经典名著系列

DYNAMIC ASSET PRICING THEORY

THIRD EDITION

DARRELL DUFFIE

动态资产价格理论

第3版

世界图书出版公司
www.wpcbj.com.cn

Dynamic Asset Pricing Theory

THIRD EDITION

Darrell Duffie

Princeton University Press
Princeton and Oxford

书　　名：Dynamic Asset Pricing Theory 3rd ed.

作　　者：Darrell Duffie

中 译 名：动态价格理论 第3版

责任编辑：高　蓉

出 版 者：世界图书出版公司北京公司

印 刷 者：北京世图印刷厂

发　　行：世界图书出版公司北京公司（北京朝内大街137号　100010）

联系电话：010-64015659, 64038347

电子信箱：kjsk@vip.sina.com

开　　本：24开

印　　张：20.5

版　　次：2007年1月第1次印刷

版权登记：图字:01-2006-6881

书　　号：978-7-5062-8234-5/ O·563　　定　　价：68.00元

Published by Princeton University Press, 41 William Street,
Princeton, New Jersey 08540
In the United Kingdom: Princeton University Press, 3 Market Place, Woodstock,
Oxfordshire OX20 1SY

Library of Congress Cataloging-in-Publication Data

Duffie, Darrell.
Dynamic asset pricing theory / Darrell Duffie.—3rd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-691-09022-X (alk. paper)
1. Capital assets pricing model. 2. Portfolio management 3. Uncertainty.
I. Title.
HG4637.D84 2001
332.6—dc21 2001021235

British Library Cataloging-in-Publication Data is available

ISBN-13: 978-0-691-09022-1

ISBN-10: 0-691-09022-X

影 印 版 前 言

本书为 Duffie 教授著名的《动态资产定价理论》第 3 版，与前二版相比，本版主要增加的内容是第 11 章公司证券，即将公司的股权融资、债券融资、违约、破产等结合在一起来考虑定价。

本版的风格仍然与前二版相同，即用复杂的数学模型来处理金融问题，这样做的优点是可以获得较为深刻的理论结果，缺点是读起来很困难，它的起始读者群定位于金融专业的博士研究生。它像一道高高的门槛，跨过去之后会发现现代金融的广大空间，跨不过去则可能对现代金融学产生敬畏的心理，因此，Duffie 的书一直是一本风格比较独特的书籍，吸引了众多的读者试图去读懂和征服它。在这里，我热情地向广大的金融研究生推荐这本书，希望你们能勇敢地跨过这道门槛。

程 兵

中国科学院数学与系统科学研究院

2006 年 11 月 5 日

For Colin

Preface

THIS BOOK IS an introduction to the theory of portfolio choice and asset pricing in multiperiod settings under uncertainty. An alternate title might be *Arbitrage, Optimality, and Equilibrium,* because the book is built around the three basic constraints on asset prices: absence of arbitrage, single-agent optimality, and market equilibrium. The most important unifying principle is that any of these three conditions implies that there are "state prices," meaning positive discount factors, one for each state and date, such that the price of any security is merely the state-price weighted sum of its future payoffs. This idea can be traced to the invention by Arrow (1953) of the general equilibrium model of security markets. Identifying the state prices is the major task at hand. Technicalities are given relatively little emphasis so as to simplify these concepts and to make plain the similarities between discrete- and continuous-time models.ricing model.

To someone who came out of graduate school in the mid-eighties, the decade spanning roughly 1969–79 seems like a golden age of dynamic asset pricing theory. Robert Merton started continuous-time financial modeling with his explicit dynamic programming solution for optimal portfolio and consumption policies. This set the stage for his 1973 general equilibrium model of security prices, another milestone. His next major contribution was his arbitrage-based proof of the option pricing formula introduced by Fisher Black and Myron Scholes in 1973, and his continual development of that approach to derivative pricing. The Black-Scholes model now seems to be, by far, the most important single breakthrough of this "golden decade," and ranks alone with the Modigliani and Miller (1958) Theorem and the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) in its overall importance for financial theory and practice. A tremendously influential simplification of the Black-Scholes model appeared in the "binomial" option pricing model of Cox, Ross, and Rubinstein (1979), who drew on an insight of Bill Sharpe.

Working with discrete-time models, LeRoy (1973), Rubinstein (1976), and Lucas (1978) developed multiperiod extensions of the CAPM. The "Lucas model" is the "vanilla flavor" of equilibrium asset pricing models. The simplest multiperiod representation of the CAPM finally appeared in Doug Breeden's continuous-time consumption-based CAPM, published in 1979. Although not published until 1985, the Cox-Ingersoll-Ross model of the term structure of interest rates appeared in the mid-seventies and is still the premier textbook example of a continuous-time general equilibrium asset pricing model with practical applications. It also ranks as one of the key breakthroughs of that decade. Finally, extending the ideas of Cox and Ross (1976) and Ross (1978), Harrison and Kreps (1979) gave an almost definitive conceptual structure to the whole theory of dynamic security prices.

Theoretical developments in the period since 1979, with relatively few exceptions, have been a mopping-up operation. Assumptions have been weakened, there have been noteworthy extensions and illustrative models, and the various problems have become much more unified under the umbrella of the Harrison-Kreps model of equivalent martingale measures. For example, the standard approach to optimal portfolio and consumption choice in continuous-time settings has become the martingale method of Cox and Huang (1989). An essentially final version of the relationship between the absence of arbitrage and the existence of equivalent martingale measures was finally obtained by Delbaen and Schachermayer (1999).

On the applied side, markets have experienced an explosion of new valuation techniques, hedging applications, and security innovation, much of this based on the Black-Scholes and related arbitrage models. No major investment bank, for example, lacks the experts or computer technology required to implement advanced mathematical models of the term structure. Because of the wealth of new applications, there has been a significant development of special models to treat stochastic volatility, jump behavior including default, and the term structure of interest rates, along with many econometric advances designed to take advantage of the resulting improvements in richness and tractability.

Although it is difficult to predict where the theory will go next, in order to promote faster progress by people coming into the field it seems wise to have some of the basics condensed into a textbook. This book is designed to be a streamlined course text, not a research monograph. Much generality is sacrificed for expositional reasons, and there is relatively little emphasis on mathematical rigor or on the existence of general equilibrium. As its title indicates, I am treating only the theoretical side of the story. Although it might be useful to tie the theory to the empirical side of asset pricing, we have excellent treatments of the econometric modeling of financial data, such as Campbell, Lo, and MacKinlay (1997) and Gourieroux and Jasiak (2000). I also leave out some important aspects of functioning security markets, such as asymmetric information and transactions costs. I have chosen to develop only some of the essential ideas of dynamic asset pricing, and even these are more than enough to put into one book or into a one-semester course.

Other books whose treatments overlap with some of the topics treated here include Avelleneda and Laurence (2000), Björk (1998), Dana and Jeanblanc (1998), Demange and Rochet (1992), Dewynne and Wilmott (1994), Dixit and Pindyck (1993), Dothan (1990), Duffie (1988b), Harris (1987), Huang and Litzenberger (1988), Ingersoll (1987), Jarrow (1988), Karatzas (1997), Karatzas and Shreve (1998), Lamberton and Lapeyre (1997), Magill and Quinzii (1994), Merton (1990), Musiela and Rutkowski (1997), Neftci (2000), Stokey and Lucas (1989), Willmott, Dewynne, and Howison (1993), and Wilmott, Howison, and Dewynne (1995). Each has its own aims and themes. I hope that readers will find some advantage in having yet another perspective.

A reasonable way to teach a shorter course on continuous-time asset pricing out of this book is to begin with Chapter 1 or 2 as an introduction to the basic notion of state prices and then to go directly to Chapters 5 through 11. Chapter 12, on numerical methods, could be skipped at some cost to the student's ability to implement the results. There is no direct dependence of any results in Chapters 5 through 12 on the first four chapters.

For mathematical preparation, little beyond undergraduate analysis, as in Bartle (1976), and linear algebra is assumed. Some familiarity with Royden (1968) or a similar text on functional analysis and measure theory, would also be useful. Some background in microeconomics would be useful, say Kreps (1990) or Luenberger (1995). Familiarity with probability theory at the level of Jacod and Protter (2000), for example, would also speed things along, although measure theory is not used heavily. In any case, a series of appendices supplies all of the required concepts and definitions from probability theory and stochastic calculus. Additional useful references in this regard are Brémaud (1981), Karatzas and Shreve (1988), Revuz and Yor (1991), and Protter (1990).

Students seem to learn best by doing problem exercises. Each chapter has exercises and notes to the literature. I have tried to be thorough in giving sources for results whenever possible and plead that any cases in which I have mistaken or missed sources be brought to my attention for correction. The notation and terminology throughout is fairly standard. I use $\mathbb{R}$ to denote the real line and $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ for the extended real line. For any set Z and positive integer n, I use Z^n for the set of n-tuples of the form $(z_1, \ldots, z_n)$ with z_i in Z for all i. An example is $\mathbb{R}^n$. The conventions used for inequalities in any context are

- $x \geq 0$ means that x is nonnegative. For x in $\mathbb{R}^n$, this is equivalent to $x \in \mathbb{R}^n_+$;
- $x > 0$ means that x is nonnegative and not zero, but not necessarily strictly positive in all coordinates;
- $x \gg 0$ means x is strictly positive in every possible sense. The phrase "x is strictly positive" means the same thing. For x in $\mathbb{R}^n$, this is equivalent to $x \in \mathbb{R}^n_{++} \equiv \text{int}(\mathbb{R}^n_+)$.

Although warnings will be given at appropriate times, it should be kept in mind that $X = Y$ will be used to mean equality almost everywhere or almost surely, as the case may be. The same caveat applies to each of the above inequalities. A real-valued function F on an ordered set (such as $\mathbb{R}^n$) is *increasing* if $F(x) \geq F(y)$ whenever $x \geq y$ and *strictly increasing* if $F(x) > F(y)$ whenever $x > y$. When the domain and range of a function are implicitly obvious, the notation "$x \mapsto F(x)$" means the function that maps x to $F(x)$; for example, $x \mapsto x^2$ means the function $F : \mathbb{R} \to \mathbb{R}$ defined by $F(x) = x^2$. Also, while warnings appear at appropriate places, it is worth pointing out again here that, for ease of exposition, a continuous-time "process" will be defined throughout as a jointly (product) measurable function on $\Omega \times [0, T]$, where $[0, T]$ is the given time interval and $(\Omega, \mathcal{F}, P)$ is the given underlying probability space.

The first four chapters are in a discrete-time setting with a discrete set of states. This should ease the development of intuition for the models to be found in Chapters 5 through 12. The three pillars of the theory, *arbitrage, optimality,* and *equilibrium,* are developed repeatedly in different settings. Chapter 1 is the basic single-period model. Chapter 2 extends the results of Chapter 1 to many periods. Chapter 3 specializes Chapter 2 to a Markov setting and illustrates dynamic programming as an alternate solution technique. The Ho-and-Lee and Black-Derman-Toy term-structure models are included as exercises. Chapter 4 is an infinite-horizon counterpart to Chapter 3 that has become known as the *Lucas model.*

The focus of the theory is the notion of state prices, which specify the price of any security as the state-price weighted sum or expectation of the security's state-contingent dividends. In a finite-dimensional setting, there exist state prices if and only if there is no arbitrage. The same fact is true in infinite-dimensional settings under mild technical regularity conditions. Given an agent's optimal portfolio choice, a state-price vector is given by that agent's utility gradient. In an equilibrium with Pareto optimality, a state-price vector is likewise given by a representative agent's utility gradient at the economy's aggregate consumption process.

Chapters 5 through 11 develop a continuous-time version of the theory in which uncertainty is generated by Brownian motion. In Chapter 11, there is a transition to discontinuous information, that is, settings in which the conditional probability of some events does not adjust continuously with the passage of time. An example is Poisson arrival.

Chapter 5 introduces the continuous-trading model and develops the Black-Scholes partial differential equation (PDE) for arbitrage-free prices of derivative securities. The Harrison-Kreps model of equivalent martingale measures is presented in Chapter 6 in parallel with the theory of state prices in continuous time. Chapter 7 presents models of the term structure of interest rates, including the Black-Derman-Toy, Vasicek, Cox-Ingersoll-Ross, and Heath-Jarrow-Morton models, as well as extensions. Chapter 8 presents specific classes of derivative securities, such as futures, forwards, American options, and lookback options. Chapter 8 also introduces models of option pricing with stochastic volatility. The notion of an "affine" state process is used heavily in Chapters 7 and 8 for its analytical tractability. Chapter 9 is a summary of optimal continuous-time portfolio choice, using both dynamic programming and an approach involving equivalent martingale measures or state prices. Chapter 10 is a summary of security pricing in an equilibrium setting. Included are such well-known models as Breeden's consumption-based capital asset pricing model and the general equilibrium version of the Cox-Ingersoll-Ross model of the term structure of interest rates. Chapter 11 deals with the valuation of corporate securities, such as debt and equity. The chapter moves from models based on the capital structure of the corporation, in which default is defined in terms of the sufficiency of assets, to models based on an assumed process for the default arrival intensity. Chapter 12 outlines three numerical methods for calculating derivative security prices in a continuous-time setting: binomial approximation, Monte Carlo simulation of a discrete-time approximation of security prices, and finite-difference solution of the associated PDE for the asset price or the fundamental solution.

In preparing the first edition, I relied on help from many people, in addition to those mentioned above who developed this theory. In 1982, Michael Harrison gave a class at Stanford that had a major effect on my understanding and research goals. Beside me in that class was Chi-fu Huang; we learned much of this material together, becoming close friends and collaborators. I owe him a lot. I am grateful to Niko and Vana Skiadas, who treated me with overwhelming warmth and hospitality at their home on Skiathos, where parts of the first draft were written.

I have benefited from research collaboration over the years with many co-authors, including George Constantinides, Qiang Dai, Peter DeMarzo, Larry Epstein, Nicolae Gârleanu, Mark Garman, John Geanakoplos, Pierre-Yves Geoffard, Peter Glynn, Mike Harrison, Chi-fu Huang, Ming Huang, Matt Jackson, Rui Kan, David Lando, Jun Liu, Pierre-Louis Lions, Jin Ma, Andreu Mas-Colell, Andy McLennan, Jun Pan, Lasse Pedersen, Philip Protter, Rohit Rahi, Tony Richardson, Mark Schroder, Wayne Shafer, Ken Singleton, Costis Skiadas, Richard Stanton, Jiongmin Yong, and Bill Zame. I owe a special debt to Costis Skiadas, whose generous supply of good ideas has had a big influence on the result.

I am lucky indeed to have had access to many fruitful research discussions here at Stanford, especially with my colleagues Anat Admati, Steve Boyd, Peter DeMarzo, Peter Glynn, Steve Grenadier, Joe Grundfest, Peter Hammond, Mike Harrison, Ayman Hindy, Harrison Hong, Ming Huang, Jack McDonald, George Parker, Paul Pfleiderer, Balaji Prabakar, Manju Puri, Tom Sargent, Bill Sharpe, Ken Singleton, Jim Van Horne, and Jeff Zwiebel, and elsewhere with too many others to name.

I am thankful to have had the chance to work on applied financial models with Mike Burger, Bill Colgin, Adam Duff, Stenson Gibner, Elizabeth Glaeser Craig Gustaffson, Corwin Joy, Vince Kaminsky, Ken Knowles, Sergio Kostek, Joe Langsam, Aloke Majumdar, Matt Page, Krishna Rao, Amir Sadr, Louis Scott, Wei Shi, John Uglum, and Mark Williams.

I thank Kingston Duffie, Ravi Myneni, Paul Bernstein, and Michael Boulware for coding and running some numerical examples, Linda Bethel for assistance with LaTeX and graphics, and, for production editing and typesetting, Laurie Pickert of Archetype and Tina Burke of Technical Typesetting. For assistance with bibliographic research, I am grateful to Yu-Hua Chen, Melissa Gonzalez, David Lee, and Analiza Quiroz. Jack Repcheck and Peter Dougherty have been friendly, helpful, and supportive editors.

Suggestions and comments have been gratefully received from many readers, including Fehmi Ashaboglu, Robert Ashcroft, Irena Asmundsen, Alexandre d'Aspremont, Flavio Auler, Chris Avery, Kerry Back, Yaroslav

Bazaliy, Ralf Becker, Antje Berndt, Michael Boulware, John Campbell, Andrew Caplin, Karen Chaltikian, Victor Chernozhukov, Hung-Ken Chien, Seongman Cho, Fai Tong Chung, Chin-Shan Chuan, Howie Corb, Qiang Dai, Eugene Demler, Shijie Deng, Michelle Dick, Phil Dolan, Rod Duncan, Wedad Elmaghraby, Kian Esteghamat, Mark Ferguson, Christian Riis Flor, Prashant Fuloria, John Fuqua, Nicolae Gârleanu, Mark Garmaise, Filippo Ginanni, Michel Grueneberg, Bing Han, Philippe Henrotte, Ayman Hindy, Yael Hochberg, Toshiki Honda, Taiichi Hoshino, Jiangping Hu, Ming Huang, Cristobal Huneeus, Don Iglehart, Michael Intriligator, Farshid Jamshidian, Ping Jiang, Shinsuke Kambe, Rui Kan, Ron Karidi, Don Kim, Felix Kubler, Allan Kulig, Yoichi Kuwana, Piero La Mura, Yingcong Lan, Joe Langsam, Jackie Lee, André Levy, Shujing Li, Wenzhi Li, Tiong Wee Lim, Jun Liu, Leonid Litvak, Hanno Lustig, Rob McMillan, Rajnish Mehra, Sergei Morozov, Christophe Mueller, Ravi Myneni, Lee Bath Nelson, Yigal Newman, Angela Ng, Kazuhiko Ōhashi, Hui Ou-Yang, John Overdeck, Hideo Owen, Caglar Ozden, Mikko Packalen, Jun Pan, Lasse Pedersen, Albert Perez, Monika Piazzesi, Jorge Picazo, Heracles Polemarchakis, Marius Rabus, Rohit Rahi, Shikhar Ranjan, Michael Rierson, Amir Sadr, Yuliy Sannikov, Marco Scarsini, Martin Schneider, Christine Shannon, Yong-Seok Shin, Mark Shivers, Hersir Sigurgeirsson, Marciano Siniscalchi, Ravi Singh, Ronnie Sircar, Viktor Spivakovsky, Lucie Tepla, Sergiy Terentyev, Rajat Tewari, Sverrir Thorvaldsson, Alex Tolstykh, Tunay Tunca, John Uglum, Len Umantsev, Stijn Van Nieuwerburgh, Laura Veldkamp, Mary Vyas, Muhamet Yildiz, Nese Yildiz, Ke Wang, Neng Wang, Chao Wei, Wei Wei, Pierre-Olivier Weill, Steven Weinberg, Seth Weingram, Guojun Wu, Pinghua Young, Assaf Zeevi, and Alexandre Ziegler, with apologies to those whose assistance was forgotten. I am especially grateful to the expert team of Japanese translators of the second edition, Toshiki Honda, Kazuhiko Ōhashi, Yoichi Kuwana, and Akira Yamazaki, all of whom are personal friends as well.

For the reader's convenience, the original preface has been revised for this third edition. Significant improvements have been made in most chapters. Chapter 11, "Corporate Securities," has been added for this edition. Errors are my own responsibility, and I hope to hear of them and any other comments from readers.

Darrell Duffie

Contents