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CASE STUDIES IN TIME SERIES ANALYSIS

时间序列分析实例研究

Xie Zhongjie

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影印版前言

本书是一本有关时间序列分析应用于实际的实证分析研究的专著。全书分为两大部分：第一部分简要介绍了时间序列分析的基础理论和方法，这些内容是读懂本书各案例研究所必备的基本知识；第二部分是案例研究。从中读者可看出时间序列分析是如何广泛地应用于实际並成为解决各种问题的核心工具。书中的案例涉及到当年中国科学家从自己的观测记录中是如何发现天王星的光环的；滤波理论如何应用于中国东海和黄海的重力勘探；谱分析如何判别先天性愚型儿童的脑电特征、多元谱的 K-L 信息量如何应用于优秀飞行员的生理特征的检测；潜周期分析如何发现离体脑垂体仍有内分泌的节律周期；预测理论如何应用于气象的建模和预报，等等许多非常有趣而真实的研究案例。这些研究成果使作者获得了中国国家自然科学奖和国内外的多项奖项。

读者通过本书的学习不仅可以学到时间序列分析的基本理论和方法，更重要的是本书介绍了“如何将一个实际问题转化成数学问题”，然后运用数学和统计学的理论和方法加以解决，这包括最后还原到实际，用实验数据加以检验的完整过程。

本书出版以来受到校内外读者和许多著名统计学家的好评（请参见本书封底 S.N.Gupta 等人的评论）。本书可作为应用时间序列分析领域的大学生和研究生教学参考书或补充教材；也是应用统计工作者和许多学科领域的科技人员、工程师很有价值的参考资料。

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Preface

In 1958, Chinese scientists, technologists, mathematicians, etc., in answer to their Government's appeal, went out of colleges and institutes with an ardent zeal for reconstructing their motherland and joined a movement of integrating theory and practice to do real practical work in factories and farmlands. This nation-wide movement gave tremendous impetus to further developments of sciences and technologies particularly mathematics. The author, at that time, was a fourth year undergraduate in a course of five years majoring in mathematics at the Peking University. He and two other classmates, under the guidance of Professor Chiang Tse-pei went out of the University to take part in a radar project to analyze the performance of filters using their knowledge of spectral analysis learned in class. It was from this occassion that the author developed an affinity for applied research.

The real, solid and all-front development of applied probability and statistics in China started after the promulgation of a policy of reform and opening to the outside world by the Government in the 1980's. In the Second National Conference of the Chinese Society of Probability and Statistics, both Chairman T. Chiang and Vice-Chairman Z. Wei stressed the urgent need of applied research in probability and statistics with practical applications as the primary motivation. Their views were warmly received by the audience. In only a span of a few years, applied research in probability and statistics fluorished first in quantity, and then in quality, and has now attained a fairly high level. A group of devoted probabilitists and statisticians is gradually forming; so how to train good researchers with emphasis on applications has become the key item on the agenda.

With the encouragement of my colleagues in the Department of Probability and Statistics, especially the Chairman, Professor Chen Jiading, I inaugurated a course on "Case Studies of Applied Probability and Statistics" for second year post-graduate students in 1987. I discussed in class some of my case studies that were closely related to the basic principles and had achieved good results. My aim was to see that a student, after a year's training, should understand how abstract mathematical concepts are related to practical problems, learn how an applied mathematician thinks, and what methodology should be adopted to solve a practical problem.

In class, I never refrained from talking about my experiences of failure. The students liked my class. Perhaps, it was such experiences that won me the students' appreciation. In 1987, I was invited by Linz Kappler University of Austria to lecture on a course "Applied Time Series Analysis" to postgraduates, who heartily welcomed it. Many friends and colleagues, upon learning of my achievement, encouraged me to write a book on "The Case Studies of Applied Time Series"; Professor Yan Shijian, former Chairman of the Chinese Society of Probability and Statistics, gave the biggest push. Through kind arrangement of Mr. Xu Jiagu of World Scientific Publishing, the book is now published. I, as the author of the book, wish to thank them all and say the following words to the reader.

1. I must have a word of warning for scientists and technologists who, having worked with various software packages on a computer for too long, may not have bothered to find out the relevant theoretical background and/or premises, as they may then arrive at wrong conclusions. In order to help readers who are not time-series specialists to better understand the theoretical basis of these methods, I have collected a number of them, hitherto scattered in different books and journals, and explained them either in the first part of the book or when discussing the appropriate case studies. The time-series specialists, however, may skip over these explanations without loss of understanding of the context.

2. Nearly every research is constrained by a time limit, and to get the solution done before the time is up often becomes the goal of a project. It is well-known that the solutions of many practical problems are not unique. A solution given here in the book is often only one of the possible solutions, not necessarily the best. Many problems could have been done better, but shortage of time prevented me from indulging in searching for the best solutions.

3. This book is intended as a reference book in applied statistics, though it actually consists of brief reports of my own research. It may be used as a source book for student seminars.

Besides the case studies listed in the book, I have also done some other researches closely related with time series. For instance, I have collaborated with some communication engineers to study various methods of reducing intersymbol interference in a tropo-scatter communication system; such methods are quite closely connected with some theories and methods of time series. In oil exploration, to predict from a few deep-well data (e.g. permeability data) permeability rates at points in the upper space over the oil fields may be treated as a problem of modelling and prediction of a spatial series. Intermodulation communication analyses in satellite communication and predictions of workers' hearing loss due to factory noise are all related to nonlinear models. Owing to the limited time available, such topics have not been included here.

I wish to take this opportunity to express my deep indebtedness to Professor Wei Zong-shu who, not only encouraged me to write the book, but also kindly consented to read my manuscript and refine my English, at a mature age of eighty-one.

I also owe Messrs. Zheng Pingping and Zhang Dabao heavily for their patient undertaking of the tedious work of typing and I thank them for their seriousness and responsible spirit.

Finally, I wish to express my hearty thanks to the International Center, Department of Mathematics, Waseda Universityk, particularly to Professors T. Kusama and T. Suzuki for their kind support by offering me excellent working surroundings, availability of the latest reference material and use of their wonderful computer facilities for my writing of the second part of the book, which I was able to complete within such a short time.

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Zhongjie Xie

Contents

PART ONE

AN INTRODUCTION TO THE THEORY AND METHODS OF TIME SERIES ANALYSIS

CHAPTER 1

Theory of Stationary Time Series

In this chapter, we shall introduce some basic ideas and models of time series which are very often used in applications. One of the most important concepts in time series is "stationary" even though observations recorded in practice are not always stationary. The reader may easily find that many techniques and theory for analyzing nonstationary data series are based on the theory and methods of stationary time series.

ARMA series is the most important stationary time series, which plays a central role both in theory and applications since ARMA model not only covers a lot of problems in diverse fields but also relates with very deep mathematical backgrounds, such as rational spectral functions, Markovian extension problems, state space models, etc. Accordingly, we shall discuss the ARMA model in rather detail.

Some basic laws of large numbers are also introduced in this chapter which are particularly important for the estimation theory and techniques in statistical analysis of time series.

§1.1 The Definition of Stationary Stochastic Processes

The reader is presumed to have a basic knowledge of single and multiple random variables and their distribution functions. But in practice, many many problems are related with infinite random variables, at least for the convenience of theoretical analysis we need not restrict ourselves only to the finite number of random variables. Some examples will be given in the sequel.

Definition 1.1 (Stochastic Processes). Suppose that $(\Omega, \mathcal{F}, P)$ is a probability space, and T is an index set. If for any $t \in T$, there exists a random variable $\xi_t(\omega)$ defined on $(\Omega, \mathcal{F}, P)$, then the family of random variables $\{\xi_t(\omega), t \in T\}$ will be called a stochastic process.

In Definition 1.1, the index set T may be understood as any set, real or complex, finite or infinite, countable and uncountable, etc. In this book, T generally represents the time index set, such as $T = \{t : t = 1, 2, \dots\}$ or $T = \{t : a < t < b\}$ and denote the real line as R_1, the integer set as $\mathbf{Z}$. In particularly, when T is a discrete time index set, then we shall call $\{\xi_t(\omega), t \in T\}$ a time series.

The random variables $\xi_t(\omega), t \in T$ in Definition 1.1 should be understood as complex-valued variables in general, so the moments are defined as

$$\begin{aligned} E\xi_t(\omega) &= E\xi_t^{(R)}(\omega) + iE\xi_t^{(I)}(\omega) & (1.1) \\ E\xi_t(\omega)\overline{\xi_s(\omega)} &= E\xi_t^{(R)}(\omega)\xi_s^{(R)}(\omega) + E\xi_t^{(I)}(\omega)\xi_s^{(I)}(\omega) \\ &\quad + i[E\xi_t^{(I)}(\omega)\xi_s^{(R)}(\omega) - E\xi_t^{(R)}(\omega)\xi_s^{(I)}(\omega)], & (1.2) \end{aligned}$$

when (1.1), (1.2) exist, where $\xi_\cdot^{(R)}(\omega)$, $\xi_\cdot^{(I)}(\omega)$ are real and imaginary parts of $\xi_\cdot(\omega)$ respectively.

When $T = \{t : a < t < b\}$, for a realization of $\{\xi_t(\omega), t \in T\}$, our convention is to denote it as $\{X_t(\omega), t \in T\}$, which can be considered as a function of t. In practical record, different realizations $X_t(\omega_1), X_t(\omega_2), \dots, X_t(\omega_n), t \in T$, may not coincide with each other. In general, $X_t(\omega), t \in T$, presents dual specifications: on one hand, the realization given by the recorder in practical problem seems like a real curve or a deterministic function; on the other hand, theoretically, we always consider the observed sample as a stochastic process, i.e. its "randomness" still exists in mathematical analysis.

In the sequel, a succinct form of stochastic process is $\{\xi_t, t \in T\}$, the element ω will be omitted.

Definition 1.2 (Gaussian Stochastic Process). Suppose that $\{\xi_t, t \in T\}$ is a real-valued stochastic process, if for any $t_1, t_2, \dots, t_n \in T$, the n-dimensional characteristic function of $\{\xi_{t_1}, \xi_{t_2}, \dots, \xi_{t_n}\}$ can be represented in the form of

$$f(\mathbf{u}) = \exp\{i\mathbf{a}'\mathbf{u} - \tfrac{1}{2}\mathbf{u}'\Sigma\mathbf{u}\} \tag{1.3}$$

where $\mathbf{a} = (a_1, a_2, \dots, a_n)'$ is a real vector of n-dimensions, Σ is a real, non-negative definite symmetric matrix, then $\{\xi_t, t \in T\}$ will be called a Gaussian process or a Normal process.

It is very clear that if the covariance matrix is positive definite $\Sigma > 0$, then the distribution function determined by (1.3) is an n-dimensional normal distribution, with probability density $N(\mathbf{a}, \Sigma)$, i.e.

$$p(\mathbf{x}) = (2\pi)^{-\frac{n}{2}}(\det\Sigma)^{-\frac{1}{2}}\exp\{-\tfrac{1}{2}\mathbf{X}'\Sigma^{-1}\mathbf{X}\}, \tag{1.4}$$

where $\mathbf{X} = \mathbf{x} - \mathbf{a}$. Now, if $\Sigma \geq 0$, $\det\Sigma = 0$ may occur, so we cannot expect that (1.4) will always keep true, but the following derivation shows that (1.3) will still be a characteristic function of an n-variate vector.

In fact, we can put

$$\Sigma_N = \Sigma + \mathbf{I}/N \tag{1.5}$$

where $N > 0$ is a sufficiently large integer, $\mathbf{I}$ is an $n \times n$ unit matrix. Then for any non-zero n-variate vector $\alpha \in R_n$, we have

$$\alpha' \Sigma_N \alpha = \alpha' \Sigma \alpha + \tfrac{1}{N} \alpha' \alpha \geq \tfrac{1}{N} \alpha' \alpha > 0$$

which shows that

$$f_N(\mathbf{u}) = \exp\{i\mathbf{a}'\mathbf{u} - \tfrac{1}{2}\mathbf{u}'\Sigma_N \mathbf{u}\} = f(\mathbf{u}) \exp\{-\tfrac{1}{2N}\mathbf{u}'\mathbf{u}\} \tag{1.6}$$

is a characteristic function. Now, for $\mathbf{u}$ in any bounded set $\mathbf{U}$, $f_N(\mathbf{u})$ will uniformly converge to $f(\mathbf{u})$, and $f(\mathbf{u})$ is continuous at $\mathbf{u} = 0$. According to the limiting theorem of characteristic functions (see Cramér (1946)), we know that $f(\mathbf{u})$ is a characteristic function of random variables.

Now suppose that $\{\xi_t, t \in T\}$ is a complex-valued stochastic process. We shall call $\{\xi_t, t \in T\}$ a Gaussian process or Normal process if for any integer $n > 0$, the joint $2n$-variates

$$(\xi_{t_1}^{(R)}, \xi_{t_1}^{(I)}, \xi_{t_2}^{(R)}, \xi_{t_2}^{(I)}, \ldots, \xi_{t_n}^{(R)}, \xi_{t_n}^{(I)})$$

of the real and imaginary parts of complex variates $(\xi_{t_1}, \ldots, \xi_{t_n})$ are Gaussian, i.e. their characteristic function possesses the form of (1.3).

The following theorem gives the existence of Gaussian processes, its proving can be found in Doob (1953).

Theorem 1.1 (Existence of Gaussian Process). Let T be an index set, a_t a complex function defined on T, and $\sigma_{t,s}$ a bivariate function on $T \times T$. If the following conditions:

1. $\sigma_{s,t} = \overline{\sigma_{t,s}}, \qquad \forall (s,t) \in T \times T,$
2. For any $t_1, t_2, \ldots, t_n \in T$,
   $\Sigma = (\sigma_{t_i,t_j})_{1 \leq i,j \leq n} \geq 0 \qquad$ (non-negative definite),

are fulfilled, then there exist a complex Gaussian process $\{\xi_t, t \in T\}$, such that

$$E\xi_t = a_t,$$
$$E(\xi_t - a_t)(\overline{\xi_s - a_s}) = \sigma_{t,s}, \qquad \forall (t,s) \in T \times T.$$

This theorem is very important both in theoretical and methodological research in time series analysis.

Definition 1.3 (Stationary Process). Let $\{\xi_t, t \in T\}$ be a stochastic process with second ordered moment, $E|\xi_t|^2 < +\infty$, $t \in T$. ξ_t is said to be a stationary process in the wide sense if the following conditions are satisfied:

$$1.\ E\xi_t = a, \qquad \forall t \in T, \tag{1.7}$$

$$2.\ E(\xi_t - E\xi_t)(\overline{\xi_s - E\xi_s}) = R(t-s), \qquad \forall t, s \in T. \tag{1.8}$$

The condition of (1.7) shows that the mean of ξ_t is invariant for the shifting of t. In practical problems, the recording curves $\{X_t, t \in T\}$ look like a random vibration along a constant level $E\xi_t$ (see Fig. 1.1). The condition of (1.8) can also be rewritten as

$$E(\xi_{t+\tau} - E\xi_{t+\tau})(\overline{\xi_t - E\xi_t}) = R(\tau), \tag{1.9}$$

for any t, $t+\tau \in T$.

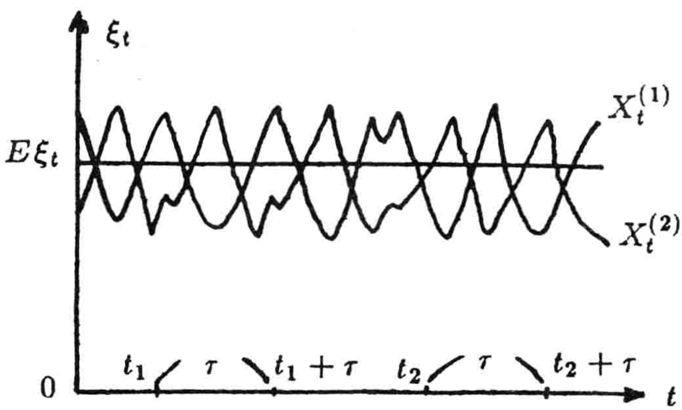

Fig. 1.1 Realization of stochastic processes

Accordingly, the second condition in the Definition 1.3 can be understood as that the covariance function of "$\xi_{t+\tau}$ vs. ξ_t" is invariant for the shifting of t. This means that the statistical linear correlation of $(\xi_{t+\tau}, \xi_t)$ only depends on the spacing τ, and is free from their initial time t.

Generally, we call

$$B(\tau) = E(\xi_{t+\tau}\overline{\xi_t}) \tag{1.10}$$

the correlation function of $\xi_t, t \in T$ and $R(\tau)$ the covariance function. No loss of generality, we shall always assume that the stationary process in the wide sense has