同济数学系列丛书
TONGJI SHUXUE XILIECONGSHU

美国数学建模竞赛
同济大学优秀论文选评（上）

同济大学数学建模组　主编

同濟大學出版社
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内容提要

截止到2013年，同济大学在美国数学建模竞赛中获得Ben Fusaro奖及入围奖1项，一等奖13项，二等奖25项，三等奖42项。本书正是精选了这些获奖论文中具有代表性的论文，每篇论文都按照竞赛论文的写作要求包含了论文的摘要、问题重述、问题分析、模型假设、模型建立和求解、模型分析和检验、模型评价等内容，在论文前列出原题，编者还在论文后给出了简要的点评，以供读者参考。

本书可供参加美国数学建模竞赛的学生学习和阅读，对于从事数学建模课程教学及指导工作的老师也有一定的参考价值，也可供相关学科的技术人员参考。

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前言

数学建模竞赛越来越成为大学生,尤其是理科学生大学学习的重要环节。许多同学通过参加数学建模竞赛让自己完成了一次从被动独自写作业的乖乖孩到主动合作解决问题的干将的涅槃。不夸张地说,数学建模竞赛就是他们的学术成人礼。然而,这个过程却是痛苦的。很多同学开始时面对问题完全不知所措,要自己去搜集资料、寻求思路、探索方案、找到答案,这其中的难度可想而知。这期间小组成员还要协调合作、反复讨论,在规定的时间里把大家的成果表达出来,完成论文。但是经过这个历练,绝大多数同学都会有更上一层楼的感觉,会以此磨亮以后走向社会打拼的利器。我们指导教师也会为同学们的进步而由衷喜悦。

通过多年努力,同济大学的数学建模教学取得了很多宝贵经验,竞赛成绩也越来越好。我们每年都能发现一批优秀的建模论文,但也有不少论文有很多待改进的地方。我们觉得竞赛的名次固然重要,但更重要的是同学们通过竞赛而成长进步,因此很有必要回顾一下他们的论文,分析成功的地方,指出待改进之处。这个工作不仅对已参赛同学而言有了深思顿悟的机会,对将参赛的同学也可获得一个借鉴的样本,使后者可以在未来的竞赛中少走弯路,最大限度地发挥自己的水平。而对指导教师而言,通过这些论文和点评,可以更多地理解参赛同学,也可以了解别的评判者的观点,对以后的指导也很有帮助。这就是我们出这本书的初衷。

本书精选了11篇同济大学近几年的参赛论文原文,文章与提交状态一致,几乎未经任何改动。文后附有他们的指导教师所做的点评。我们约定,点评只限于论文内容,而不涉及英语的语法、用法等问题。需要指出的是,由于指导教师的个人风格不同,点评的侧重点和观念会有差别。而这些点评者也不是参赛论文的评判者,甚至有可能

在某些点上与评判者的观点相左。事实上，为了尽可能地公平，竞赛评判过程采取了一些措施却还是避免不了参赛成绩因评判人而异的结果。尽管如此，优秀论文会是公认的，而不经意的论文瑕疵却有可能让参赛者痛失名次。这就是数学建模论文在评阅中所遇到的与一般数学竞赛不一样的地方。换句话说，数学建模问题没有标准答案。当然点评的角度不同并不代表论文的评判可以随心所欲。点评者会从成果、结构、表达、论述等诸方面考查论文。我们希望读者可以设身处地站在论文作者的立场上，并通过点评者的眼光审视论文，从而取得经验教训，汲取营养。

本书由周羚君、钱志坚、陈雄达、余斌、梁进、殷俊锋和项家樑为优秀论文进行了点评，最后由陈雄达统稿并修改定稿。

最后我们感谢贡献论文的参赛者，他们将自己仓促而不成熟但原汁原味的参赛作品和大家分享。更感谢参加点评的指导教师们，学生的成长离不开大家的辛勤工作。也感谢同济大学出版社让我们的想法得以实现。

梁　进

2014 年 5 月

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第一章

赛题：The Ultimate Brownie Pan

When baking in a rectangular pan heat is concentrated in the 4 corners and the product gets overcooked at the corners (and to a lesser extent at the edges). In a round pan the heat is distributed evenly over the entire outer edge and the product is not overcooked at the edges. However, since most ovens are rectangular in shape using round pans is not efficient with respect to using the space in an oven.

Develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes—rectangular to circular and other shapes in between.

Assume

(1) A width to length ratio of W/L for the oven which is rectangular in shape.

(2) Each pan must have an area of A.

(3) Initially two racks in the oven, evenly spaced.

Develop a model that can be used to select the best type of pan (shape) under the following conditions:

(1) Maximize number of pans that can fit in the oven (N).

(2) Maximize even distribution of heat (H) for the pan.

(3) Optimize a combination of conditions (1) and (2) where weights p and $1-p$ are assigned to illustrate how the results vary with different values of W/L and p.

In addition to your MCM formatted solution, prepare a one to two page advertising sheet for the new Brownie Gourmet Magazine highlighting your design and results.

当我们用一个长方形的锅烘烤食物的时候，热量总是集中在四个角上，从而导致四个角上的食物被过度烹饪，而边上的则烹饪程度不够。在一个圆形的锅里，热量均匀分布在整个外部边上，那么在边上的食物就不会被过度烹饪。但是，大多数的烤箱仍然是长方形的，从形状上来讲用圆形的烤盘并不能有效率地运用烤箱的空间。

试建立一个模型来说明在不同形状(如长方形，圆形或者介于两者之间的其他某个形状)的锅里，锅的外部边的热量的分布情况。

假定：

(1) 形状为长方形的烤箱的宽度与长度之比为 W/L。

(2) 每个锅的面积都是 A。

(3) 最初，烤箱中有两层烤箱架，并且在空间上是均匀分布的。

建立一个模型，使得我们能用该模型选择满足以下条件的最佳类型(形状)的锅：

(1) 希望能放在烤箱中的锅的数目(N)最大。

(2) 希望锅里的平均分布热量(H)最大。

(3) 对条件(1)与(2)分别赋予权重 p 与 $1-p$,希望将条件(1)与(2)的组合尽量优化,并分析说明在不同的 W/L 及 p 值的变化下,结果分如何变化。

特别的,除了你的 MCM 的结论外,请为 BG 杂志准备一到两页的广告页,以凸显你的设计和结果。

第二章

论文 1: The Ultimate Brownie Pan

by Yafeng Wang, Xiaxia Cui and Chao Li

Summary

In order to find an ultimate shape of the pan to achieve the highest utility ratio of space without overcooking, two models are developed.

In the first part, the pan is preliminarily simplified into a 2-D figure. And to get an intermediate shape between rectangle and circle, curve chamfer is used, in which every corner of the initial rectangle is replaced by a quarter of a circle with a certain radius. As the radius increases, the shape of pan changes gradually from rectangle to circle. On condition that heat conduction is the primary factor, the heat distribution of pan with the change of time is illustrated based on *heat conduction equation*. To solve the equations, the *finite volume element method* (*FVEM*) is applied to get the numerical solution of the partial differential equation (PDE).

Furthermore, since a pan is three-dimensional, we develop our model into a 3-D one by the software—ANSYS. In both the cases, the same conclusion is drawn that the evenness reaches the maximum when circular and the minimum when square. And the heat distribution is quite similar in both the 2-D case and the 3-D case, which proves our simplification feasible and reasonable.

In the second part of optimization, the multi-objective programming is converted into a single-objective one with linear weighted method. Firstly, the utility ratio of space is defined to measure the number of pans in the oven and the variance of heat distribution to measure the evenness. Then, the two indexes are normalized to achieve the combination of objectives. In addition, *a dynamic programming* (*DP*) is applied to deal with the configuration of pans in the oven. Thus the optimal solution about the shape and the number of pans is to obtained once the weight p and the ratio W/L is given. In particular, when two objects are of the same importance, the optimal radius is always around the half of the maximum radius.

§ 2.1 Introduction

Defining the Problem

If you are a chocolate lover there is nothing better than a delicious, home made brownie with a glass of ice cold milk when your sweet tooth is making you crave chocolate. However, the baking pans in our mom's kitchen are often rectangular, in which the brownies are often overcooked at the four corners. Meanwhile, if we choose a circular one to achieve even distribution of heat, the space in an oven will not be efficiently made use of.

In this case, it is necessary to find the best shape of pan to maximize both the utilization rate of the space in an oven and the even distribution of heat for the pan. Before this task, the distribution of heat across the outer edge of a pan should be illustrated. Thus, two tasks will be done in this paper:

- Develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes-rectangular to circular and other shapes in between.
- Develop a model that can be used to select the best type of pan (shape) under the following conditions:

1. Maximize number of pans that can fit in the oven;
2. Maximize even distribution of heat for the pan;
3. Optimize a combination of conditions (1) and (2) where two weights are assigned to them.

Model Overview

At first, we use curve chamfer (**Figure 2. 1**) to describe the intermediate shapes between rectangular and circle. When the radius r changes from 0 to maximum, the shape changes from square(Here we simplify the rectangle into a square, and we will discuss this treatment in the part of assumptions) to circle.

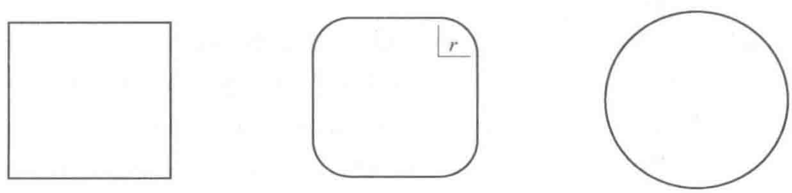

Figure 2. 1 The sketch map of curve chamfer

Then, in order to solve the problems above, we build two models. The first model which is based on the *heat conduction equation* and solved by the means of *finite volume element method*(*FVEM*) can illustrate the distribution of heat across the pan. The second model which is based on *the linear weighted method and dynamic programming* (*DP*) that deals with the optimization of rectangular parts nesting.

In the first model, *the heat conduction equation* is used respectively in a 2-D case and a 3-D case, and in both the cases the distribution of heat across the pan can be illustrated. In the second model, the multi-objective programming is converted into a single-objective programming by using the *analysis of variance* (*ANOVA*), data normalization and linear weighted method. Thus we can get the best shape of the pan to optimize the utilization rate of the space in an oven and the distribution of heat across the pan.

§ 2.2 Assumptions

The pan is initially to a cuboid

In reality, the shape of the pan used in an oven is similar to a flat box without lid. Nevertheless, considering that the heat distribution of brownie baking in the pan is mainly influenced by the pan and the need of simplifying the model, we initially regard the pan as a homogeneous cuboid and determine it as our object of study.

The space in oven is constant temperature field

According to the technical parameters published by some household oven manufacturers, the temperature range in the oven is about 2 Degree Celsius, a relatively smaller value, so we decide to ignore the range and treat the inner space of an oven as a constant temperature field.

The process of the the thermal transmission is a transient process

Before the pan is put inside, the oven is often preheated[1]. Thus when the pan enters, there is a temperature difference between the pan and its surroundings. So it takes time for the pan to get heated gradually to a certain temp. And we believe that the heat distribution of cuboid in this process is most of the time non-uniform, which leads to the overcooking at the corners.

Only square pan is considered, or ignore the rectangle pan whose length is unequal to its width

In order to make calculation easier, we simplify the rectangle into square.

Using different chamfer to describe the different shape between the square and circle

As for the study of intermediate shape between rectangle and circle, basically two ways are available. One is to use regular polygons and the other is curve chamfer. However, the angle of polygons does no good to heat distribution because of tip thermal effect[2], we decide to abandon the disadvantageous regular polygon and to apply curve chamfer only.

Each pan has the same area

Initially two racks in the oven, evenly spaced and the distribution on each rack are the same

The distribution of heat across the pans on the upper rack and on the under rack is the same. Therefore, we can only study the pans on one of the racks.

§ 2.3 Notations

Table 2.1 **Notations and Descriptions**

Notations	Descriptions
W/L	Width to length ratio of the oven
A	The area of pan
N	The number fit in the oven
H	The distribution of heat across the pan
r	The radius of the curve chamfer
p	Weight giving to the first optimization objective
T_t	The distribution of temperature on the pan at time t
T_{out}	The temperature out of the pan
f	Energy source inside the heat carrier
λ	Heat conductivity of the pan
h	Coefficient of heat convection
ρ	The density of the pan
c	Specific heat capacity
$\sigma^2(H)$	The variance of the distribution of heat across the pan
ξ	Utilization rate of the area in the oven
μ	The standardized value of ξ
η	Evenness of the distribution of heat across the pan
OBJ	Objective function of the multiple objective programming

§ 2.4 Part I

Model I: The Distribution of Heat

Newton articulated some principles of heat flow through solids, but it was Fourier who created the correct systematic theory. Inside a solid there is no convective transfer of heat energy and little radiative transfer, so temperature changes only by conduction, as the energy we now recognize as molecular kinetic energy flows from hotter regions to cooler regions. The basic principles of heat are:

- The heat energy contained in a material is proportional to the temperature, the density of the material, and a physical characteristic of the material called the specific heat capacity.
- The heat transfer through the boundary of a region is proportional to the heat conductivity, to the gradient of the temperature across the region, and to the area of contact.