

實 用

# 土木工程計算圖表

楊文淵編

改版本

龍門聯合書局出版

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本書取材注重實用，內容包括數學公式、數表、單位換算、應用力學、水力學、測量、工程材料、土工及爆破、混凝土及鋼筋混凝土工、基礎工、道路工、土木機械等十二編，均為土建工程中的重要部分。

全書以簡明手冊的方式編排，計有實用公式、數據、圖表、圖解等六百餘種，除採用祖國最新資料外，並將蘇聯的先進經驗儘量列入，俾在設計施工時，藉以獲得參考與檢查上的便利。

本書初版於1948年，至1953年共印行兩萬六千冊，其間雖經數次修訂，但均屬局部增刪，為適應國家大規模經濟建設的迅速發展，經編者將原書予以改編，內容較前為充實，它使我們在工程實際工作中能廣泛地被應用。

本書可作為土建工程從業人員的實用手冊，亦適用於教學時的一般參考。

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# 第 1 編 數學公式

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## 1—1 代數

## I. 乘冪及根

1.  $a^m \times a^n = a^{m+n}$

2.  $a^m \div a^n = a^{m-n}$

3.  $(a^m)^n = (a^n)^m = a^{mn}$

4.  $\sqrt[n]{a} = a^{1/n}$

5.  $\sqrt[n]{a} = a^{-1/n}$

6.  $\sqrt[n]{a} \times \sqrt[n]{a} = \sqrt[n]{a^{m+n}}$

7.  $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

8.  $\sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{a}$

## II. 恒等式及因數分解

1.  $a^2 - b^2 = (a+b)(a-b)$

2.  $a^2 + b^2 = (a+b\sqrt{-1})(a-b\sqrt{-1})$

3.  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

4.  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

5.  $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$

6.  $a^n - b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 + \dots - b^{n-1})$  ( $n$ =偶數)

7.  $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \dots + b^{n-1})$  ( $n$ =奇數)

8.  $(x+a)(x+b) = x^2 + (a+b)x + ab$

9.  $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (bc+ca+ab)x + abc$

10.  $(a+b)^2 = a^2 + 2ab + b^2$

11.  $(a-b)^2 = a^2 - 2ab + b^2$

12.  $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

13.  $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

14.  $a^4 - a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

15.  $a^4 - a^2b^2 + b^4 = (a^2 - ab\sqrt{2} + b^2)(a^2 + ab\sqrt{2} + b^2)$

16.  $(a+b+c)^2 = a^2 + b^2 + c^2 + 2bc + 2ca + 2ab$

17.  $(a+b+c)^3 = a^3 + b^3 + c^3 + 3(b^2c + bc^2 + c^2a + ca^2 + a^2b + ab^2) + 6abc$

18.  $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - bc - ca - ab)$

## III. 對數

1. 設  $a$  為異於 1 之有限正數, 且  $a^x = N$ , 則  $x$  是以  $a$  為底的  $N$  之對數。  
或  $\log_a N = x$ . 設  $\log_a N = x$ , 則  $a^x = N$ .

2.  $\log_a a = 1$ .

3.  $\log_a 1 = 0$ .

4.  $\log_a 0 = -\infty$

5.  $\log_a (A \times B) = \log_a A + \log_a B$

6.  $\log_a(A \div B) = \log_a A - \log_a B$ .  
 7.  $\log_a A^n = n \log_a A$ .      8.  $\log_a \sqrt[n]{A} = \frac{1}{n} \log_a A$ .  
 9.  $\log_a A = \log_b A \times \log_a b$ .      10.  $\log_a b \times \log_b a = 1$ .  
 11. 普通对数(布立格茲, Briggsian)——以10為底.  
 12. 自然对数(納百尔或双曲线, Napierian or hyperbolic)——以2.7183為底——(用e或 $\epsilon$ 表示).  
 (本編普通对数以Log表之, 自然对数单以log表之)

13.  $\text{Log } N = \log N \times \text{Log } e$       14.  $\text{Log } e = 0.43429448 \dots\dots$   
 15.  $\text{Log } N = \log N \times \log 10$ .      16.  $\log 10 = 2.30258509 \dots\dots$   
 17.  $\text{Log } e \times \log 10 = 1$       18.  $\text{Log } 10 = 1$   
 19.  $\text{Log } 10^n = n$

#### IV. 方程式

1. 二次方程式  $ax^2 + bx + c = 0$ :

$$x = \frac{1}{2a}(-b \pm \sqrt{b^2 - 4ac})$$

2. 三次方程式  $ax^3 + bx^2 + cx + d = 0$  設  $z = x - \frac{b}{3a}$  可化為  
 $x^3 + 3px + 2q = 0$

3. 三次方程式  $x^3 + 3px + 2q = 0$  之根

$$x_1 = u + v, \quad x_2 = w_1 u + w_2 v, \quad x_3 = w_2 u + w_1 v$$

$$\text{設 } w_1 = w_2^2 = \frac{1}{2}(-1 + \sqrt{-3}), \quad w_2 = w_1^2 = \frac{1}{2}(-1 - \sqrt{-3})$$

$$u = \sqrt[3]{-q + \sqrt{q^2 + p^3}}, \quad v = \sqrt[3]{-q - \sqrt{q^2 + p^3}}$$

4. 四次方程式  $ax^4 + bx^3 + cx^2 + dx + e = 0$        $z = x - \frac{b}{4a}$  可化為  
 $x^4 + px^2 + qx + r = 0$  其四次方程式之根.

$$x_1 = \frac{1}{2}(\sqrt{y_1} + \sqrt{y_2} + \sqrt{y_3}), \quad x_2 = \frac{1}{2}(-\sqrt{y_1} + \sqrt{y_2} - \sqrt{y_3})$$

$$x_3 = \frac{1}{2}(\sqrt{y_1} - \sqrt{y_2} - \sqrt{y_3}), \quad x_4 = \frac{1}{2}(-\sqrt{y_1} - \sqrt{y_2} + \sqrt{y_3})$$

設  $y$  為三次方程式  $y^3 + 2py^2 + (p^2 - 4r)y - q^2 = 0$  之3根, 以  $\sqrt{y}$  表之.

$$\text{則 } \sqrt{y_1} \times \sqrt{y_2} \times \sqrt{y_3} = -q$$



## V. 級數之和

1. 等差級數  $a+(a+d)+(a+2d)+\dots+(a+nd)=\frac{n+1}{2}(2a+nd)$
2. 等比級數  $a+ar+ar^2+\dots+ar^n=a\frac{1-r^{n+1}}{1-r}=a\frac{r^{n+1}-1}{r-1}$
3.  $1+2+3+\dots+n=\frac{1}{2}n(n+1)$
4.  $1^2+2^2+3^2+\dots+n^2=\frac{1}{6}n(n+1)(2n+1)$
5.  $1^3+2^3+3^3+\dots+n^3=\frac{1}{4}n^2(n+1)^2$
6.  $1^4+2^4+3^4+\dots+n^4=\frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$
7.  $1+3+5+\dots+(2n-1)=n^2$
8.  $1^2+3^2+5^2+\dots+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$
9.  $1^3+3^3+5^3+\dots+(2n-1)^3=n^2(2n^2-1)$
10.  $1\times 2+2\times 3+3\times 4+\dots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$
11.  $1\times 2\times 3+2\times 3\times 4+3\times 4\times 5+\dots+n(n+1)(n+2)$   
 $=\frac{1}{4}n(n+1)(n+2)(n+3)$
12.  $1+2x+3x^2+4x^3+\dots+nx^{n-1}=\frac{1-x^{n+1}}{(1-x)^2}-\frac{nx^{n+1}}{1-x}$

## VI. 無限級數及函數之展式

1.  $(1\pm x)^n=1\pm nx+\frac{n(n-1)}{1\times 2}x^2\pm\frac{n(n-1)(n-2)}{1\times 2\times 3}x^3+\dots$ , 設  $n$  為任意值,  
 $|x|<1$ .
2.  $\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+\frac{7}{256}x^5-\frac{21}{1024}x^6+\dots$
3.  $\frac{1}{\sqrt{1+x}}=1-\frac{1}{2}x+\frac{3}{8}x^2-\frac{5}{16}x^3+\frac{35}{128}x^4-\frac{63}{256}x^5+\frac{231}{1024}x^6+\dots$
4.  $\sqrt[3]{1+x}=1-\frac{1}{3}x+\frac{1}{9}x^2-\frac{5}{81}x^3+\frac{10}{243}x^4-\frac{22}{729}x^5+\frac{154}{6561}x^6+\dots$
5.  $\frac{1}{\sqrt[3]{1+x}}=1+\frac{1}{3}x+\frac{2}{9}x^2+\frac{14}{81}x^3+\frac{35}{243}x^4+\frac{91}{729}x^5+\frac{728}{6561}x^6+\dots$
6.  $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots$  (設  $n!=1\times 2\times 3\times\dots\times n$ )

$$7. a^x = 1 + \frac{x \log a}{1!} + \frac{(x \log a)^2}{2!} + \frac{(x \log a)^3}{3!} + \dots$$

$$8. \log x = 2 \left[ \frac{x-1}{x+1} + \frac{1}{3} \left( \frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left( \frac{x-1}{x+1} \right)^5 + \dots \right], (x > 0)$$

$$9. \log(1 \pm x) = \pm x - \frac{x^2}{2} \pm \frac{x^3}{3} - \frac{x^4}{4} \pm \frac{x^5}{5} - \dots, (|x| < 1)$$

$$10. \log \frac{1+x}{1-x} = 2 \left( x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \right), (|x| < 1)$$

$$11. \log \frac{x+1}{x-1} = 2 \left( \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \dots \right), (|x| > 1)$$

$$12. \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$13. \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$14. \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots, (|x| < \frac{\pi}{2})$$

$$15. \cot x = \frac{1}{x} - \frac{x}{3} + \frac{x^3}{45} - \frac{2x^5}{945} + \frac{x^7}{4725} - \dots, (|x| < \pi)$$

$$16. \sin^{-1} x = x + \frac{1}{2 \times 3} x^3 + \frac{1 \times 3}{2 \times 4 \times 5} x^5 + \frac{1 \times 3 \times 5}{2 \times 4 \times 6 \times 7} x^7 + \dots, (|x| < 1)$$

$$17. \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots, (|x| < 1)$$

$$18. \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$19. \cosh x = x + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

## 1-2 平面三角法

## I. 同一角之三角函数間之關係

|                                 | $\sin \alpha = x$            | $\cos \alpha = x$            | $\tan \alpha = x$            | $\cot \alpha = x$            | $\sec \alpha = x$            | $\operatorname{cosec} \alpha = x$ |
|---------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|-----------------------------------|
| $\sin \alpha =$                 | $x$                          | $\pm \sqrt{1-x^2}$           | $\pm \frac{x}{\sqrt{1+x^2}}$ | $\pm \frac{1}{\sqrt{1+x^2}}$ | $\pm \frac{\sqrt{x^2-1}}{x}$ | $\frac{1}{x}$                     |
| $\cos \alpha =$                 | $\pm \sqrt{1-x^2}$           | $x$                          | $\pm \frac{1}{\sqrt{1+x^2}}$ | $\pm \frac{x}{\sqrt{1+x^2}}$ | $\frac{1}{x}$                | $\pm \frac{\sqrt{x^2-1}}{x}$      |
| $\tan \alpha =$                 | $\pm \frac{x}{\sqrt{1-x^2}}$ | $\pm \frac{\sqrt{1-x^2}}{x}$ | $x$                          | $\frac{1}{x}$                | $\pm \sqrt{x^2-1}$           | $\pm \frac{1}{\sqrt{x^2-1}}$      |
| $\cot \alpha =$                 | $\pm \frac{\sqrt{1-x^2}}{x}$ | $\pm \frac{x}{\sqrt{1-x^2}}$ | $\frac{1}{x}$                | $x$                          | $\pm \frac{1}{\sqrt{x^2-1}}$ | $\pm \sqrt{x^2-1}$                |
| $\sec \alpha =$                 | $\pm \frac{1}{\sqrt{1-x^2}}$ | $\frac{1}{x}$                | $\pm \sqrt{1+x^2}$           | $\pm \frac{\sqrt{1+x^2}}{x}$ | $x$                          | $\pm \frac{x}{\sqrt{x^2-1}}$      |
| $\operatorname{cosec} \alpha =$ | $\frac{1}{x}$                | $\pm \frac{1}{\sqrt{1-x^2}}$ | $\pm \frac{\sqrt{1+x^2}}{x}$ | $\pm \sqrt{1+x^2}$           | $\pm \frac{x}{\sqrt{x^2-1}}$ | $x$                               |

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad 1 + \tan^2 \alpha = \sec^2 \alpha \quad 1 + \cot^2 \alpha = \operatorname{cosec}^2 \alpha$$

## II. 倍角之三角函数及三角函数之幂

1.  $\sin 2\alpha = 2\sin \alpha \cos \alpha$
2.  $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
3.  $\sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$
4.  $\cos 3\alpha = 4\cos^3 \alpha - 3\cos \alpha$
5.  $\sin 4\alpha = 2\sin 2\alpha \cos 2\alpha$
6.  $\cos 4\alpha = 1 - 8\cos^2 \alpha + 8\cos^4 \alpha$
7.  $\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$
8.  $\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha)$
9.  $\sin^3 \alpha = \frac{1}{4}(3\sin \alpha - \sin 3\alpha)$
10.  $\cos^3 \alpha = \frac{1}{4}(3\cos \alpha + \cos 3\alpha)$
11.  $\sin^4 \alpha = \frac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$
12.  $\cos^4 \alpha = \frac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$
13.  $\tan 2\alpha = \frac{2\tan \alpha}{1 - \tan^2 \alpha}$
14.  $\tan 3\alpha = \frac{3\tan \alpha - \tan^3 \alpha}{1 - 3\tan^2 \alpha}$

## III. 两角之和或差之函数

1.  $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$

2.  $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$

3.  $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$

4.  $\cot(\alpha \pm \beta) = \frac{\cot\alpha \cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$

IV. 二函数之和差及积

1.  $\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

2.  $\sin\alpha - \sin\beta = 2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$

3.  $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

4.  $\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$

5.  $\tan\alpha \pm \tan\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha \cos\beta}$

6.  $\cot\alpha \pm \cot\beta = \frac{\sin(\beta \pm \alpha)}{\sin\alpha \sin\beta}$

7.  $\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

8.  $\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

9.  $\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$

V. 三角形之性质

$a, b, c =$  边;  $A, B, C =$  角;  $R =$  外切圆之半径;  
 $s = \frac{1}{2}(a+b+c)$ .

1. 正弦法则  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

2. 余弦法则  $c = a \cos B + b \cos A$  等.

3. 由二边及夹角求他边

$$c^2 = a^2 + b^2 - 2ab \cos C \text{ 等.}$$

4.  $\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}$ ,  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

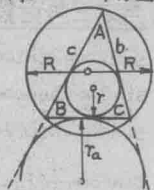
$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}, \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

5. 面积  $= \frac{bc}{2} \sin A = \frac{abc}{4R} = \sqrt{s(s-a)(s-b)(s-c)}$

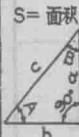
6. 外切圆半径  $R = \frac{abc}{4 \times \text{面积}} = \frac{a}{2 \sin A}$

7. 内切圆半径  $r = \frac{\text{面积}}{s} = (s-a) \tan \frac{A}{2}$

8. 傍切圆半径  $r_a = \frac{\text{面积}}{s-a} = s \tan \frac{A}{2}$

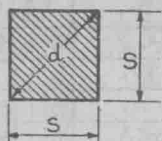


## VI. 三角形公式

| 图形                                                                                 | 已知                                                                                                          | 求     | 公式                                                                                                               | 简号                    |
|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|-------|------------------------------------------------------------------------------------------------------------------|-----------------------|
|    | a, c,                                                                                                       | A, B, | $\sin A = \frac{a}{c}, \quad \cos B = \frac{b}{c},$                                                              | 直角三角形表解               |
|                                                                                    |                                                                                                             | b, s, | $b = \sqrt{c^2 - a^2}, \quad s = \frac{a}{2} \sqrt{c^2 - a^2},$                                                  | 横高 倾斜角 斜边长            |
|                                                                                    | a, b,                                                                                                       | A, B, | $\tan A = \frac{a}{b}, \quad \tan B = \frac{b}{a},$                                                              | b a A c               |
|                                                                                    |                                                                                                             | c, s, | $c = \sqrt{a^2 + b^2}, \quad s = \frac{ab}{2},$                                                                  | 1/2   85°14' 1.0035   |
|                                                                                    | A, a,                                                                                                       | B, b  | $B = 90^\circ - A, \quad b = a \cdot \cot A,$                                                                    | 1/1   84°50' 1.0041   |
|                                                                                    |                                                                                                             | c, s, | $c = \frac{a}{\sin A}, \quad s = \frac{a^2 \cot A}{2},$                                                          | 1/0   84°17' 1.0050   |
|                                                                                    | A, b,                                                                                                       | B, a, | $B = 90^\circ - A, \quad a = b \cdot \tan A = c \cdot \cos B,$                                                   | 1/9   83°40' 1.0061   |
|                                                                                    |                                                                                                             | c, s, | $c = \frac{b}{\cos A}, \quad s = \frac{b^2 \tan A}{2},$                                                          | 1/8   82°53' 1.0060   |
|                                                                                    | A, c,                                                                                                       | B, a, | $B = 90^\circ - A, \quad a = c \cdot \sin A,$                                                                    | 1/7   81°52' 1.0102   |
|                                                                                    |                                                                                                             | b, s  | $b = c \cdot \cos A, \quad s = \frac{c^2 \sin A \cos A}{2} = \frac{c^2 \sin 2A}{4},$                             | 1/6   80°31' 1.0138   |
|  | a, b, c,                                                                                                    | A     | $\sin \frac{1}{2} A = \sqrt{\frac{p(p-a)}{bc}}, \quad \cos \frac{1}{2} A = \sqrt{\frac{p(p-a)}{bc}},$            | 1/2   63°26' 1.1180   |
|                                                                                    |                                                                                                             |       | $\tan \frac{1}{2} A = \sqrt{\frac{p(p-a)}{p(p-a)}}, \quad \sin \frac{1}{2} B = \sqrt{\frac{p(p-b)}{ac}},$        | 3/4   53°08' 1.2500   |
|                                                                                    |                                                                                                             | B     | $\cos \frac{1}{2} B = \sqrt{\frac{p(p-b)}{ac}}, \quad \tan \frac{1}{2} B = \sqrt{\frac{p(p-b)}{p(p-b)}},$        | 1   45°00' 1.4142     |
|                                                                                    |                                                                                                             | C     | $\sin \frac{1}{2} C = \sqrt{\frac{p(p-c)}{ab}}, \quad \cos \frac{1}{2} C = \sqrt{\frac{p(p-c)}{ab}},$            | 1/4   38°40' 1.6000   |
|                                                                                    |                                                                                                             | s     | $\tan \frac{1}{2} C = \sqrt{\frac{p(p-c)}{p(p-c)}}, \quad s = \sqrt{p(p-a)(p-b)(p-c)},$                          | 1/2   33°42' 1.8000   |
|                                                                                    | a, A, B,                                                                                                    | b, c, | $b = \frac{a \cdot \sin B}{\sin A}, \quad c = \frac{a \cdot \sin C}{\sin A} = \frac{a \cdot \sin(A+B)}{\sin A},$ | 1/3   29°44' 2.0160   |
|                                                                                    |                                                                                                             | s,    | $s = \frac{1}{2} ab \cdot \sin C = \frac{a^2 \sin B \sin C}{2 \sin A},$                                          | 2   26°34' 2.2360     |
|                                                                                    | a, b, A,                                                                                                    | B     | $\sin B = \frac{b \cdot \sin A}{a},$                                                                             | 2 1/2   21°50' 2.6880 |
|                                                                                    |                                                                                                             | c     | $c = \frac{a \cdot \sin C}{\sin A} = \frac{b \cdot \sin C}{\sin B} = \sqrt{a^2 + b^2 - 2ab \cos C},$             | 3   18°26' 3.1620     |
|                                                                                    | a, b, C,                                                                                                    | s     | $s = \frac{1}{2} ab \cdot \sin C,$                                                                               | 3 1/2   16°00' 3.6880 |
|                                                                                    |                                                                                                             | A     | $\tan A = \frac{a \cdot \sin C}{b - a \cdot \cos C},$                                                            | 4   14°02' 4.1240     |
|                                                                                    |                                                                                                             |       | $\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \cot \frac{1}{2} C,$                                                    | 4 1/2   12°32' 4.6081 |
|                                                                                    |                                                                                                             | c     | $c = \sqrt{a^2 + b^2 - 2ab \cos C} = \frac{a \cdot \sin C}{\sin A},$                                             | 5   11°18' 5.1033     |
|                                                                                    |                                                                                                             | s     | $s = \frac{1}{2} ab \cdot \sin C = \frac{1}{2} bc \cdot \sin A,$                                                 | 5 1/2   10°18' 5.6000 |
|                                                                                    | A+B+C=180°, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$                                       |       |                                                                                                                  | 6   9°25' 6.1121      |
|                                                                                    | $\sin A + \sin B + \sin C = 4 \cdot \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2},$        |       |                                                                                                                  | 6 1/2   8°45' 6.5733  |
|                                                                                    | $\cos A + \cos B + \cos C = 1 + 4 \cdot \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2},$    |       |                                                                                                                  | 7   8°08' 7.0680      |
|                                                                                    | $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C,$                                              |       |                                                                                                                  | 7 1/2   7°36' 7.5601  |
|                                                                                    | $a^2 = b^2 + c^2 - 2bc \cos A, \quad b^2 = a^2 + c^2 - 2ac \cos B,$                                         |       |                                                                                                                  | 8   7°07' 8.0717      |
|                                                                                    | $c^2 = a^2 + b^2 - 2ab \cos C, \quad \frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$ |       |                                                                                                                  |                       |

## 1—3 平面圖形之求積

正方形



A=面積

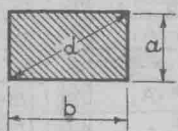
$$A = s^2$$

$$A = \frac{1}{2} d^2$$

$$s = 0.7071 d = \sqrt{A}$$

$$d = 1.414 s = 1.414 \sqrt{A}$$

矩形



A=面積

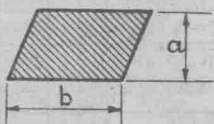
$$A = a \cdot b \quad A = a \sqrt{d^2 - a^2} = b \sqrt{d^2 - b^2}$$

$$d = \sqrt{a^2 + b^2}$$

$$a = \sqrt{d^2 - b^2} = A \div b$$

$$b = \sqrt{d^2 - a^2} = A \div a$$

平行四邊形



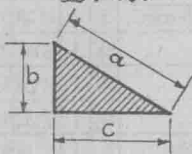
A=面積

$$A = a \cdot b$$

$$a = A \div b$$

$$b = A \div a$$

直角三角形



A=面積

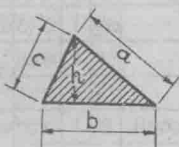
$$A = \frac{bc}{2}$$

$$a = \sqrt{b^2 + c^2}$$

$$b = \sqrt{a^2 - c^2}$$

$$c = \sqrt{a^2 - b^2}$$

銳角三角形



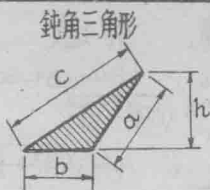
A=面積

$$A = \frac{bh}{2} = \frac{b}{2} \sqrt{a^2 - \left( \frac{a^2 - b^2 + c^2}{2b} \right)^2}$$

如  $s = \frac{1}{2}(a+b+c)$  則

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

## 1—3 平面圖形之求積

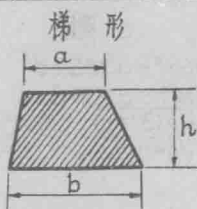


A=面積

$$A = \frac{bh}{2} = \frac{b}{2} \sqrt{a^2 - \left( \frac{c^2 - a^2 + b^2}{2b} \right)^2}$$

若  $s = \frac{1}{2}(a+b+c)$  則

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$



A=面積

$$A = \frac{(a+b)h}{2}$$



A=面積

$$A = \frac{(H+h)a + bh + cH}{2}$$



A=面積

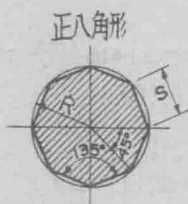
R=外切圓半徑

r=內切圓半徑

$$A = 2.598s^2 = 2.598R^2 = 3.464r^2$$

$$R = s = 1.155r$$

$$r = 0.866s = 0.866R$$



A=面積

R=外切圓半徑

r=內切圓半徑

$$A = 4.828s^2 = 2.828R^2 = 3.314r^2$$

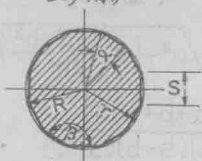
$$R = 1.307s = 1.082r$$

$$r = 1.207s = 0.924R$$

$$s = 0.765R = 0.828r$$

## 1—3 平面圖形之求積

正多角形



A=面積, n=邊數.

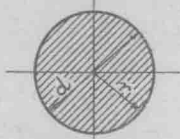
$$\alpha = 360^\circ \div n \quad \beta = 180^\circ - \alpha$$

$$A = \frac{n s r}{2} = \frac{n s}{2} \sqrt{R^2 - \frac{s^2}{4}}$$

$$R = \sqrt{r^2 + \frac{s^2}{4}}, \quad r = \sqrt{R^2 - \frac{s^2}{4}}$$

$$s = 2\sqrt{R^2 - r^2}$$

圓



A=面積, C=圓周.

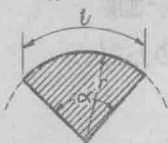
$$A = \pi r^2 = 3.1416 r^2 = 0.7854 d^2$$

$$C = 2\pi r = 6.2832 r = 3.1416 d$$

$$r = C \div 6.2832 = \sqrt{A \div 3.1416} = 0.564 \sqrt{A}$$

$$d = C \div 3.1416 = \sqrt{A \div 0.7854} = 1.128 \sqrt{A}$$

分圓



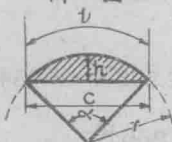
A=面積, l=弧長, alpha=角度.

$$l = \frac{r \times \alpha \times 3.1416}{180} = 0.01745 r \alpha = \frac{2A}{r}$$

$$A = \frac{1}{2} r l = 0.008727 \alpha r^2$$

$$\alpha = \frac{57.296}{r}, \quad r = \frac{2A}{l} = \frac{57.296 l}{\alpha}$$

割圓



A=面積, l=弧長, alpha=角度.

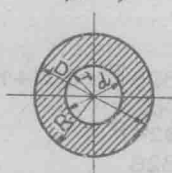
$$c = 2\sqrt{h(2r-h)}$$

$$A = \frac{1}{2} [r l - c(r-h)]$$

$$r = \frac{c^2 + 4h^2}{8h}, \quad l = 0.01745 \alpha$$

$$h = r - \frac{1}{2} \sqrt{4r^2 - c^2}, \quad \alpha = \frac{57.296 l}{r}$$

環形



A=面積

$$A = \pi(R^2 - r^2) = 3.1416(R^2 - r^2)$$

$$= 3.1416(R+r)(R-r)$$

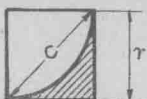
$$= 0.7854(D^2 - d^2)$$

$$= 0.7854(D+d)(D-d)$$



## 1—3 平面圖形之求積

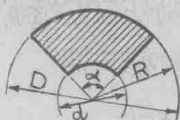
角 緣



A=面積

$$A = r^2 - \frac{\pi r^2}{4} = 0.215 r^2 = 0.1075 c^2$$

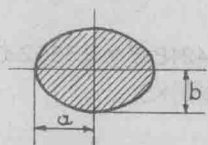
扇 形

A=面積,  $\alpha$ =角度.

$$A = \frac{\alpha \pi}{360} (R^2 - r^2) = 0.00873 \alpha (R^2 - r^2)$$

$$= \frac{\alpha \pi}{4 \times 360} (D^2 - d^2) = 0.00218 \alpha (D^2 - d^2)$$

橢 圓



A=面積, P=橢圓周圍.

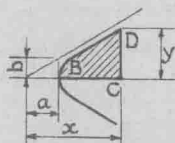
$$A = \pi a b = 3.1416 a b$$

求P之近似公式:

$$1. P = 3.1416 \sqrt{2(a^2 + b^2)}$$

$$2. P = 3.1416 \sqrt{2(a^2 + b^2 - \frac{(a-b)^2}{22})}$$

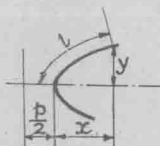
雙 曲 線



A=面積, BCD

$$A = \frac{xy}{2} - \frac{ab}{2} \log \left( \frac{x}{a} + \frac{y}{b} \right)$$

拋 物 線



$$l = \text{弧長} = \frac{p}{2} \left[ \sqrt{\frac{2x}{p}} \left( 1 + \frac{2x}{p} \right) + \text{hyp. log} \left( \sqrt{\frac{2x}{p}} + \sqrt{1 + \frac{2x}{p}} \right) \right]$$

x比y較小時之近似公式:

$$l = y \left[ 1 + \frac{2}{3} \left( \frac{x}{y} \right)^2 - \frac{2}{5} \left( \frac{x}{y} \right)^4 \right]$$

$$l = \sqrt{y^2 + \frac{4}{3} x^2}$$