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离散系统网络化控制理论

——传输速率定理

刘庆泉 金 芳 丁国华 袁志华 / 著

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Data-Rate Theorem

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内 容 简 介

基于物联网的网络化控制技术正在逐渐形成，并受到关注，网络化控制技术主要应用于大城市交通系统的实时指挥和控制，航空工业中的飞机导航自动化控制，石油化工和冶金等连续流程工业的生产控制和调整，战术导弹的数字化制导控制，网络赋能弹药控制等。不同于以往的研究成果，本书针对网络通信信道传输速率受限的情况，研究了线性离散系统的网络化控制问题，在多种情况下证明了确保系统可镇定的传输速率下界，给出了控制性能与传输速率之间固有的平衡关系，提出了新的量化、编码和控制策略，实现了控制与通信一体化设计。

本书内容新颖，理论深入，系统性强，可作为高等学校自动化、检测技术、通信工程等相关专业的教学用书，也可作为工程设计人员的参考书。

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前　言

基于物联网的网络化控制技术正在逐渐形成，并成为近年来受到关注的热点研究问题。促使这个技术迅速发展的一个重要原因是来自社会生活、工业控制及军事领域的技术需求，如大城市交通系统的实时指挥和控制，航空工业中的飞机导航自动化控制，石油化工和冶金等连续流程工业的生产控制和调整，战术导弹的数字化制导控制等。采用拥有自主知识产权的网络化控制技术，将有助于在石油、化工、国防等国家重点工业行业内降低企业相关产品生产和使用的风险，保障产业安全，扩大我国具有核心技术的相关领域自动化产品所占据的市场份额，为工业化和信息化的融合提供高端解决方案，促进我国工业节能减排目标的实现。工业无线网络重点关注“高可靠”、“高安全”、“实时性”和“节能”等问题。

虽然采用网络化控制具有许多优点，但是也要看到，无线网络的引入也给控制系统的分析与综合带来了新的挑战。例如，数据包丢失、数据包传输时序错乱、传输时间延迟、信号量化、系统不确定性等因素的存在，使得传统的控制系统设计方法不再有效。针对在数据包丢失、数据包传输时序错乱、传输时间延迟等条件下的网络化控制系统研究，已经有了大量的研究成果。不同于以往的研究结果，本书对线性控制系统进行了研究，考虑了通信网络信道传输速率受限给控制系统分析与设计所带来的影响，提出了许多行之有效的理论方法。

在具有较大通信带宽的工程系统中，常常把控制和通信作为两个分离的步骤，单独完成设计。这种处理方法简化了整个网络控制系统的设计。但近年来随着系统规模的不断扩大及多个系统共享网络资源，这种分离的设计方法已不能使整个系统更加有效地运行了。系统状态经过传感器测量所获得的信号需要经过一个数字化的通信网络传输给控制器。因此，系统的测量输出需要被量化、编码，再通过信道传输给接收端的解码器。解码器对其解码，获得测量输出的估计值，并发送给控制器。对于含有成千上万个传感器的系统来说，虽然总的通信带宽很大，但分配给每个分量的可能是很小的一部分，即只能采用有限的传输速率来传递每个分量的信息。这将导致控制器只能获得非常不精确的系统测量输出估计值，从而剧烈地影响了系统的控制性能。如果多个大规模系统共享同一通信网络资源，将导致这个问题更为突出。因此，在传输速率受限的情况下，如何设计更加有效的量化、编码和控制策略来确保系统的控制性能，成为一个具有重要理论意义和实践应用价值的研究课题。

本书第 1 章研究了在传输速率受限情况下，基于信息论的线性系统量化器、编码器/解码器和控制器的设计问题。其中，假设传感器与控制器在地理上是分离的，并通过一个传输速率受限的数字通信通道连接起来。针对多输入、多输出线性系统，分别采用基于系统输出量和基于状态观测器估计量的量化、编码策略，给出了确保系统可镇定的量化器、编码器/解码器和控制器的设计方法。采用信息理论的分析方法，证明了与信道传输速率相关的确保系统可镇定的充分条件。另外，针对多输入、多输出线性系统，还给出了一种新的量化、编码和控制策略，从而实现确保系统可镇定的最小传输速率。

第 2 章研究了在传输速率受限的情况下，线性系统的量化反馈控制问题。需要解决的关键问题是：针对每个状态量，采用什么样的比特分配策略，才能给出确保系统可镇定的传输速率下界。目前的研究结果中，通常采用一种基于系统矩阵特征值的速率分配策略，但这种方法只适合于系统矩阵可对角化的情况。无论怎么样，对于绝大部分情况，系统矩阵是不可对角化的。为了解决这个问题，与以往的研究成果不同，本章给出了一种基于系统矩阵奇异值的速率分配策略，从而给出了一个更加一般化的结果。

第 3 章研究了在传输速率受限的情况下，线性时变不确定系统的鲁棒控制问题。针对线性时变不确定系统，在传输速率受限的情况下，给出了一种量化、编码和控制策略，确保系统的鲁棒可镇定性。并分别针对完全可观测系统和部分可观测系统，证明了确保系统鲁棒可镇定性的充分条件，给出了信道必须提供的传输速率下界。

第 4 章研究了在传输速率受限，数据包丢失的情况下，线性时不变系统的可镇定性问题。一般情况下，通信网络都会受到噪声干扰的作用，这会造成部分数据包在传输过程中丢失，从而给控制系统的可镇定性带来影响。本章针对这种情况，采用了差分量化编码策略和预测控制方法，证明了确保系统可镇定的充分条件。

第 5 章研究了在传输速率受限，信道传输时延的情况下，线性时不变系统的可镇定性问题。信道传输时延常常会大大降低系统的控制性能。本章分别针对有干扰系统和无干扰系统，完全可观测系统和部分可观测系统，证明了确保系统可镇定性的充分条件，给出了信道必须提供的传输速率下界。

第 6 章研究了在传输速率受限的情况下，线性时变不确定系统的 LQG 控制问题。以往的结果已经表明，数据包丢失、传输时延等都会造成系统控制性能下降。本章主要考虑了传输速率对系统 LQG 性能的影响。并分别针对完全可观测系统和部分可观测系统，证明了与传输速率有关的、确保系统可镇定的充分条件，给出了信道传输速率与系统 LQG 性能之间的平衡关系。

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作 者

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目　录

1 Networked Control Schemes on The Basis of Information Theory

1.1 Stabilization of Stochastic Linear Systems With Data–Rate Constraints

1.1.1 Introduction

Our focus in this note is on stabilization of stochastic linear systems with limited data rates. A high-water mark in the study of quantized feedback using data-rate limited feedback channels is known as the data-rate theorem, which states the larger the magnitude of the unstable poles, the larger the required data rate through the feedback loop (see [1-4]).

Information theory was employed in control systems as a powerful conceptual aid, which extended existing fundamental limitations of feedback systems, and was used to derive necessary and sufficient conditions for robust stabilization of uncertain linear systems and unstructured uncertain systems (see [5-6]). The result on continuous-time linear Gaussian systems was derived in [8]. The result on time-varying communication channel was derived in [9]. The decentralized control schemes were addressed in [7].

Furthermore, [14] was concerned with the networked control problem for linear time delay systems. Fault detection for continuous-time networked control systems with non-ideal network Quality of Service (QoS) was studied in [16]. [12] mainly investigated a sampled-data control approach to deal with the stabilization problem of networked control systems with packet losses and bounded time varying delays. In [13], robust state feedback and observer-based fuzzy controllers were developed for uncertain T-S fuzzy systems with input delay. [17] was concerned with the stability analysis and controller design for a class of MIMO networked control systems with communication constraints. The control algorithm of a controller-observer scheme for robust optimal stabilizing control of a decentralized stochastic singularly-perturbed computer controlled systems with multiple time-varying delays was designed in [15]. [11] considered the control of the class of continuous-time linear Markov jump systems with Wiener process and partial information on the transition jump rates. [2] proposed the notion that the system is stabilizable if and only if the dynamical increase in "uncertainty volume" due to unstable dynamics is outweighed by the partitioning induced by the coder. However, in this note, it is derived that distributions determine not only "uncertainty volume" but also "uncertainty degree" in the same volume, such that distributions also determine the amount of information to be transmitted in one sampling period. Thus, distributions have to be taken into account in the calculation of data rates for the stabilization.

Here, the most attention problem is whether there exists a feedback data rate smaller than that of the literatures above, which still ensures stabilization of stochastic linear systems. In this note, it is shown that for stochastic systems with known distribution, the lower bound of data rates for the stabilization may be smaller.

1.1.2 Problem Formulation

Consider a stochastic linear system described by

$$X_{t+1} = AX_t + BU_t + FW_t \qquad (1.1)$$

$$Y_t = CX_t + V_t$$

where $X_t \in \mathbb{R}^n$, $U_t \in \mathbb{R}^m$, $Y_t \in \mathbb{R}^p$, $V_t \in \mathbb{R}^p$ and $W_t \in \mathbb{R}^q$ are the state process, control input, observation output, measurement noise and process disturbance, respectively. A, B, F and C are known constant matrices with appropriate dimensions (see Fig. 1.1).

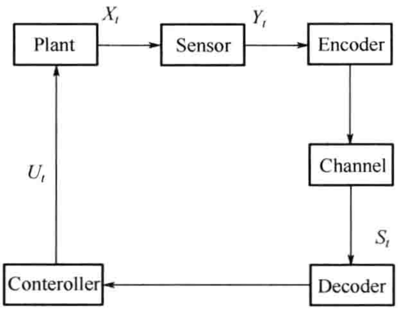

Figure 1.1 Networked control systems

Assume without loss of generality that the pair (A, B) is controllable, and the pair (A, C) is observable. The matrix A is uniquely composed by unstable modes. The initial condition X_0 and disturbance W_t are possibly non-Gaussian and mutually independent random variables with zero mean, satisfying $E\|X_0\|_2^2<\phi_0<\infty$ and $E\|W_t\|_2^2<\phi_W<\infty$ respectively. The sensors and the controller are geographically separated and connected by noiseless communication channels.

We summarize the main definitions, used throughout the section, and adopt [10] as a primary reference. Let X and Y denote two random variables. The following notations are adopted throughout this section:

- Upper case variables, like X, represent random vectors.
- We write $\log_2(\cdot)$ simply as $\log(\cdot)$.
- Let $p(X)$ denote the probability density function of X and $p(X|Y)$ denote the conditional probability density function of X given Y.
- $\|\cdot\|$ represents either the Euclidean norm on a real vector space or the matrix norm induced by it.
- $E_X[\cdot]$ denotes expectation on X.

We summarize the main definitions of Information Theory used throughout the section, and adopt [10] as a primary reference. The definitions listed in this section hold under general assumptions:

Definition 1.1: Let X and Y denote two random variables. The differential entropy $h(X)$ is defined as

$$h(X) := E_X[\log\frac{1}{p(X)}]$$

The conditional differential entropy of X given Y is defined as

$$h(X|Y) := E_{X,Y}[\log\frac{1}{p(X|Y)}]$$

Definition 1.2: The mutual information between X and Y is defined as

$$I(X;Y) := h(X)-h(X|Y)=E_{X,Y}[\log\frac{p(X|Y)}{p(X)}]$$

Definition 1.3: The information rate distortion function between X and Y is defined as

$$R(D) := \inf_{p(X|Y)\in\Xi}\{I(X;Y)\}$$

with $\Xi := \{P(Y|X): E_{X,Y}[d(X,Y)] \leqslant D\}$.Therein, $d(X;Y)$ denotes a distortion function or distortion measure, which is a mapping d : $X\times Y\rightarrow\mathbb{R}$ and D is a given constant.

Definition 1.4: Over communication channels, there are two different methods to denote data rates. One is the code element rate $R_c(t)$ that denotes the number of code element transmitted in unit time, and the other is the information transmission rate $R_e(t)$ that denotes the amount of information transmitted in unit time. $R_e(t)$ may or may not be time-varying. We write $R_e(t)$ simply as $R(t)$. Therefore $R(t)$ is given by

$$R(t) = \frac{1}{h} I(X;Y)$$

where h is the transmission time for $I(X; Y)$.

Remark 1.1: The above definition of data rates states that data rates are determined by not the quantization scheme but the amount of information transmitted in unit time. Thus, it is a more general definition.

The main problem here is to present a lower bound of the data rates of the channel, above which there exists a quantization, coding and control scheme to stabilize the unstable plant in the mean square sense

$$\sup_{t\in N} E \| X_t \|_2^2 < \infty. \tag{1.2}$$

1.1.3 Lower Bound of Data Rates for Stabilization

This section deals with the stabilization problem for stochastic linear systems with limited data rates. First, we give the following lemma.

Lemma 1.1: Consider the system (1.1). Let $\hat{X}_t$ denote an estimate of X_t. Then, $\Sigma_{X|\hat{X}_t} = E[(X_t - \hat{X}_t)(X_t - \hat{X}_t)^{\mathrm{T}}]$ denotes a covariance matrix of X_t given $\hat{X}_t$ Assume $d(X_t, \hat{X}_t) = (X_t - \hat{X}_t)(X_t - \hat{X}_t)^{\mathrm{T}}$ is a distortion measure, satisfying

$$|\det(E_{X_t,\hat{X}_t}[d(X_t,\hat{X}_t)])| = |\det(\Sigma_{X_t|\hat{X}_t})| \leqslant D$$

Thus, the rate distortion function is given by

$$R(D) \geqslant h(X_t) - \frac{1}{2}\log(2\pi e)^n D(\text{bits}) \tag{1.3}$$

Proof: Let Q be the unitary matrix that diagonalizes

$$\Sigma_{X_t} = Q^{\mathrm{T}} \Sigma_{X_t} Q$$

Here, we define

$$\bar{X}_t = QX_t$$

$$\tilde{X}_t = Q\hat{X}_t$$

Then, notice that

$$I(X_t;\hat{X}_t)=I(\overline{X}_t;\tilde{X}_t)^{[10]}$$

and

$$\Sigma_{X_t|\hat{X}_t}=Q^{\mathrm{T}}\Sigma_{\overline{X}_t|\tilde{X}_t}Q$$

Therefore, we have

$$|\det(\Sigma_{X_t|\hat{X}_t})|=|\det(\mathrm{diag}[E(\overline{x}_{t_1},-\tilde{x}_{t_1})^2,\cdots,E(\overline{x}_{t_n},-\tilde{x}_{t_n})^2])|$$

We set

$$d(\overline{x}_{t_i},\tilde{x}_{t_i})=(\overline{x}_{t_i}-\tilde{x}_{t_i})^2$$

satisfying

$$E_{\tilde{X}_{ti},\hat{X}_{ti}}[d(\overline{x}_{t_i},\tilde{x}_{t_i})]\leqslant D_i$$

where

$$D=\prod\nolimits_{i=1}^{n}D_i(i=1,2,\cdots,n)$$

It follows that

$$|\det(E[d(X_t,\hat{X}_t)])|\leqslant D$$

Therefore, the following holds:

$$I(\overline{X}_t;\tilde{X}_t)=\sum\nolimits_{i=1}^{n}\int_R\int_R p(\overline{x}_{t_i})p(\overline{x}_{t_i}\mid\tilde{x}_{t_i})\log\frac{p(\tilde{x}_{t_i},\overline{x}_{t_i})}{p(\tilde{x}_{t_i})}\mathrm{d}\overline{x}_{t_i}\mathrm{d}\tilde{x}_{t_i}$$

$$=\sum\nolimits_{i=1}^{n}I(\tilde{x}_{t_i};\overline{x}_{t_i})$$

Note that

$$R(D)=\inf_{p(X_t|\hat{X})_{t\in\Xi}}\{I(X_t;\hat{X}_t)\}$$

$$=\inf_{p(X_t|\hat{X})_{t\in\Xi}}\sum\nolimits_{i=1}^{n}I(\tilde{x}_{t_i};\overline{x}_{t_i})\}$$

where

$$\Xi=\{p(\tilde{x}_{t_i}\mid\overline{x}_{t_i}):E[\mathrm{d}(\overline{x}_{t_i},\tilde{x}_{t_i})]\leqslant D_i\}$$

satisfying the following conditions:

$$p(\tilde{x}_{t_i}\mid\overline{x}_{t_i})\geqslant 0$$

$$\int_R p(\tilde{x}_{t_i}\mid\overline{x}_{t_i})d\hat{x}_{t_i}=1 \tag{1.4}$$

$$\int_R \int_R p(\bar{x}_{t_i}) p(\tilde{x}_{t_i} \mid \bar{x}_{t_i}) \mathrm{d}(\tilde{x}_{t_i}, \bar{x}_{t_i}) \mathrm{d}\tilde{x}_{t_i} \mathrm{d}\bar{x}_{t_i} = D$$

We set up the functional

$$\Gamma = \sum_{i=1}^{n} \Gamma_i = \sum_{i=1}^{n} [I(\bar{x}_{t_i}, \tilde{x}_{t_i})] - \mu_i \int_R p(\tilde{x}_{t_i} \mid \bar{x}_{t_i}) \mathrm{d}x_{t_i} - s_i D_i$$

In the following proof, we employ the techniques following from [10]. Note that $I(\bar{x}_{t_i}, \tilde{x}_{t_i})$ is a convex function. Then, differentiating with respect to $p(\tilde{x}_{t_i} \mid \bar{x}_{t_i})$ and setting $\frac{\partial \Gamma_i}{\partial p(\tilde{x}_{t_i} \mid \bar{x}_{t_i})} = 0$, we have

$$p(x_{t_i}) \log \frac{p(\tilde{x}_{t_i} \mid \bar{x}_{t_i})}{p(\tilde{x}_{t_i})} - s_i p(\bar{x}_{t_i}) \mathrm{d}(\tilde{x}_{t_i}, \bar{x}_{t_i}) - \mu_i = 0$$

Setting

$$\mu_i = p(\bar{x}_{t_i}) \log \lambda_i$$

we obtain

$$\log \frac{p(\tilde{x}_{t_i} \mid \bar{x}_{t_i})}{p(\tilde{x}_{t_i})} - s_i \mathrm{d}(\tilde{x}_{t_i} \mid \bar{x}_{t_i}) - \log \lambda_i = 0$$

or

$$p(\tilde{x}_{t_i} \mid \bar{x}_{t_i}) = p(\tilde{x}_{t_i}) \lambda_i 2^{s_i \mathrm{d}(\tilde{x}_{t_i}, \bar{x}_{t_i})} \tag{1.5}$$

Since $\int_R p(\tilde{x}_{t_i} \mid \bar{x}_{t_i}) \mathrm{d}_{\tilde{x}_{t_i}} = 1$, we must have

$$\lambda_i = \frac{1}{\int_R p(\tilde{x}_{t_i}) 2^{s_i \mathrm{d}(\tilde{x}_{t_i}, \bar{x}_{t_i}) \mathrm{d}\bar{x}_{t_i}}}$$

Combining (1.4) and (1.5) together, we get

$$D_i = \int_R \int_R p(\tilde{x}_{t_i}) p(\bar{x}_{t_i}) \lambda_i \mathrm{d}(\tilde{x}_{t_i}, \bar{x}_{t_i}) 2^{s_i \mathrm{d}(\bar{x}_{t_i}, \tilde{x}_{t_i})} \mathrm{d}\tilde{x}_{t_i} \mathrm{d}\bar{x}_{t_i}$$

Define

$$R_{M_i} := \sup_{s \leqslant 0} s_i D_i + \int_R p(\bar{x}_{t_i}) \log \lambda_i \mathrm{d}\tilde{x}_{t_i} \tag{1.6}$$

Since

$$\int_R p(\bar{x}_{t_i}) p(\tilde{x}_{t_i} \mid \bar{x}_{t_i}) \mathrm{d}_{\tilde{x}_{t_i}} = \int_R p(\bar{x}_{t_i}) p(\tilde{x}_{t_i}) \mathrm{d}_{\tilde{x}_{t_i}} \lambda_i 2^{s_i \mathrm{d}(\tilde{x}_{t_i}, \bar{x}_{t_i})} \mathrm{d}_{\tilde{x}_{t_i}}$$

we have

$$\int_R p(\bar{x}_{t_i})p(\tilde{x}_{t_i})\mathrm{d}_{\bar{x}_{t_i}}\lambda_i 2^{s_i \mathrm{d}(\tilde{x}_{t_i},\bar{x}_{t_i})}\mathrm{d}_{\tilde{x}_{t_i}}=1 \quad (when\ p(\tilde{x}_{t_i})\neq 0) \tag{1.7}$$

Setting

$$\Delta_i=\frac{K_i}{p(\tilde{x}_{t_i})},\Delta=\bar{x}_{t_i}-\tilde{x}_{t_i}$$

and substituting them into (1.6) and (1.7), we obtain

$$K_i\int_R 2^{s_i \mathrm{d}(\tilde{x}_{t_i},\bar{x}_{t_i})}\mathrm{d}_{\bar{x}_{t_i}}=K_i\int_R 2^{s_i \mathrm{d}(\Delta)}\mathrm{d}\Delta=1$$

We know that

$$\frac{\partial R_{M_i}}{\partial s_i}=D_i-\int_R g_i(\Delta)\mathrm{d}_i(\Delta)\mathrm{d}\Delta=0$$

$$\frac{\partial^2 R_{M_i}}{\partial s_i^2}=-\int_R g_i(\Delta)\mathrm{d}_i^2(\Delta)\mathrm{d}\Delta+[g_i(\Delta)\mathrm{d}_i(\Delta)\mathrm{d}\Delta]^2<0$$

where let $g_i(\Delta)=\dfrac{2^{s_i(\Delta)d_i(\Delta)}}{\int_R 2^{s_i(\Delta)d_i(\Delta)}d\Delta}$ and $\int_R g_i(\Delta)\mathrm{d}\Delta=1$. R_{M_i} is the convex function of s_i, we have

$$\int_R g_i(\Delta)\mathrm{d}_i(\Delta)\mathrm{d}\Delta=D_i$$

Substituting them into (6), we obtain

$$R_{M_i}=h(\bar{x}_{t_i})+\int_R(s_i\mathrm{d}_i(\Delta)-\log\int_R 2^{s_i\mathrm{d}_i(\Delta)}\mathrm{d}\Delta)g_i(\Delta)\mathrm{d}\Delta$$

$$=h(\bar{x}_{t_i})+\int_R \log g_i(\Delta)\mathrm{d}\Delta$$

$$=h(\bar{x}_{t_i})-h(g_i(\Delta))$$

Define $G:=[g_1g_2\cdots g_n]^{\mathrm{T}}$. Note that

$$R(D)=\sum_{i=1}^n R(D_i)\geqslant\sum_{i=1}^n R_{M_i}=h(X_t)-h(G)$$

where

$$h(X_t)=-\int\cdots\int_R p(x_{t_1},x_{t_2},\cdots,x_{t_n})\log p(x_{t_1},x_{t_2},\cdots,x_{t_n})\mathrm{d}x_{t_1},\mathrm{d}x_{t_2},\cdots,\mathrm{d}x_{t_n}$$

$$h(G)=-\sum_{i=1}^n\int_R \log g_i(\Delta)\mathrm{d}(\Delta)$$

By the assumption, we know that