

STEVEN WEINBERG

# THE QUANTUM THEORY OF FIELDS

Vol. 2 Modern Applications

量子场论

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# The Quantum Theory of Fields

Volume II  
Modern Applications

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## Preface To Volume II

This volume describes the advances in the quantum theory of fields that have led to an understanding of the electroweak and strong interactions of the elementary particles. These interactions have all turned out to be governed by principles of gauge invariance, so we start here in Chapters 15–17 with gauge theories, generalizing the familiar gauge invariance of electrodynamics to non-Abelian Lie groups.

Some of the most dramatic aspects of gauge theories appear at high energy, and are best studied by the methods of the renormalization group. These methods are introduced in Chapter 18, and applied to quantum chromodynamics, the modern non-Abelian gauge theory of strong interactions, and also to critical phenomena in condensed matter physics. Chapter 19 deals with general spontaneously broken global symmetries, and their application to the broken approximate  $SU(2) \times SU(2)$  and  $SU(3) \times SU(3)$  symmetries of quantum chromodynamics. Both the renormalization group method and broken symmetries find some of their most interesting applications in the context of operator product expansions, discussed in Chapter 20.

The key to the understanding of the electroweak interactions is the spontaneous breaking of gauge symmetries, which are explored in Chapter 21 and applied to superconductivity as well as to the electroweak interactions. Quite apart from spontaneous symmetry breaking is the possibility of symmetry breaking by quantum-mechanical effects known as anomalies. Anomalies and various of their physical implications are presented in Chapter 22. This volume concludes with a discussion in Chapter 23 of extended field configurations, which can arise either as new ingredients in physical states, such as skyrmions, monopoles, or vortex lines, or as non-perturbative quantum corrections to path integrals, where anomalies play a crucial role.

It would not be possible to provide a coherent account of these developments if they were presented in a historical order. I have chosen instead to describe the material of this book in an order that seems to me to work best pedagogically — I introduce each topic at a point where

the motivation as well as the mathematics can be understood with the least possible reference to material in subsequent chapters, even where logic might suggest a somewhat different order. For instance, instead of having one long chapter to introduce non-Abelian gauge theories, this material is split between Chapters 15 and 17, because Chapter 15 provides a motivation for the external field formalism introduced in Chapter 16, and this formalism is necessary for the work of Chapter 17.

In the course of this presentation, the reader will be introduced to various formal devices, including BRST invariance, the quantum effective action, and homotopy theory. The Batalin–Vilkovisky formalism is presented as an optional side track. It is introduced in Chapter 15 as a compact way of formulating gauge theories, whether based on open or closed symmetry algebras, and then used in Chapter 17 to study the cancellation of infinities in ‘non-renormalizable’ gauge theories, including general relativity, and in Chapter 22 to show that certain gauge theories are anomaly-free to all orders of perturbation theory. The effective field theory approach is extensively used in this volume, especially in applications to theories with broken symmetry, including the theory of superconductivity. I have struggled throughout for the greatest possible clarity of presentation, taking time to show detailed calculations where I thought it might help the reader, and dropping topics that could not be clearly explained in the space available.

The guiding aim of both Volumes I and II of this book is to explain to the reader why quantum field theory takes the form it does, and why in this form it does such a good job of describing the real world. Volume I outlined the foundations of the quantum theory of fields, emphasizing the reasons why nature is described at accessible energies by effective quantum field theories, and in particular by gauge theories. (A list of chapters of Volume I is given at the end of the table of contents of this volume.) The present volume takes quantum field theory and gauge invariance as its starting points, and concentrates on their implications.

This volume should be accessible to readers who have some familiarity with the fundamentals of quantum field theory. It is not assumed that the reader is familiar with Volume I (though it wouldn’t hurt). Aspects of group theory and topology are explained where they are introduced.

Some of the formal methods described in this volume (such as BRST invariance and the renormalization group) have important applications in speculative theories that involve supersymmetry or superstrings. I am enthusiastic about the future prospects of these theories, but I have not included them in this book, because it seems to me that they require a whole book to themselves. (Perhaps supersymmetry and supergravity will be the subjects of a Volume III.) I have excluded some other interesting topics here, such as finite temperature field theory, lattice gauge calcula-

tions and the large  $N_c$  approximation, because they were not needed to provide either motivation or mathematical techniques for the rest of the book, and the book was long enough.

The great volume of the literature on quantum field theory and its applications makes it impossible for me to read or quote all relevant articles. I have tried to supply citations to the classic papers on each topic, as well as to papers that describe further developments of material covered here, and to references that present detailed calculations, data, or proofs referred to in the text. As before, the mere absence of a citation should not be interpreted as a claim that the material presented is original, but some of it is.

In my experience this volume provides enough material for a one-year course for graduate students on advanced topics in quantum field theory, or on elementary particle physics. Selected parts of Volumes I and II would be suitable as the basis of a compressed one-year course on both the foundations and the modern applications of quantum field theory. I have supplied problems for each chapter. Some of these problems aim simply at providing exercise in the use of techniques described in the chapter; others are intended to suggest extensions of the results of the chapter to a wider class of theories.

\* \* \*

I must acknowledge my special intellectual debt to colleagues at the University of Texas, notably Luis Boya, Phil Candelas, Bryce and Cecile De Witt, Willy Fischler, Joaquim Gomis, and Vadim Kaplunovsky, and especially Jacques Distler. Also, Luis Alvarez-Gaumé, Sidney Coleman, John Dixon, Tony Duncan, Jürg Fröhlich, Arthur Jaffe, Marc Henneaux, Roman Jackiw, Joe Polchinski, Michael Tinkham, Cumrun Vafa, Don Weingarten, Edward Witten and Bruno Zumino gave valuable help with special topics. Jonathan Evans read through the manuscript of this volume, and made many valuable suggestions. For corrections to the first printing of this volume I am indebted to several students and colleagues, including Mark Byrd, Vincent Liu, Chun-yen Wang, and especially Michio Masujima. Thanks are due to Alyce Wilson, who prepared the illustrations and typed the  $\text{\LaTeX}$  input files until I learned how to do it, to Terry Riley for finding countless books and articles, and to Jan Duffy for many helps. I am grateful to Maureen Storey and Alison Woollatt of Cambridge University Press for working to ready this book for publication, and especially to my editor, Rufus Neal, for his continued friendly good advice.

STEVEN WEINBERG

Austin, Texas  
December, 1995

# Notation

Latin indices  $i, j, k$ , and so on generally run over the three spatial coordinate labels, usually taken as 1, 2, 3. Where specifically indicated, they run over values 1, 2, 3, 4, with  $x^4 = it$ .

Greek indices  $\mu, \nu$ , etc. from the middle of the Greek alphabet generally run over the four spacetime coordinate labels 1, 2, 3, 0, with  $x^0$  the time coordinate.

Greek indices  $\alpha, \beta$ , etc. from the beginning of the Greek alphabet generally run over the generators of a symmetry algebra.

Repeated indices are generally summed, unless otherwise indicated.

The spacetime metric  $\eta_{\mu\nu}$  is diagonal, with elements  $\eta_{11} = \eta_{22} = \eta_{33} = 1$ ,  $\eta_{00} = -1$ .

The d'Alembertian is defined as  $\square \equiv \eta^{\mu\nu} \partial^2 / \partial x^\mu \partial x^\nu = \nabla^2 - \partial^2 / \partial t^2$ , where  $\nabla^2$  is the Laplacian  $\partial^2 / \partial x^i \partial x^i$ .

The 'Levi-Civita tensor'  $\epsilon^{\mu\nu\rho\sigma}$  is defined as the totally antisymmetric quantity with  $\epsilon^{0123} = +1$ .

Spatial three-vectors are indicated by letters in boldface.

Three-vectors in isospin space are indicated by arrows.

A hat over any vector indicates the corresponding unit vector: Thus,  $\hat{\mathbf{v}} \equiv \mathbf{v}/|\mathbf{v}|$ .

A dot over any quantity denotes the time-derivative of that quantity.

Dirac matrices  $\gamma_\mu$  are defined so that  $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu}$ . Also,  $\gamma_5 = i\gamma_0 \gamma_1 \gamma_2 \gamma_3$ , and  $\beta = i\gamma^0 = \gamma_4$ .

The step function  $\theta(s)$  has the value +1 for  $s > 0$  and 0 for  $s < 0$ .

The complex conjugate, transpose, and Hermitian adjoint of a matrix or vector  $A$  are denoted  $A^*$ ,  $A^T$ , and  $A^\dagger = A^{*T}$ , respectively. The Hermitian adjoint of an operator  $O$  is denoted  $O^\dagger$ , except where an asterisk is used to emphasize that a vector or matrix of operators is not transposed. +H.c. or +c.c. at the end of an expression indicates the addition of the Hermitian adjoint or complex conjugate of the foregoing terms. A bar on a Dirac spinor  $u$  is defined by  $\bar{u} = u^\dagger \beta$ . The antifield of a field  $\chi$  in the Batalin–Vilkovisky formalism is denoted  $\chi^\ddagger$  rather than  $\chi^*$  to distinguish it from the ordinary complex conjugate or the antiparticle field.

Units are usually used with  $\hbar$  and the speed of light taken to be unity. Throughout  $-e$  is the rationalized charge of the electron, so that the fine structure constant is  $\alpha = e^2/4\pi \simeq 1/137$ .

Numbers in parenthesis at the end of quoted numerical data give the uncertainty in the last digits of the quoted figure. Where not otherwise indicated, experimental data are taken from ‘Review of Particle Properties,’ *Phys. Rev. D***50**, 1173 (1994).



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Sections marked with an asterisk are somewhat out of the book's main line of development and may be omitted in a first reading.

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