of moments (MoM); the seventh chapter introduces the high frequency methods in the solution of electromagnetic radiation and scattering problems, including the basic principles of physical optics (PO), geometrical optics (GO) and geometrical theory of diffraction (GTD); the eighth chapter presents the Green's function method (GFM) for solving the boundary value problems of electromagnetic radiation and scattering, namely the dyadic Green's function method (DGFM).

This book can be used in the teaching of the graduate students and senior undergraduate students of related majors in the electronic science and technology disciplines and also can be used for reference of relevant professionals and the technical personnel. Besides the basic theory of electromagnetic fields for the undergraduate curriculum, the prerequisites to learn the contents of this book also include the essential knowledge of the mathematical physics equations and special functions.

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徐晓文

Xiaowen XU

May 10, 2017

CONTENTS

Chapter One

Some Basic Principles [1]

1.1 Electromagnetic Duality

If the time factor is taken as $e^{j\omega t}$, the frequency domain of Maxwell's equations in the linear space can be written as follows, i.e.,

$$\nabla \times \boldsymbol{E} = -\mathrm{j}\omega\mu \boldsymbol{H} - \boldsymbol{K}, \quad (1.1a)$$

$$\nabla \bullet \boldsymbol{E} = \frac{\rho}{\varepsilon}, \quad (1.1b)$$

$$\nabla \bullet \boldsymbol{J} = -\mathrm{j}\omega\rho, \quad (1.1c)$$

$$\nabla \times \boldsymbol{H} = \mathrm{j}\omega\varepsilon \boldsymbol{E} + \boldsymbol{J}, \quad (1.1d)$$

$$\nabla \bullet \boldsymbol{H} = \frac{m}{\mu}, \quad (1.1e)$$

$$\nabla \bullet \boldsymbol{K} = -\mathrm{j}\omega m, \quad (1.1f)$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are respectively the electric and magnetic field in the space, $\boldsymbol{J}$ and $\boldsymbol{K}$ respectively the electric and magnetic current density, ρ and m respectively the electric and magnetic charge density, ε and μ respectively the permittivity and permeability, and ω the angular frequency.

According to Eq.(1.1), we can obtain the following electromagnetic duality relationships, i.e., in the case of replacing the electric quantities with the magnetic ones, they should be

$$\boldsymbol{E} \to \boldsymbol{H} \quad \boldsymbol{J} \to \boldsymbol{K} \quad \rho \to m \quad \varepsilon \leftrightarrow \mu \quad \eta \to 1/\eta \quad k \to k; \quad (1.2)$$

in the case of replacing the magnetic quantities with the electric ones, then they should be

$$\boldsymbol{H} \to -\boldsymbol{E} \quad \boldsymbol{K} \to -\boldsymbol{J} \quad m \to -\rho \quad \mu \leftrightarrow \varepsilon \quad \eta \to 1/\eta \quad k \to k . \tag{1.3}$$

The electromagnetic duality is very helpful to the memory of formulas and the simplification of analysis. For example, suppose that when there is a certain uniform electric current distribution, $\boldsymbol{J} = \hat{x}J_0$, in the xOy ($z=0$) plane, the electric field in the space is obtained as $E_x = \begin{cases} -\eta \dfrac{J_0}{2} \mathrm{e}^{-\mathrm{j}kz} & (z>0) \\ -\eta \dfrac{J_0}{2} \mathrm{e}^{\mathrm{j}kz} & (z<0) \end{cases}$. Then, according to the duality principle, for a certain uniform magnetic current distribution, $\boldsymbol{K} = \hat{x}K_0$, in the xOy ($z=0$) plane, one can directly write out the magnetic field in the space as $H_x = \begin{cases} -\dfrac{K_0}{2\eta} \mathrm{e}^{-\mathrm{j}kz} & (z>0) \\ -\dfrac{K_0}{2\eta} \mathrm{e}^{\mathrm{j}kz} & (z<0) \end{cases}$.

1.2 Theorem of Uniqueness

It is very important to know whether or not a problem has a unique solution. The reasons for this can be summarized as follows.

1) The theorem of uniqueness points out the essential conditions to obtain such a solution.

2) The theorem of uniqueness allows several different methods to be used in the evaluation, so as to increase the efficiency.

3) The theorem of uniqueness establishes the field-to-source one-by-one corresponding conditions, so as to be able to evaluate the source (or sources) from the fields, or vice versa.

The theorem of uniqueness can be obtained as follows.

Consider the following closed surface S in a linear medium space, within which a group of electric and magnetic sources $\boldsymbol{J}$ and $\boldsymbol{K}$ are enclosed, as shown in Fig.1.1.

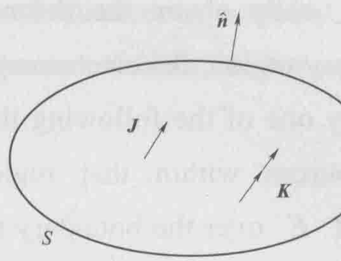

Fig.1.1 A closed surface S enclosing the sources $\boldsymbol{J}$ and $\boldsymbol{K}$ in the linear space

Suppose that in the space there are two groups of possible field solutions $(\boldsymbol{E}^a, \boldsymbol{H}^a)$ and $(\boldsymbol{E}^b, \boldsymbol{H}^b)$. Substitute them respectively in the curl equations in Eq.(1.1) and subtract the corresponding two results, and then we have

$$-\nabla \times \delta \boldsymbol{E} = \mathrm{j}\omega\mu\delta\boldsymbol{H}, \tag{1.4a}$$

$$\nabla \times \delta \boldsymbol{H} = \mathrm{j}\omega\varepsilon\delta\boldsymbol{E}, \tag{1.4b}$$

where $\delta\boldsymbol{E} = \boldsymbol{E}^a - \boldsymbol{E}^b$ and $\delta\boldsymbol{H} = \boldsymbol{H}^a - \boldsymbol{H}^b$ stand for the electric and magnetic difference fields, respectively.

By applying the complex Poynting theorem to the difference fields, we can obtain

$$\oint\!\!\!\oint (\delta\boldsymbol{E} \times \delta\boldsymbol{H}^*) \bullet \mathrm{d}\boldsymbol{s} + \iiint (\tilde{z}\left|\delta H\right|^2 + \tilde{y}^*\left|\delta E\right|^2)\mathrm{d}v = 0, \tag{1.5}$$

where $\tilde{z} = \mathrm{j}\omega\mu$, $\tilde{y} = \mathrm{j}\omega\varepsilon$, and the asterisk "*" represents the conjugate operation. If on the surface S

$$\oint\!\!\!\oint (\delta\boldsymbol{E} \times \delta\boldsymbol{H}^*) \bullet \mathrm{d}s = 0, \tag{1.6}$$

then we have

$$\iiint [\mathrm{Re}(\tilde{z})\left|\delta H\right|^2 + \mathrm{Re}\,(\tilde{y})\left|\delta E\right|^2]\,\mathrm{d}v = 0, \tag{1.7a}$$

$$\iiint [\mathrm{Im}(\tilde{z})\left|\delta H\right|^2 - \mathrm{Im}(\tilde{y})\left|\delta E\right|^2]\,\mathrm{d}v = 0. \tag{1.7b}$$

For lossy media, $\mathrm{Re}(\tilde{z})$ and $\mathrm{Re}(\tilde{y})$ are always positive. Therefore, as long as there is some loss in the space (no matter how small), it is required that $\delta \boldsymbol{E} = \delta \boldsymbol{H} = 0$ everywhere inside *S*.

Consequently, we can finally obtain the theorem of uniqueness. This theorem states that, in the lossy region, the electromagnetic field solution will be uniquely specified in any one of the following three situations. The first situation is when the sources within that region and the tangential components of electric field $\boldsymbol{E}$ over the boundary surface *S* are given. The second one is when the sources within that region and the tangential components of magnetic field $\boldsymbol{H}$ over the boundary surface *S* are obtained. The last one is when the sources within that region and the tangential components of electric field $\boldsymbol{E}$ over part of the boundary surface *S* and the tangential components of magnetic field $\boldsymbol{H}$ over the rest of it are presented.

According to the concept of limit, it is easily shown that the above theorem also applies to all of the cases including that for the lossless region, the singular sources and the open space.

1.3 Principle of Equivalence

The two or more sources that can produce the same fields in a certain space are referred to as equivalent in that space. The principle of equivalence is based on the previously discussed theorem of uniqueness.

First, let's consider the general case of this principle. Suppose in the linear medium space there exist some electromagnetic current sources, as shown in Fig.1.2 (a) and Fig.1.2 (b).

We may establish such an equivalent problem that it is identical to problem (a) in the outer space of *S* and at the same time to problem (b) inside *S*, as shown in Fig.1.2(c). This can be done as follows, that is, in the outer space maintain the same fields, medium (or media) and sources as in problem (a), and in the inner space maintain the same fields, medium (or media) and sources as in problem (b). To support such field distributions, the equivalent current densities $\boldsymbol{J}_s$ and $\boldsymbol{K}_s$ on the surface have to be introduced by which will, according to the theorem of uniqueness, certainly present the supposed field distributions. Similarly, we may establish another equivalent problem as shown in Fig.1.2 (d).

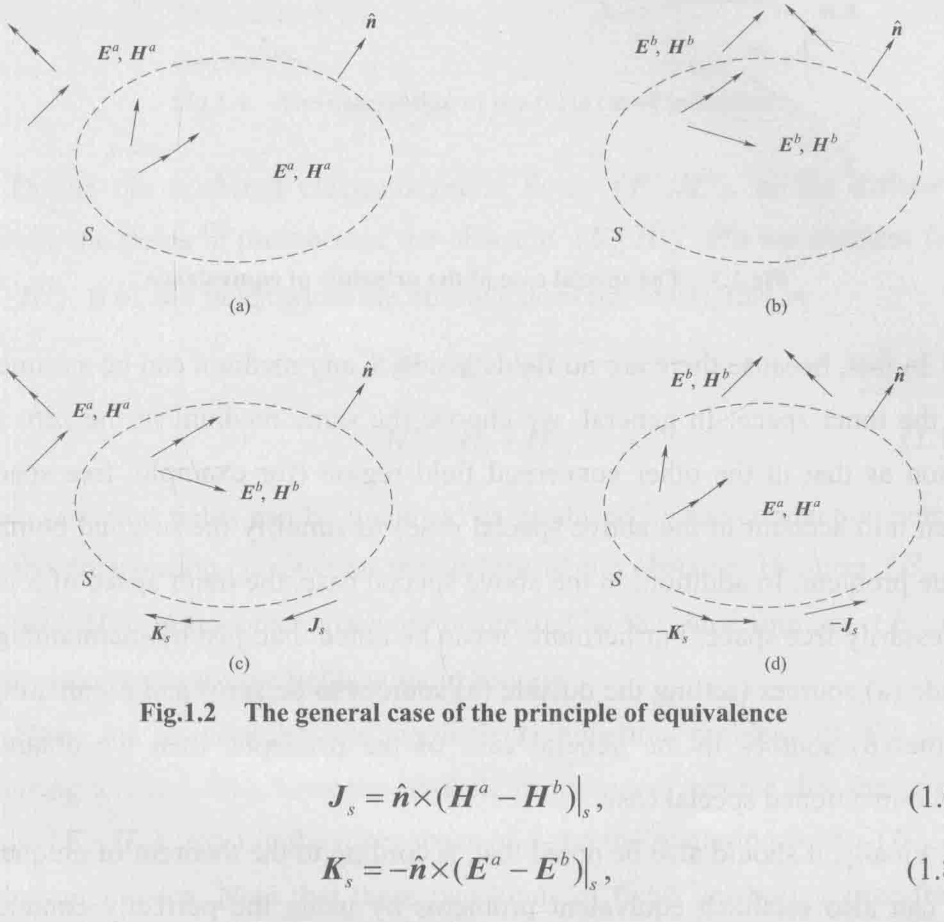

Fig.1.2 The general case of the principle of equivalence

$$\boldsymbol{J}_s = \hat{\boldsymbol{n}} \times (\boldsymbol{H}^a - \boldsymbol{H}^b)\big|_S, \tag{1.8a}$$

$$\boldsymbol{K}_s = -\hat{\boldsymbol{n}} \times (\boldsymbol{E}^a - \boldsymbol{E}^b)\big|_S, \tag{1.8b}$$

Next, let's consider the more often used special case of the principle. As shown in Fig.1.3(a), in a closed boundary surface S there are some sources (e.g. transmitters or antennas), and the outside of S is free space. Now, if we only care about the distributions of the electromagnetic fields outside S, then we can establish the equivalent problem by maintaining the original outside fields in Fig.1.3(a), setting zero fields inside S and considering everywhere being in free space, as shown in Fig.1.3(b). Similarly, to support such field distributions, on the surface S there must exit the following equivalent surface current, i.e.,

$$\boldsymbol{J}_s = \hat{\boldsymbol{n}} \times \boldsymbol{H}|_s , \tag{1.9a}$$

$$\boldsymbol{K}_s = -\hat{\boldsymbol{n}} \times \boldsymbol{E}\big|_s . \tag{1.9b}$$

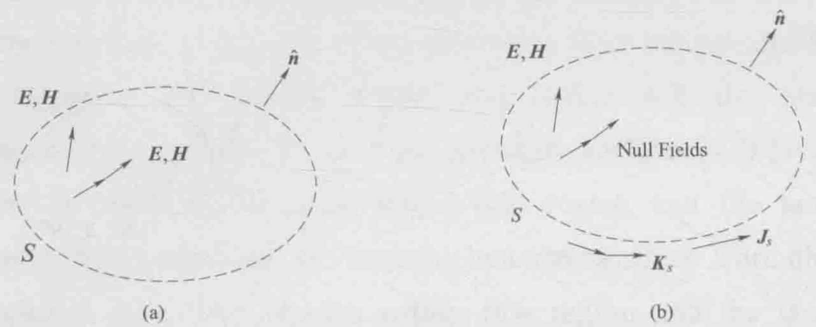

Fig.1.3 The special case of the principle of equivalence

In fact, because there are no fields inside S, any medium can be assumed to fill the inner space. In general, we choose the same medium in the zero field region as that in the other concerned field region (for example, free space is taken into account in the above special case) to simplify the original boundary value problem. In addition, in the above special case, the outer space of S is not necessarily free space. Furthermore, it can be noted that, just by maintaining the inside (a) sources (setting the outside (a) sources to be zero) and eliminating all of the (b) sources in the general case of the principle, then we obtain the above-mentioned special case.

Finally, it should also be noted that, according to the theorem of uniqueness, we can also establish equivalent problems by using the perfectly conducting electric wall and the equivalent magnetic current, or by using the perfectly conducting magnetic wall and the equivalent electric current.

1.4 Theorem of Induction

Consider the following radiation problem, where a set of sources are in presence of an obstacle, as shown in Fig.1.4(a).

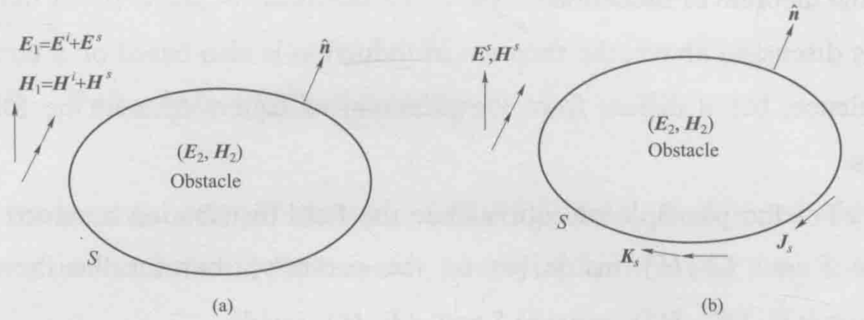

Fig.1.4 The description of the theorem of induction

Define the scattered electromagnetic fields $(\boldsymbol{E}^s, \boldsymbol{H}^s)$ as the differences between the fields in presence of the obstacle $(\boldsymbol{E}_1, \boldsymbol{H}_1)$ and the incident fields $(\boldsymbol{E}^i, \boldsymbol{H}^i)$ (i.e., the fields when the obstacle does not exist), that is,

$$\boldsymbol{E}^s = \boldsymbol{E}_1 - \boldsymbol{E}^i, \tag{1.10a}$$

$$\boldsymbol{H}^s = \boldsymbol{H}_1 - \boldsymbol{H}^i. \tag{1.10b}$$

These scattered fields can be imagined as produced by the conduction currents and the polarization currents on the surface of the obstacle. Because $(\boldsymbol{E}_1, \boldsymbol{H}_1)$ and $(\boldsymbol{E}^i, \boldsymbol{H}^i)$ in the outer space are originated by the same sources, $(\boldsymbol{E}^s, \boldsymbol{H}^s)$ in the outer space are the fields without sources.

Now, let us establish the equivalent induction problem of Fig.1.4(a). Referring to Fig.1.4(b), keep the obstacle in presence, assume that the original fields $(\boldsymbol{E}_2, \boldsymbol{H}_2)$ exist in the inner space of *S* and the scattering fields $(\boldsymbol{E}^s, \boldsymbol{H}^s)$ in the outer space. Note that these two kinds of fields are both source-free in their corresponding existing region. To support such field distributions, the equivalent surface currents on *S* have to be defined by

$$\boldsymbol{J}_s = \hat{\boldsymbol{n}} \times (\boldsymbol{H}^s - \boldsymbol{H}_2)\Big|_S = \hat{\boldsymbol{n}} \times (\boldsymbol{H}^s - \boldsymbol{H}_1)\Big|_S = -\hat{\boldsymbol{n}} \times \boldsymbol{H}^i\Big|_S, \tag{1.11a}$$

$$\boldsymbol{K}_s = -\hat{\boldsymbol{n}} \times (\boldsymbol{E}^s - \boldsymbol{E}_2)\Big|_S = -\hat{\boldsymbol{n}} \times (\boldsymbol{E}^s - \boldsymbol{E}_1)\Big|_S = \hat{\boldsymbol{n}} \times \boldsymbol{E}^i\Big|_S. \tag{1.11b}$$

According to the theorem of uniqueness, the above-defined $\boldsymbol{J}_s$ and $\boldsymbol{K}_s$ must be able to produce the assumed field distributions shown in Fig.1.4(b), and this gives the theorem of induction.

As discussed above, the theorem of induction is also based on a concept of equivalence, but it differs from the principle of equivalence in the following aspects.

(1) For the principle of equivalence the field distribution assumed is zero outside S and $(\boldsymbol{E},\boldsymbol{H})$ inside (or on the contrary), but for the theorem of induction it is $(\boldsymbol{E}^s,\boldsymbol{H}^s)$ outside S and $(\boldsymbol{E},\boldsymbol{H})$ inside.

(2) For the principle of equivalence the equivalent surface currents $\boldsymbol{J}_s=-\hat{\boldsymbol{n}}\times\boldsymbol{H}\big|_s$ and $\boldsymbol{K}_s=\hat{\boldsymbol{n}}\times\boldsymbol{E}\big|_s$ are unknown, but for the theorem of induction the equivalent surface currents $\boldsymbol{J}_s=-\hat{\boldsymbol{n}}\times\boldsymbol{H}^i\big|_s$ and $\boldsymbol{K}_s=\hat{\boldsymbol{n}}\times\boldsymbol{E}^i\big|_s$ are known.

(3) For the principle of equivalence the medium outside S can be chosen as the same as that of the obstacle, so that the original boundary problem reduces to an unbounded uniform space radiation problem. However, for the theorem of induction the boundary still exists, or in other words, the original boundary problem is only changed into another boundary problem.

When the obstacle is a perfect electric conductor, we can obtain the simplified theorem of induction as shown in Fig.1.5, where $\boldsymbol{K}_s=\hat{\boldsymbol{n}}\times\boldsymbol{E}^i\big|_s$.

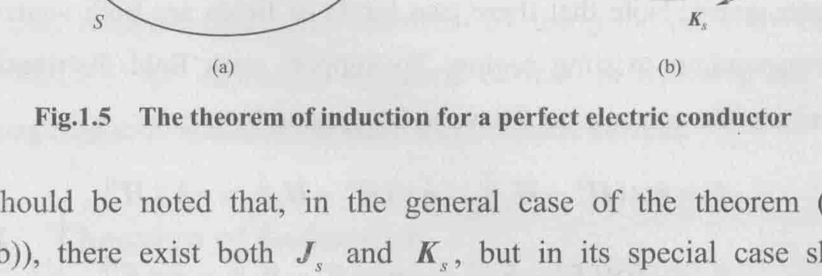

Fig.1.5 The theorem of induction for a perfect electric conductor

It should be noted that, in the general case of the theorem (refer to Fig.1.4(b)), there exist both $\boldsymbol{J}_s$ and $\boldsymbol{K}_s$, but in its special case shown in Fig.1.5(b), we only present the equivalent magnetic current $\boldsymbol{K}_s$. This is not because the equivalent electric current $\boldsymbol{J}_s$ does not exist, but because it makes no contribution to the total fields. The proof of this conclusion will be presented in section 1.5 by using the principle of reciprocity.

1.5 Principle of Reciprocity

Consider two sets of sources $(\boldsymbol{J}^a, \boldsymbol{K}^a)$ and $(\boldsymbol{J}^b, \boldsymbol{K}^b)$ of the same frequency in the same medium space. According to Maxwell's equations, we obtain

$$\nabla \times \boldsymbol{H}^a = \tilde{y}\boldsymbol{E}^a + \boldsymbol{J}^a, \tag{1.12a}$$

$$-\nabla \times \boldsymbol{E}^a = \tilde{z}\boldsymbol{H}^a + \boldsymbol{K}^a, \tag{1.12b}$$

$$\nabla \times \boldsymbol{H}^b = \tilde{y}\boldsymbol{E}^b + \boldsymbol{J}^b, \tag{1.12c}$$

$$-\nabla \times \boldsymbol{E}^b = \tilde{z}\boldsymbol{H}^b + \boldsymbol{K}^b, \tag{1.12d}$$

where $(\boldsymbol{E}^a, \boldsymbol{H}^a)$ and $(\boldsymbol{E}^b, \boldsymbol{H}^b)$ are the electromagnetic fields produced by $(\boldsymbol{J}^a, \boldsymbol{K}^a)$ and $(\boldsymbol{J}^b, \boldsymbol{K}^b)$, respectively. Making the scalar product of $\boldsymbol{E}^b$ and Eq.(1.12a) and the scalar product of $\boldsymbol{H}^a$ and Eq.(1.12d), adding the two results, and making use of the identity

$$\nabla \bullet (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \bullet (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \bullet (\nabla \times \boldsymbol{B}), \tag{1.13}$$

we further obtain

$$-\nabla \bullet (\boldsymbol{E}^b \times \boldsymbol{H}^a) = \tilde{y}\boldsymbol{E}^a \bullet \boldsymbol{E}^b + \tilde{z}\boldsymbol{H}^a \bullet \boldsymbol{H}^b + \boldsymbol{E}^b \bullet \boldsymbol{J}^a + \boldsymbol{H}^a \bullet \boldsymbol{K}^b. \tag{1.14}$$

Exchanging a and b in Eq.(1.14), we also have

$$-\nabla \bullet (\boldsymbol{E}^a \times \boldsymbol{H}^b) = \tilde{y}\boldsymbol{E}^a \bullet \boldsymbol{E}^b + \tilde{z}\boldsymbol{H}^a \bullet \boldsymbol{H}^b + \boldsymbol{E}^a \bullet \boldsymbol{J}^b + \boldsymbol{H}^b \bullet \boldsymbol{K}^a. \tag{1.15}$$

Making the subtraction of Eq.(1.14) and Eq.(1.15), we finally obtain

$$-\nabla \bullet (\boldsymbol{E}^a \times \boldsymbol{H}^b - \boldsymbol{E}^b \times \boldsymbol{H}^a) = \boldsymbol{E}^a \bullet \boldsymbol{J}^b + \boldsymbol{H}^b \bullet \boldsymbol{K}^a - \boldsymbol{E}^b \bullet \boldsymbol{J}^a - \boldsymbol{H}^a \bullet \boldsymbol{K}^b, \tag{1.16}$$

from which we can further deduce the principle of reciprocity.

1.5.1 Principle of Reciprocity in Source-Free Space

The source-free principle of reciprocity is also referred to as Lorenz

Principle of Reciprocity, which can be obtained by setting $\boldsymbol{J}=\boldsymbol{K}=0$ in Eq.(1.16), that is,

$$\nabla \cdot (\boldsymbol{E}^a \times \boldsymbol{H}^b - \boldsymbol{E}^b \times \boldsymbol{H}^a) = 0. \tag{1.17}$$

By using the theorem of divergence, the integral form of the principle can be written as

$$\oiint (\boldsymbol{E}^a \times \boldsymbol{H}^b - \boldsymbol{E}^b \times \boldsymbol{H}^a) \cdot \mathrm{d}\boldsymbol{s} = 0. \tag{1.18}$$

1.5.2 General Principle of Reciprocity

By integrating both sides of Eq.(1.16) in the entire space containing sources, we obtain

$$-\oiint (\boldsymbol{E}^a \times \boldsymbol{H}^b - \boldsymbol{E}^b \times \boldsymbol{H}^a) \cdot \mathrm{d}\boldsymbol{s} = \iiint (\boldsymbol{E}^a \cdot \boldsymbol{J}^b - \boldsymbol{H}^a \cdot \boldsymbol{K}^b - \boldsymbol{E}^b \cdot \boldsymbol{J}^a + \boldsymbol{H}^b \cdot \boldsymbol{K}^a)\mathrm{d}v. \tag{1.19}$$

Suppose that all the sources and media exist in a limited region, the left side of Eq.(1.19) then becomes zero when the radius of the closed integration surface S extends to the infinity ($r \to \infty$). Therefore, we have

$$\iiint (\boldsymbol{E}^a \cdot \boldsymbol{J}^b - \boldsymbol{H}^a \cdot \boldsymbol{K}^b)\mathrm{d}v = \iiint (\boldsymbol{E}^b \cdot \boldsymbol{J}^a - \boldsymbol{H}^b \cdot \boldsymbol{K}^a)\,\mathrm{d}v, \tag{1.20}$$

where the integration includes the total space.

In fact, Eq.(1.20) is also true for a finite region as long as Eq.(1.18) holds true.

1.5.3 Reaction

The integral in Eq.(1.20) does not stand for the power because it does not include the conjugation. We define the following integral as the reaction of field a to source b, i.e.,

$$\langle a,b \rangle = \iiint (\boldsymbol{E}^a \cdot \boldsymbol{J}^b - \boldsymbol{H}^a \cdot \boldsymbol{K}^b)\mathrm{d}v. \tag{1.21}$$