A Concise Bilingual Course of Advanced Mathematics (Part Ⅱ)

高等数学简明双语教程

（下册）

平艳茹 姚海楼 编著

$$S_{\Delta}=\sqrt{p(p-a)\cdot(p-b)\cdot(p-c)}=p\cdot r$$

北京工业大学出版社

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内 容 简 介

本书按照教育部对理工类和金融类本科生高等数学课程的要求编写，满足高等数学课程的双语教学要求。本书分上、下两册。下册的内容是微分方程、无穷级数、向量与空间解析几何、多元函数微分学、多重积分。

本书可作为高校工科本科生一年级高等数学双语课程的教材，也可作为数学专业的专业英语教材，还可供相关专业的教师和科研人员参考。

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Preface

Advanced Mathematics is a basic course for the college students, and is also a discipline with highly logical, abstract and rigorous academic curriculum system. At the same time, the level of acquiring the curriculum knowledge directly affects the college students' subsequent academic study. With the higher education becoming increasingly international and trying to be in line with international standards, it is extremely necessary to carry out bilingual teaching in colleges and universities. Because of the strong logicality in the Advanced Mathematics, the bilingual teaching can enable students to change the inherent passive exam – oriented thinking mode. Cultivating talents is the starting point and also the ultimate goal of bilingual teaching. In the world of natural science, knowledge updates at an alarming rate. The most part of information and materials on science and technology is published in English globally. A good command of English in terms of the mathematical knowledge helps to keep up with the latest achievements in the natural science abroad. By means of bilingual teaching the students can enlarge their vocabulary, especially the mathematical terms in foreign language, and show keen interest in learning. In this way, the students can apply the knowledge of English to math learning. What's more important, they can realize that learning is very practical, helping lay the foundation referring to data in foreign language in the future. Bilingual teaching of Advanced Mathematics can not only improve students' communication skills, but also stimulate their learning potentials to acquire mathematical knowledge when they use English as a tool. Offering the bilingual teaching in Advanced Mathematics helps cultivate the students' abilities both in Mathematics and English. The book is completed under this background.

This book is exclusively designed for undergraduates major in engineering as bilingual Advanced Mathematics course. It can also be used as a reference book for teachers and students of the similar level and interests.

The outstanding advantage of this textbook lies in its brevity and clarity. We try to combine the advantages of foreign original teaching materials and the domestic textbooks, making it possible for the students to learn with ease and interest. Because of our efforts in carefully selecting the materials with the proper level, the students can enhance their reading ability, the ability to think and the ability to solve practical problems. The textbook is also characterized by its integrity, being systematic as well as intuitive and practical.

The textbook emphasizes the basic ideas in calculus, such as the concept of local linearization, the method of approximation, the method of optimization, the micro – element method and variable substitutions. In order to cultivate the students' consciousness, interest and ability to solve practical problems, examples and exercises are chosen to relate to practical problems.

The books learn from the original books, showing some strengths of foreign materials, considering the actual situation of the students, embodying the editors' rich experience in several rounds of bilingual teaching. But owing to our limitation in some aspects, the books are not perfect in a way. So, comments are welcomed from readers. At the same time, the books are on the way to be improved constantly through the editor's practice in bilingual teaching.

The books are supported by textbook publishing fund in Beijing University of Technology. In the end, we would like to acknowledge those who offer help to the completion of the books.

前　言

高等数学是一门公共基础课，也是一门逻辑性很强、高度抽象且具有严谨课程体系的学科。同时，该课程知识掌握的程度直接关系到大学生后续课程的学习。随着高校国际化办学的日益发展，以及高等教育与国际接轨，在高校里开展双语教学显得尤为重要。由于高等数学的逻辑性强，通过双语教学能促进学生改变固有的、被动的、应试式的思维模式。培养优秀人才是双语教学的出发点，也是双语教学的最终目标。在自然科学领域，知识更新速度飞快，国际上的科技资料绝大部分是用英语发表的，掌握英语中有关数学的相关知识，有助于吸收国外优秀自然科学成果。高等数学双语教学不但能使学生在英语交流能力上获得提高，还可以使其以英语为工具获得数学知识，更能够激发学生的学习潜能。开展高等数学双语教学有助于数学与英语的相互促进，本书正是在这种背景下完成的。

本书主要是为高校工科本科生开设的高等数学双语课程而编写的，可作为高等数学双语课程的教材或参考用书。

本书借鉴了国外原版教材浅显易懂的优势，解决了国外教材课外阅读量大和国内教材偏难的问题，简洁易懂，可帮助学生轻松进入阅读原版数学书籍的状态。本书在内容上紧紧围绕工科数学的特点，既照顾到数学内容的完整性与系统性，又照顾到工科学生要求教材讲解内容的直观性与实用性的特点。本教材强调了微积分的基本思想与方法：局部线性化方法、逼近的思想、最优化方法、微元法及变量代换的思想和方法等。为了培养学生应用数学解决实际问题的意识、兴趣和能力，本书中还挑选了一些与实际问题相关的例题和习题。

本书吸收了国外教材的一些长处，结合了学生的实际情况，融入了编者长期双语教学的经验。限于编者的水平，书中难免有不妥之处，恳请读者批评指正。同时，本书在编者今后的双语教学实践中有待不断完善与改进。

本书的出版得到了北京工业大学教材出版基金的资助。最后，衷心感谢为本书的完成做出奉献的人们。

目　录

Chapter 6 Differential Equations

Perhaps the most important of all the applications of calculus is to differential equations. When social or physical scientists use calculus, more often than not it is to analyze a differential equation that has arisen in the process of modeling some phenomena they are studying. The primary object of this chapter is to develop techniques for solving basic types of ordinary differential equations.

6.1 Basic Concepts of Differential Equations

6.1.1 Examples of Differential Equations

Example 6.1.1.1 (Models of Population Growth) The growth of a population is based on the assumption that the population grows at a rate directly proportional to the size of the population, which is based on ideal conditions (unlimited environment, adequate nutrition, absence of predators, immunity from disease).

Let t stand for time, and p stand for the size of population, then the $\frac{dp}{dt}$ is the rate of growth of the population. So the assumption that the rate of growth of the population is proportional to the population size is written as the equation

$$\frac{dp}{dt} = kp, \tag{6.1.1}$$

where k is the proportionality constant.

Example 6.1.1.2 Suppose that a body of mass m falls freely from somewhere at a standstill, if we can ignore the air resistance, try to find the relation between the displacement s

and time t.

Solution Take s axis downward, and build up coordinate system as Figure 6.1.1. By Newton's Second Law ($F = ma$, Force equals mass times acceleration), $a = \frac{d^2s}{dt^2}$, $F = mg$, we have

$$m \cdot \frac{d^2s}{dt^2} = mg, \text{ i.e. } s'' = g \tag{6.1.2}$$

and s also satisfies the condition

$$\begin{cases} s(0) = 0, \\ s'(0) = 0. \end{cases} \tag{6.1.3}$$

Equation(6.1.3) is called initial conditions of equation(6.1.2).

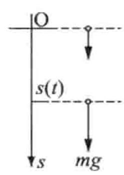

Figure 6.1.1 Free fall

6.1.2 Basic Concepts

Definition 6.1.2.1 (Differential Equation) An equation that connects the independent variable, unknown function and its derivatives is called a differential equation.

Generally, a differential equation can be expressed as follows

$$F(x, y, y', y'', \cdots, y^{(n)}) = 0.$$

If the unknown function y is of one variable, the differential equation is called ordinary equation.

Definition 6.1.2.2 (The Order of An Equation) The order of the differential equation is the order of the highest derivative that appears.

For example, $s'' = g$ is of second order.

Definition 6.1.2.3 (Solution, General Solution and Initial Conditions, Particular Solution) A solution of an equation is any function $y = f(x)$, which satisfies the equation.

The general solution of a n-th order differential equation is a family of solutions, which depend on n arbitrary and mutually independent constants.

If a solution without any arbitrary constant, then the solution is called a particular solution.

The supplementary conditions to determine a particular solution are called the initial conditions.

Example 6.1.2.1 Check whether $y = c_1\cos x + c_2\sin x$ is the general solution of the differential equation $y'' + y = 0$ or not.

Solution $y' = -c_1\sin x + c_2\cos x$, $y'' = -c_1\cos x - c_2\sin x$, $y'' + y = 0$ is satisfied. So $y = c_1\cos x + c_2\sin x$ is the general solution.

Exercise 6.1

1. Try to find the orders of the following differential equations.

(1) $x(y')^2 - 2y'y'' + xy = 0$;

(2) $x^2y'' - xy' + y = 0$;

(3) $x^2y''' + 4y'' - (\sin x)y = 0$;

(4) $\frac{dy}{dx} + y = \sin^2 x$.

2. Find a differential equation of the curve that passes through the point $(0,1)$ and whose slope of the tangent line at point (x,y) is xy.

6.2 First-order Differential Equations

In this section, we will deal with four kinds of first-order differential equations.

6.2.1 First-order Separable Differential Equation

Definition 6.2.1 (First-order Separable Differential Equation) A first-order separable differential equation is of the form

$$y' = g(x) \cdot h(y). \tag{6.2.1}$$

To solve a separable differential equation, we just follows two steps.

Step 1 Separate variables, that is, transform $y' = g(x) \cdot h(y)$ into the following

$$\frac{dy}{h(y)} = g(x)\,dx.$$