

ALM 28

Advanced Lectures in Mathematics

Selected Expository Works of Shing-Tung Yau with Commentary

(Vol. I)

丘成桐综述文章选集
附评论 (第 I 卷)

Editors: Lizhen Ji · Peter Li · Kefeng Liu · Richard Schoen



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With Steve Nadis as a consulting editor on
language for commentary

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Playing Taichi on a beach of the New Territory of Hong Kong in 1969



Receiving Bachelor degree from Chung Chi College in CUHK in 1969



In the front of Prof. Yau's apartment in Stony Brook in New York in 1972



Standing on Massachusetts Avenue near Harvard

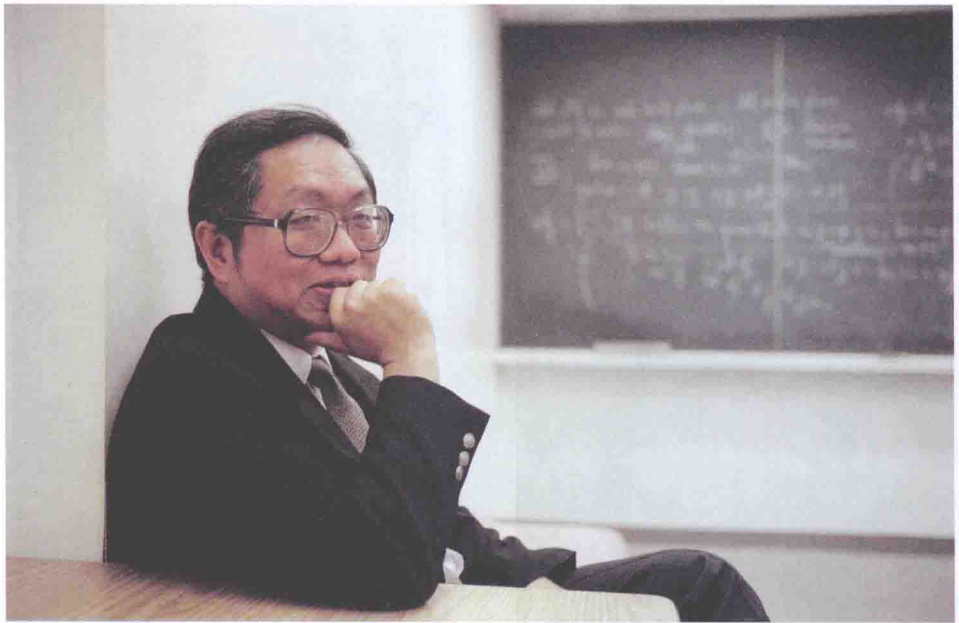


Photo by Kris Snibbe
at the Science Center of Harvard University on May 19, 2008

CURRICULUM VITAE

Shing-Tung Yau (丘成桐)

Last updated on Jun 12, 2014

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Harvard University
Cambridge, MA 02138, U.S.A.

BORN: Shantou, Guangdong, China, April 4, 1949

EDUCATION: Ph.D., Mathematics, University of California, Berkeley, 1971

HONORARY DEGREE:

May 11, 2014 Doctor of Humane Letters, University of Illinois at Chicago
Jun 17, 2011 Doctor of Mathematics, University of Waterloo
Jun 05, 2010 Doctor of Science, Cheng Kung University
May 18, 2009 Doctor of Science, Lehigh University
Nov 2005 Doctor of Science, Taiwan University
May 22, 2005 Doctor of Science, Polytechnic University in Brooklyn
Nov 2004 Doctor of Science, Hong Kong University of Science
and Technology
Jul 2004 Doctor of Science, Central University (Jhongli)
Mar 2003 Doctor of Science, Zhejiang University
Oct 16, 2002 Doctor of Science, Macau University
Jun 24, 2000 Doctor of Science, Tsing Hua University (Hsinchu)
Jun 21, 1997 Doctor of Science, Chiao Tung University (Hsinchu)
Sept 1987 Master of Arts, Harvard University
Dec 1980 Doctor of Science, The Chinese University of Hong Kong

RESEARCH INTERESTS:

Differential geometry, differential equations, and general relativity

POSITIONS:

- 2014–Present Director of the Center for Mathematical Sciences and Applications, Harvard University
- 2013–Present Professor of Physics, Harvard University
- 2008–2012 Chair of Department of Mathematics, Harvard University
- 2003–Present Distinguished Professor-at-Large, The Chinese University of Hong Kong
- 2000–Present William Caspar Graustein Professor of Mathematics, Harvard University
- 2009–Present Director of the Mathematical Sciences Center, Tsinghua University
- 2002–Present Director of the Centre of Mathematical Sciences, Zhejiang University
- 1996–Present Director of the Morningside Center of Mathematics at Chinese Academy of Sciences
- 1994–Present Director of The Institute of Mathematical Sciences, The Chinese University of Hong Kong
- 1987–Present Professor of Mathematics, Harvard University
- 1997–2000 Higgins Professor of Mathematics (Chair Professor), Harvard University
- 1994–2003 Adjunct Professor of Mathematics, The Chinese University of Hong Kong
- 1984–1987 Chancellor Associate Chair and Professor of Mathematics, University of California, San Diego
- 1979–1984 Professor of Mathematics, Institute for Advanced Study, Princeton University
- 1974–1979 Professor of Mathematics, Stanford University
- 1972–1973 Assistant Professor of Mathematics, State University of New York, Stony Brook
- 1971–1972 Member of the School of Mathematics, Institute for Advanced Studies, Princeton University

VISITING PROFESSOR:

- Apr–May 2007 Visiting University of California, Los Angeles
- Jan–Mar 2007 Visiting University of California, Irvine
- Jan–Jun 2002 Gordon Moore Visiting Professor, Department of Mathematics, California Institute of Technology
- 1999 Samuel Eilenberg Visiting Professor, Department of Mathematics, Columbia University
- 1996 John Harvard Fellow, Issac Newton Institute for Mathematical Sciences and Faculty of Mathematics, University of Cambridge, U.K.
- 1991–1992 Special Chair, Department of Mathematics, Tsing Hua University (Hsinchu)

- 1991–1992 Wilson T. S. Wang Distinguished Visiting Professor, Department of Mathematics, The Chinese University of Hong Kong
- Sept 1990 Distinguished Visiting Professor, Department of Mathematics, State University of New York, Stony Brook
- 1989 Sherman Fairchild Distinguished Visiting Scholar, Department of Mathematics, California Institute of Technology
- Fall 1986 Sid Richardson Centennial Chair in Mathematics and Visiting Professor, Department of Mathematics, University of Texas at Austin

ADDITIONAL AFFILIATIONS:

- 2009–present Honorary Professor, Wuhan University
- 2009–present Honorary Professor, Hunan Normal University
- 2009–present Honorary Professor, North University of China
- 2009–present Honorary Professor, Northwest University
- 2006–present Honorary Professor, Huazhong University of Science and Technology
- 2002–present Honorary Professor, Zhejiang University
- 1999–present Honorary Professor, University of Science and Technology of China
- 1998–present Honorary Professor, Peking University
- 1993–present Honorary Professor, Nankai University
- 1987–present Honorary Professor, Tsinghua University
- 1987–present Honorary Professor, Hangzhou University
- 1983–present Honorary Professor, Fudan University
- 1983–present Honorary Professor, Chinese Academy of Sciences

AWARDS and FELLOWSHIPS:

- 2010 The Wolf Prize
- 2010 AAEOY Distinguished Science & Technology Award
- 2003 2003 International Scientific and Technological Cooperation Award
- 1997 National Medal of Science, U.S.A
- 1994 Crafoord Prize, The Royal Swedish Academy of Sciences
- 1991 Humboldt Research Award, The Alexander von Humboldt Foundation, Germany
- 1985 John D. and Catherine T. MacArthur Fellowship
- 1984 One of Americas 100 brightest scientists under 40, Science Digest
- 1982 Fields Medal, International Congress of Mathematicians
- 1981 The John J. Carty Award for the Advancement of Science, National Academy of Sciences, U.S.A.

- 1981 The Oswald Veblen Prize in Geometry, American Mathematical Society
 1980 John Simon Guggenheim Fellowship
 1979 California Scientist of the Year, California Science Center
 1975–1976 Alfred P. Sloan Fellow

MEMBERSHIPS:

- Jan 2013 Fellow, American Mathematical Society
 2009 Member of the Selection Committee for Distinguished Research Achievement Award (DRAA) of the University of Hong Kong
 2008 Foreign Member, Indian National Academy of Science
 2005 Foreign Member, National Academy of Lincei, Italy
 Jul 2005 Member of the Overseas Expert Consultant Committee of Overseas Chinese Affairs Office of State Council
 2003 Foreign Member, Russian Academy of Sciences
 1995 Foreign Member, Chinese Academy of Sciences
 1993 Member, National Academy of Sciences, U.S.A.
 1993 Fellow, American Association for the Advancement of Science
 1990–1992 Member-at-Large, Council of the American Mathematical Society
 1989 Member, Scientific Advisory Council, Mathematical Sciences Research Institute
 1989 Member, Board of Mathematical Sciences, National Academy of Science, U.S.A.
 1985 Member, American Physical Society
 1985 Member, Society for Industrial and Applied Mathematics, U.S.A.
 1984 Academician, Academia Sinica, Taiwan, China
 1983 Fellow, New York Academy of Sciences
 1982 Member, American Academy of Arts and Sciences
 1980 Honorary Member, Academic Committee, Institute of Mathematics, Chinese Academy of Sciences, Beijing, China
 1971 Member, American Mathematical Society

EDITORSHIPS:

- Present Editorial Boards, Editor, *American Journal of Mathematics*
 2007–Present Editors-in-Chief, *Communications in Number Theory and Physics*
 2005–Present Editors-in-Chief, *Pure and Applied Mathematics Quarterly*
 2004–Present Editor-in-Chief, *Dynamics of Partial Differential Equations*
 2001–Present Honorary Editor, *Communications in Information and Systems*
 1997– Editorial Boards, Editor, *Journal of Mathematical Physics*

- 1997–Present Editor-in-Chief, *Advances in Theoretical Mathematics and Physics*
- 1997–Present Editor-in-Chief, *Asian Journal of Mathematics*
- 1994–Present Editor-in-Chief, *Methods and Application of Analysis*
- 1994– Editorial Boards, Editor, *Advances in Mathematics*
- 1993–Present Editorial Boards, Editor, *Mathematical Research Letters*
- 1993–Present Editorial Boards, Advisor, *Communications in Analysis and Geometry*
- 1993–Present Editorial Boards, Advisor, *Methods and Applications of Analysis*
- 1982–1999 Editor, *Communication in Mathematical Physics*
- 1981–1988 Editor, *Inventiones Mathematicae*
- 1980–Present Editor-in-Chief, *Journal of Differential Geometry*

INVITED LECTURESHIPS:

- Oct 5, 2012 The Edmund R. Michalik Distinguished Lecture Series, University of Pittsburgh, U.S.A.
- Jan 17, 2012 2012 McKnight-Zame Distinguished Lecture at University of Miami, U.S.A.
- Jan 20–21, 2011 The Fields Institute Distinguished Lecture Series at University of Toronto, Canada
- Jul 13, 2010 Distinguished Applied Math. Lecture at Chiao Tung University (Hsinchu), China
- Jul 11, 2010 2010 NCTS International Conference on “Several Complex Variables and Complex Geometry” at the Center for Theoretical Sciences, Taiwan, China
- Jun 2010 Workshop “Geometric Analysis and General Relativity” at The Banff International Research Station for Mathematical Innovation and Discovery (BIRS), Canada
- Nov 10, 2009 Invariants in Algebraic Geometry at The University of Tokyo, Japan
- Sept 7–8, 2009 Keynote Speaker for the 13th IMA Conference on The Mathematics of Surfaces at York, England, U.K.
- Jun 30, 2009 Distinguished Lecture Series Conference at Taida Institute for Mathematical Sciences, China
- Jun 18, 2008 ICTP Trieste, Italy
- May 30, 2008 Scientist Forum at Yunnan, China
- May 21, 2008 80th Hirzebruch Conference, Bar Ilan University, Tel Aviv, Israel
- Mar 11, 2008 IPMU Opening Symposium, Japan
- Dec 26, 2007 Indian Mathematics Society, India
- Dec 17–22, 2007 International Congress of Chinese Mathematicians, China
- Aug 27–31, 2007 Geometrie differentielle, Physique mathematique, Mathematiques and societe, IHES, France
- May 2007 Distinguished Lecture Series (DLS) 2006-2007 at UCLA, U.S.A.

- Jan 2006 Brauer Lectures, The University of North Carolina at Chapel Hill, U.S.A.
- 2005 String 2005, Fields Institute, Toronto, Canada
- May–Jun 2005 Andrejewski Lectures, Gottingen, Germany
- Sept–Dec 2004 Eilenberg Lectures, Department of Mathematics, Columbia University, U.S.A.
- May 2004 Bloomberg Lecture, University of Texas at Austin, U.S.A.
- 2003 Distinguished Lecture Series, Department of Mathematics, University of California at LA, U.S.A.
- Oct 2003 Andre Aisenstadt Chair Lecture Series, Department of Mathematics, Université de Montréal, Canada
- 1999 Hans Rademacher Lecture, Department of Mathematics, University of Pennsylvania, U.S.A.
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- 1998 Run Run Shaw Distinguished Lecture, The Chinese University of Hong Kong, China
- 1997 Rufus Bowen Lecture, Department of Mathematics University of California, Berkeley, U.S.A.
- 1988 American Mathematical Society Colloquium, U.S.A.
- 1983 James K. Whittemore Lecture, Department of Mathematics, Yale University, U.S.A.
- 1982 Alexander Ziwet Lecture, Department of Mathematics, University of Michigan, Ann Arbor, U.S.A.
- 1982 International Mathematical Union Lecture, Zurich, Switzerland
- 1981 The 33rd British Mathematical Colloquium, London Mathematical Society, U.K.
- 1979 Milton Brouckett Porter Lecture, Department of Mathematics, Rice University, U.S.A.
- 1978 International Congress of Mathematicians, Plenary Speaker, Helsinki, Finland

Preface

In the early spring of 2013, Lizhen Ji asked me to write comments about my collected or selected works. I was too busy at the time to take on such a task. At one point, however, I gave in to his request and decided to write comments about my survey articles. Upon tallying them up, I was surprised to see that I had written far more survey articles than I had remembered.

Since I was a child, I have always been interested in history. Hence when I started to write these commentaries, I tried to stick to the facts to the best of my memory. I also consulted friends who participated in these events and looked at letters and emails that I had kept over the past forty years.

This does not mean that there are absolutely no mistakes in the statements. Nevertheless, I believe that these accounts can be interesting – and maybe even important – for students who'd like to know something about how the various papers were written and what my friends and I thought about the approaches we took.

In the course of putting together this collection, I received strong support from Lizhen Ji, Hao Xu, Kefeng Liu, Shiu-Yuen Cheng, and Hung-Hsi Wu. I am also very grateful to the publishers led by Liping Wang, Yushan Deng, and others. My friend Steve Nadis agreed to be the consulting editor for this project. I am extremely thankful for all of their help, without which this project likely would not have materialized.

Shing-Tung Yau
June 30, 2014

Preface

Shing-Tung Yau, His Mathematics and Writings

1 Why selected works

There has been a long tradition of publishing collected or selected works of distinguished mathematicians. There are several good reasons for doing this, and it has served many purposes. Probably the most obvious one is that collected and selected works provide an easy access to papers that are scattered in different journals, some of which are not easily accessible to many people. Otherwise, few people, if any, will take the time and trouble to dig up all the papers of their admired mathematicians—especially not those papers that are far away from their interests, of their admired mathematicians and read them. On the other hand, reading papers of a master dealing with different subjects or areas conveys the underlying unity and hence a big picture of mathematics, and it also allows one to gain a historical perspective (or to enter the history). In other words, collected and selected works are more than the simple sum of individual papers.

Indeed, as Abel said famously, we learn “by studying the masters, not their pupils.” Even though the world is becoming smaller, few people have many chances to interact with masters who are alive. Of course, the next best way to learn from masters is to read and study their collected works.

Naturally, publishing collected or selected works is also an honor to the authors of these papers. It should be mentioned that collected works of some people can bring honor of the genre of collected works.

Now, with the wide and easy use of e-papers and e-books, most papers in journals can be obtained easily online, and a mere reprinting of papers is probably not as valuable as before. Of course, the value of selected works still stands. For example, holding and reading a beautifully printed book is definitely different from viewing papers online or on e-book readers. But they should also provide something else. Several additional things seem to be reasonable: descriptions of how ideas in the paper were formed and time and place the papers were written, relations between papers with the advantage of hindsight, and developments of subjects after the papers were published, and visions for the future. In other words, they should explain the circumstances of the birth of papers and proper, impacts of the papers, and fitting these papers in the grand scheme of mathematics.

These additional things are especially important to beginners, non-experts and even some experts. Most people often concentrate on the best known theorems and most important papers of great mathematicians, but even masters struggled and stumbled sometimes on their mathematical trips. How they found good problems and their ways in their careers, made progress and reached peaks is best described by their own papers, recollections and commentaries, but not textbooks where everything is polished and presented in a streamlined matter, without mentioning that textbooks and research books might not cover some gems in the original papers that are not directly related to the themes of the books. But many people, especially younger ones, often prefer to read polished textbooks. Of course, reading mathematics papers can be difficult (more difficult than reading textbooks), and proper arrangement of related papers and additional guides from the masters are certainly valuable and helpful. Such collected or selected works of distinguished mathematicians often tell good stories of the authors and their mathematics, and browsing through them can be enjoyable and beneficial to people who are not interested in some specific results in the papers.

In these works of expository writings of Shing-Tung Yau, all these things are printed together with his survey papers and papers on open problems. One reason for restricting these volumes to expository papers of Yau is practical. Yau has been very creative and prolific. The collected works including all his papers (both research and expository papers) up to now will occupy too many volumes. Besides, he is also still very active and productive, and the time for collected works may not be ripe yet.

2 Why expository writing

Probably some explanation is needed for publishing these volumes of expository writings of Yau now. Briefly, it is the right time for Yau to share his perspectives and his vision on the broad area of geometric analysis, and his expository writings provide a unique means to this end. They will render a valuable service to the mathematics community.

Colloquium talks have been a common means of communication between mathematicians from different subjects, after they were made successful and popular by Klein and Hilbert in Göttingen about 100 years ago. More expository talks such as *Basic notion seminars* and *What is ...* have also sprung up in many places. They provide effective ways for people to learn and enjoy some beautiful pieces of mathematics, which are outside their fields of speciality. Though there are many books and papers dealing with all kinds of subjects in mathematics, one difficulty is that there are too many of them. It is difficult for people to find the right books and papers, and people may lack the motivation to read mathematics outside their specialities, especially when they involve difficult and technical material. Many people choose to study mathematics not for fame or fortune, but for the beauty and enjoyment of the discipline. To really appreciate the beauty and power of mathematics, one has to roll up one's sleeves and do the work. But not many people can work in many different subjects in mathematics. In the history