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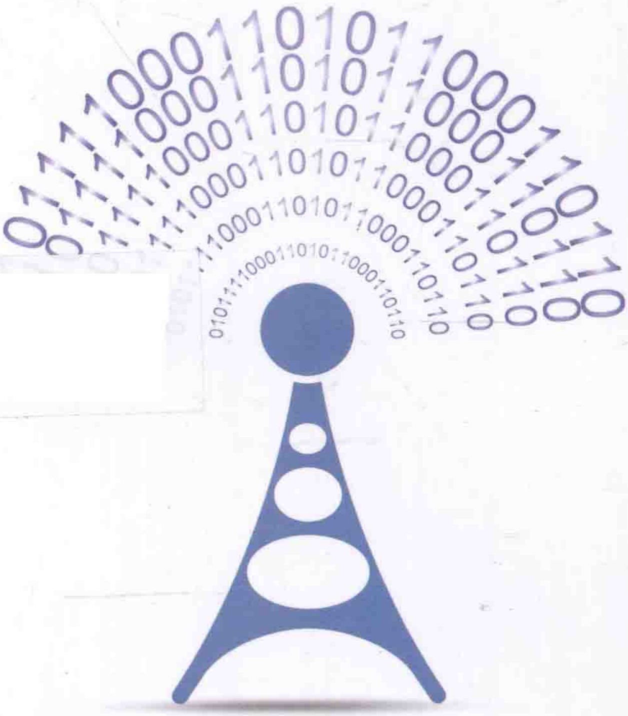
高等学校电子信息类专业系列教材

Fundamentals of Signals and Systems

信号与系统基础

李刚 常丽萍 李胜 编著
Li Gang Chang Liping Li Sheng



清華大学出版社

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FUNDAMENTALS OF SIGNALS AND SYSTEMS

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Introduction

Fundamentals of Signals and Systems deals with the concepts of signals and systems, which are the corner-stone of a wide variety of areas ranging from home-oriented consumer electronics and multimedia entertainment products to sophisticated communications, aeronautics and astronautics, and control. A course on signals and systems is fundamental and compulsory for an engineering under-graduate curriculum in any well-established tertiary (education) institutions.

This book provides readers with the basic principles and methods underlying the analysis of signals and systems in the context of both continuous- and discrete-time, including Fourier analysis based representations for signals and both time- and transform-domain approaches as well as state-space approach to (linear time-invariant) systems. Great emphasis is placed on being *concise, easy for self-study,* and *rigorous.* Compared with most of the existing textbooks on the same topics, more effort is made in this book to provide a clear picture of the relationship between linear constant differential/difference equations and linear time-invariant systems. Another significant feature of this book is a separated chapter (Chapter 8) that contains a set of comprehensive exercises, each of which usually involves mathematical deviations and more sophisticated application of the concepts and approaches provided in the entire text book rather than an individual chapter.

This book is primarily designed for undergraduate students majoring electrical & electronics and information engineering, the concepts and techniques introduced in the book, however, are of fundamental importance in all other engineering disciplines. Readers of this book are assumed to have a basic background in engineering mathematics, including calculus, complex functions, and linear differential equations. Some knowledge on circuit theory, though not a prerequisite, would be helpful for the study.

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Preface

The concepts of *signals and systems* are the corner-stone of a wide variety of areas, ranging from home-oriented consumer electronics and multimedia entertainment products to sophisticated communications, aeronautics and astronautics, and control. The ideas and approaches associated with these concepts have great effects on our life in one way or another. Although the signals and systems which arise across those fields are naturally different in their physical make-up and application, the principles and tools for analyzing signals and systems are the same and hence applicable to all of them. Therefore, an introductory course on signals and systems is fundamental and compulsory for an engineering under-graduate curriculum in any well-established tertiary (education) institutions. Such a course is commonly designed in one of the two forms below:

- a one-semester subject that intends to provide students with a rich set of concepts and tools for analyzing deterministic signals and an important class of systems known as linear time-invariant systems;
- a two-semester subject that expands on the one-semester course by adding more detailed treatment of signal processing and systems for specified applications such as communications, multimedia signal processing and control engineering.

This book takes the first form and assumes that the students have a background in calculus and introductory physics.

Why another "Signals and Systems"?

Given that there are many well-written textbooks on signals and systems available, our readers would be screaming when seeing this textbook: Why another "Signals and Systems"?

The thought of writing such a textbook was stimulated in 2007 when the first author conducted a 2nd year course on signals and systems in Zhejiang University of Technology (ZJUT) in Hangzhou, which he just joined from Nanyang Technological University (NTU), Republic of Singapore. The course was designed as "shuang yu ke" spelled in Chinese, meaning that the teaching materials such as the textbooks and slides are all in English and as to the class language, the lecturers can choose either Chinese, English or a mixture of the two—a typical Chinese style! The textbooks adopted were those popularly used in the world but were found very difficult to most of our students as their English is not good enough to deal with those textbooks of one thousand pages or so in length.[1] This motivates him to write a new "Signals and Systems"

[1] Here, we have those big masters of *Signals and Systems* to "blame"—why not writing their textbook in Chinese!

with a primary objective of providing a condensed version of "Signals and Systems" in English, while keeping the important technical materials as much as possible.

Most of us agree that for a university study it is more important to teach students how to learn and analyze than what to be learned and analyzed. The first author still remembers what was said by Professor Y. Liu who taught him calculus in 1978 in Beijing Institute of Technology (BIT) (now, known as Bei Li Gong) that *a good textbook, say of ten chapters, should be written in such a way that after the first four chapters taught by the lecturer, the rest can be studied by students themselves easily.* What Prof. Liu really meant is that a textbook should be written to facilitate and reinforce self-study. This is another objective that this textbook is intended to achieve. Given the mathematical nature of this subject, rigidity should be sustained as much as possible, which is something very important to engineering students to learn. This is the third objective of this textbook.

How the book is structured ?

One of the reasons for the existing textbooks of signals and systems to have easily over eight hundred pages is due to the continuous- and discrete-time forms of signals and systems. The success of a *signals and systems* that achieves our primary object lies in how to provide a balanced and integrated treatment of the two forms in a pedagogical way that can help students see the fundamental similarities and differences between the two without too much repeating. With all of these in mind, this book is organized as follow.

- *Chapter 1* is aimed to provide an overview of signals and systems. Compared with the one in most of the *signals and systems*, it is condensed with an emphasis on the periodical signals and linear time-invariant (LTI) systems.
- *Chapter 2* deals with the time-domain approach to the LTI systems. Since the key to the development of this chapter is to exploit the properties of *linearity* and *time-invariance* as well as signal decompositions, the concepts of unit impulse response and convolution are developed in details for the discrete-time case, while the continuous-time counterparts are directly given. One of the remarkable points emphasized in this chapter is the equivalence between the LTI systems and convolutions, which relates the physical interpretation and mathematical expression. Another important technical point in this chapter is the establishment of a conclusion which states that any complete solution of a linear constant coefficient differential/difference equation (LCCDE) can be characterized by the sum of the output of an LTI system excited by the force signal and a homogeneous solution of the LCCDE. This conclusion yields a very clear picture of the relationship between an LCCDE and the systems that it can characterize. With the help of the

transforms to be discussed in Chapter 6, it also provides an easy way to find a particular solution and hence the set of complete solutions of the LCCDE.

- *Chapter 3* is the biggest chapter, dealing with four signal representations, namely, Fourier series (FS), discrete-time FS (DTFS), Fourier transform (FT), and discrete-time FT (DTFT). The most important concept in this chapter is signal decomposition. Along this line, the four signal transforms can be unified as a linear combination of basis signals. This unification allows us to condense the text significantly as one can just focus on the development and the properties of FT and DTFT. Great effort has been made to explain why a signal should be transformed, which is one of the difficult points for most of the 2nd year students to understand given that they usually do not have a relevant background.
- Equipped with Chapters 1–3, a top student should be able to study on ones own the rest of this book, including *Chapter 4* that studies the class of LTI systems using Fourier analysis techniques developed in Chapter 3. The key points of the chapter are the frequency response and the equivalence between convolution in time-domain and multiplication in frequency-domain and the development of the concepts in both continuous- and discrete-time domains are rotated to avoid unnecessary repetition.
- *Chapter 5* mainly focuses on discrete processing of continuous-time signals. The development flows mainly from the techniques derived in Chapters 2–4. Starting from sampling a signal that is continuous in time-domain, the relationship between the spectrum of the continuous-time signal and that of the discrete-time signal obtained is established, based on which the famous sampling theorem and hence the ideal reconstruction can be derived. Unlike most of the existing textbooks, this relation is derived directly using a simple mathematical procedure rather than the one that is obtained with the help of impulse train. Note that the DTFT of a discrete-time signal is a continuous function in frequency. Sampling (in frequency) of such a function leads to a new transform-discrete Fourier transform (DFT) that is popularly used in many applications of digital signal processing, including linear filtering, correlation analysis, and spectral analysis. More profound discussions on the DFT, however, are beyond the scope of this book as the relevant topics are parts of the core contents for the subjects on *digital signal processing.*
- *Chapter 6* deals with transform-domain approaches to signals and systems. The Laplace transform and z-transform are studied in a parallel way, while the applications of the two transforms to LTI systems and finding the complete solution of an LCCDE are given in a unified manner, which once again allows us to condense the textbook. As two of the new features of this book, the problems of finding inverse systems of a given LTI system & de-convolutions and decomposing system

responses are carefully treated. The former is rarely found in most of the existing textbooks, while the latter is intended organized to help students have an easy understanding of the unilateral transforms.

- The advantage of the unified/integrated treatment of the continuous- and discrete-time forms is particularly demonstrated in *Chapter 7* in study of block-diagram representations and structures of the LTI systems in transform-domain. As a special class of system structures, the state-space realizations of LTI systems are introduced.
- Following the traditional pedagogy, the tutorial questions given in each chapter are mainly designed for helping students to enhance the understanding of the concepts in an individual chapter *only*. This, however, would make most of the students problem-result-oriented and hence loses the real purpose of learning. To overcome this, *Chapter 8* is devoted to providing a set of comprehensive exercises, each of which usually involves mathematical deviations and a more sophisticated application of the concepts and approaches provided in the entire text book rather than an individual chapter.
- *Appendices* are used to make the book concise and self-contained, and also sustain rigidity of the development.

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Liping Chang is grateful to her Ph.D supervisor, academician Z.Q. Lin, SIOM of Chinese Academy of Science, Shanghai, who taught her how to explore and solve

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We thank many students who made contributions to this book with suggestions and questions that helped us over years to refine and rethink the organization and presentation of the materials in the book. Particularly, we would like to thank Chaogeng Huang, who has just obtained his Ph.D under the guidance of the 1st author, and Huang Bai for the excellent job done in terms of research and administration, and two very talent undergraduate students (Year 2007) Yue Wang and Dehui Yang who helped us with simulations, figures and tutorial problems. Special thanks go to our teaching assistants Zhihui Zhu, Tao Hong, Qiuwei Li, Shuang Li, and Liang Zhu for their help in preparing tutorial solutions and some of the computer experiments. We are grateful to our students for their active participation and intensive interaction with us, which makes this book even more student-oriented.

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Lastly, we would like to thank the support from our families, which is the driving force for us to write this book. It may be felt unusual by those not close to Gang Li that the first one among his family members to mention is his brother-in-law Aiguo

Li who has done everything possible for him and his family. Liping Chang wishes to thank her family members, particularly her husband Jia Li and their four-month old son for their deep love. Sheng Li has the pleasure to say "I love you much more than myself" to his darling wife Xiaolei Zhang and his parents.

Gang Li (ieligang@zjut.edu.cn)
Liping Chang (clping@zjut.edu.cn)
Sheng Li (shengli@zjut.edu.cn)
July 01, 2014
at Xiao He Shan, Hangzhou

Contents

Chapter 1 Introduction

This chapter is devoted to providing students a set of general concepts of signals and systems. We will begin our development with the intuitive questions that may be raised by most of the students at the beginning of the first class of this course: what is a signal and what is a system? Mathematical descriptions and representations of signals and systems are the most important concepts throughout this course and also play a role of corn-stone for other more advanced subjects. We will build on the foundation for developing these concepts and discuss some properties of systems as well as the relationship between signals and systems in this chapter.

1.1 Overview of signals and systems

Signals and *systems* are two of the words which are heard most frequently in our daily life. These concepts arise in virtually all areas, and particularly, play a very important role in engineering. In fact, it can be argued that much of the recent development of high technology, which has brought our life to a new dimension, is a result of advancements in the theory and techniques of signals and systems.

1.1.1 What is a signal?

Examples of signals we encounter frequently include speech music signals, and, picture image signals, which, in the *signal processing* community, are usually referred to as *audio* and *video* signals, respectively. A signal is actually a variable used to carry *information*. For example, a speech signal from the speaker of a research seminar represents air pressure that varies with time and stimulates the audience's ears, and the information is reflected by the way how the air pressure (i.e., the signal) changes with time. A Signal can be represented in different forms Figure 1.1 shows the waveform of a recorded speech signal displayed on a computer screen. The physical meaning (i.e., information) carried by this signal is "di qiu" in Mandarin Chinese (it means "earth" in English) spoken by a Chinese male.

An image signal is the light intensity, also called *gray level*, that varies with two spatial coordinates (see Figure 1.2), while video signal in the televisions consists of a set of pictures that occur in sequence and hence are the light intensity that changes with the two spatial coordinates as well as the time.

A *signal* is formally defined as a *function* of one or more *independent* variables.

The speech signal in Figure 1.1 can then be denoted as $s(t)$ with the independent variable t representing the time, while the image signal in Figure 1.2, as $p(x, y)$ with x, y representing the horizontal and vertical coordinates, respectively.

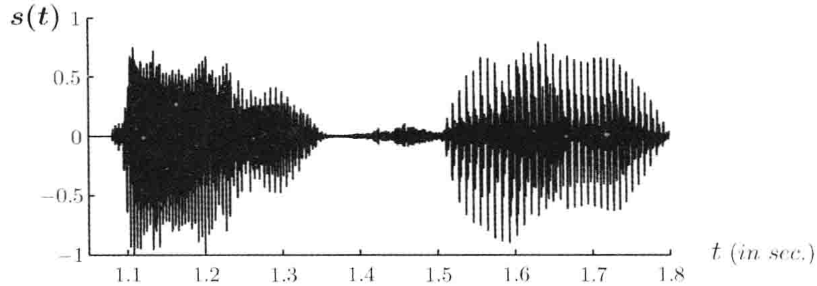

Figure 1.1 A recording of the speech for "di qiu" in Chinese (meaning "earth" in English) spoken by a Chinese male.

Figure 1.2 The scene of *female teacher* in San Qing Mountain.

It should be pointed out that most signals in our life are generated by natural means and hence correspond to physical phenomena. However, a signal can also be produced artificially, say by computer simulations. Such a signal does not have a specified physical meaning as it is not born from any natural phenomenon but we can always use it to represent a certain meaning (i.e., information). In fact, a signal, no matter how it is generated, is just a carrier that can be used to represent different information.

1.1.2 What is a system?

In the broadest sense, a *system* is an *entity* that is used to achieve a specified function. Figure 1.3 depicts a simplified rectifier circuit.

Such a system consists of two resistors r, R, one diode D, and a capacitor C as components. Denote $y(t)$ as the voltage across the capacitor, which is the response to the voltage source $x(t) = A\cos(2\pi F_0 t + \phi)$ applied. The function of this system is to make $y(t)$ constant, namely, invariant with time t. In this system, the source $x(t)$ and the capacitor voltage $y(t)$ are referred to as the *input signal* and *output signal*, respectively.

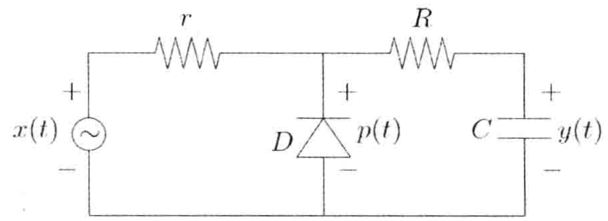

Figure 1.3 Block diagram of a rectifier circuit.

A system is an interconnection of components or parts with terminals or access ports through which signals can be applied and extracted.

Let y be the output *responding to* the input x of a system. This fact is denoted as

$$x \rightarrow y$$

Such a notation focuses on the relationship between the input and output of the system rather than how all the components are connected. Frequently, a system can be viewed as a *black box*, in which the *input* signals are *processed* in some manner to yield the *output* signals. Figure 1.4(a) is the black box representation of the system $x(t) \rightarrow y(t)$, whose detailed structure is specified by Figure 1.3.

Figure 1.4 Black box representations of the circuit by Figure 1.3.

It should be pointed out that if we are also interested in the voltage $p(t)$ across the diode D in Figure 1.3, then the same circuit can be viewed as a system of one input and two outputs. See Figure 1.4(b).

A system may have M inputs and N outputs. Such a system is usually referred to as a multi-input multi-output (MIMO) system when M, N are all bigger than one. If $M = N = 1$, the system is a single-input single-output (SISO) system. In this book, most of the systems in our discussions belong to the catalog of SISO.

Examples of much more complicated systems can be easily found. Below are three important classes of systems that find a lot of applications in our daily life.

A. Communication systems

Figure 1.5 shows a simplified structure of communication systems. The function of such a system is to convey information from one point (the sender) to the other point (the destination).

Every communication system consists of three basic *sub-systems*: the emitter, the

Figure 1.5 A block-diagram of communication systems.

channel, and the receiver. The emitter, located at one point in space, is to generate a signal $x(t)$ that contains the message signal $s(t)$ (say a speech signal) produced by a source of information and to transmit it efficiently to the channel. The channel, as the physical medium, can be an optical fiber, a coaxial cable, or simply the air, and is to guide the transmitted signal $x(t)$ to the receiver that is located at some other point separate from the emitter. As the transmitted signal propagates over the channel, it is distorted before reaching the receiver due to many factors including the physical characteristics of the channel, noise and interfering signals from other sources. The objective of the receiver is to process the received signal $r(t)$, a distorted version of the transmitted signal $x(t)$, so as to yield a signal $y(t)$ which is in a recognizable form of the original message signal $s(t)$.

B. Control systems

Control engineering is another important area in which the theory and technology of *signals and systems* find successful applications. Such examples are ranged from simple appliances such as air-conditioners and refrigerators found in homes to very sophisticated engineering innovations such as aircraft autopilots, robots, paper mills, mass-transit vehicles, oil refineries, automobile engines, nuclear reactors, and power plants.

A block-diagram for a class of control systems is depicted in Figure 1.6:

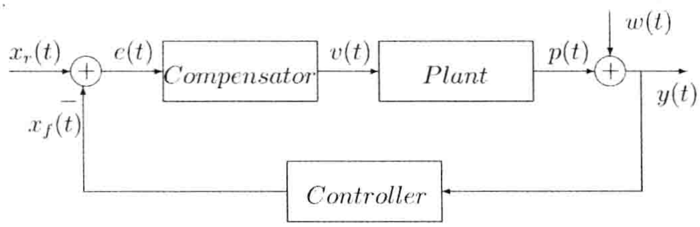

Figure 1.6 A block-diagram for a class of feedback control systems.

where the *plant* is the system to be controlled, $w(t)$ is the disturbance signal which plus the plant output forms the measurement signal $y(t)$, and the (feedback) *controller* is a system that consists of a sensor to collect the signal $y(t)$ and a micro-processor to generate the feedback signal $x_f(t)$. The latter is then to be compared with a *reference* signal $x_r(t)$ to produce an error signal $e(t) = x_r(t) - x_f(t)$. This error signal is then fed into the *compensator*, a system used to generate a signal $v(t)$ to control the plant such that a desired plant output $p(t)$ is achieved.

In an aircraft landing system, the plant refers to the aircraft's body and actuator.

The sensor system is used by the pilot to measure the lateral position of the aircraft. In this situation, $w(t)$ is the measurement error, while the reference input $x_r(t)$ corresponds to the desired landing path of the vehicle and the compensator is designed such that the output of the plant tracks $x_r(t)$ well.

C. Signal processing systems

As stated by Simon Haykin[1], signal processing is at its best when it successfully combines the unique ability of mathematics to generalize with both the insight and prior information gained from the underlying physics of the problem at hand.

In general, the functional form of a signal does not directly reveal the embedded information. One of the reasons for this is that due to factors such as measurement noise and channel distortion, the signal under processing is usually a corrupted version of the one which contains information. *Filtering*, one of the most important tools of signal processing, is a set of signal operations used to get rid of the disturbance such that information can be extracted easily.

In the system depicted in Figure 1.7, the input is a corrupted music signal $s(t) = s_0(t) + e(t)$, where $s_0(t)$ is the desired music signal and $e(t)$ is the noise attached somehow.

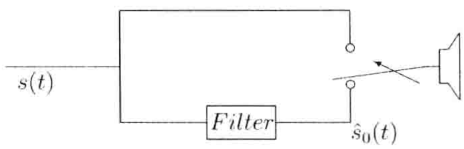

Figure 1.7 Block-diagram of a simplified audio play system.

None would enjoy listening to the sound from the louder speaker driven directly by $s(t)$. The filter is a system that is intended to block the noise $e(t)$ and let $s_0(t)$ pass through. It would be a completely different story if the speaker is excited by the output $\hat{s}_0(t) \approx s_0(t)$ of the filter.

A digital mixer is a more sophisticated audio instrument used for audio signal processing such as equalization, noise gating, and dynamic control. One of the most important parts of such a system is a set of filters, called filter bank.

1.2 Description and classification of signals

A signal, represented mathematically as a function of M independent variables, is usually referred to as an M-dimensional (M-D) signal. The speech signal $s(t)$ shown in Figure 1.1 is a 1-D signal, while the light density $p(x, y)$ of a picture (see Figure 1.2) is a 2-D signal.

[1]S.S. Haykin, "Signal processing: where physics and mathematics meet," *IEEE Signal Processing Magazine*, vol. 18, Issue 4, pp. 6-7, Jul., 2001.