BILINGUAL COURSE FOR COLLEGE PHYSICS EXPERIMENTS

大学物理实验双语教程

主编 史金辉 邢健 张晓峻 朱正
主审 孙晶华

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内容简介

本书是根据高等院校物理实验课程的基本要求，结合哈尔滨工程大学近年来针对留学生物理实验教学的经验，在历年来所用中文实验教材的基础上编写而成的。本书介绍了有关物理实验的数据处理知识，精选了力学、热学、电磁学、光学及近代物理共20个实验。

本书可作为理工科类来华留学生的物理实验教材，也可作为双语教材供中国学生选用。

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前　言

PREFACE

物理实验是我国高等理工科院校必开的一门基础课程，是培养学生实验能力和科学素质的基石。随着国际交流日益频繁，越来越多的留学生到中国各大高校交流学习。但是国内适合于留学生物理实验教学的双语教材并不多见。因此，我们根据教育部2008年颁发的“理工科类大学物理实验课程教学基本要求”，结合哈尔滨工程大学近年来对留学生物理实验教学的经验，在历年来所用中文实验教材的基础上，吸收国外经典物理实验的内容，编写了这本大学物理实验双语教材。

本书主要分为两部分内容：第一部分介绍了误差理论、不确定度的概念及测量结果的评定，数据处理的基本方法和常用物理实验仪器介绍；第二部分共精选了20个实验，包括基础实验、设计性实验和综合性实验，涉及力学、热学、光学、电磁学和近代物理的内容。本书可作为理工科类来华留学生的物理实验教材，也可作为双语教材供国内学生选用。

参加本书编写工作的有：史金辉（实验3、7、10、13、15、17），邢健（实验4、8、9、11、16），张晓峻（实验12、14、18~20），朱正（绪论、实验1、2、5、6）。全书由孙晶华老师主审，修改定稿。

本书的编写离不开物理实验中心教师的大力支持，凝聚着全体任课教师的共同努力和辛勤劳动。在编写过程中，参考了一些兄弟院校的教材及国内的文献资料，在此谨致深切的谢意。

由于编者的水平有限，书中难免有错误和不妥之处，敬请读者批评指正。

编　者

2014年5月于哈尔滨

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Introduction

Introduction to Physical Experiment

1.1 Basic concepts of measurement and error

Physical quantities are often needed to be measured and usually quantitative relations between physical quantities are needed to be found out in the process of production, life and scientific research. If related information of their big or small quantity values is to be obtained, people need have the aid of a tool or instrument, in this way there is a problem of confidence level about measured values that are obtained. So concepts of the measurement and error are introduced.

1. Basic concept of measurement

So-called measurement is to compare the physical quantity to be measured with a uniformity quantity as a standard to obtain multiple relation between them, and the multiplication result is the measured value to be measured. The uniformity quantity as the standard is called as "Unit". For example, use a standard meter scale to measure the track length of sports ground, and the length from starting point to finishing point is just 100 standard meter scale length, so the length of track is equal to $1\ m \times 100 = 100\ m$, here 100 is as the multiple of unit 1 m. As required, for a measured result usually there are a few expression methods: Values having confirmed unit, curves on confirmed coordinate, figure drawn out as a certain proportion and collected waveforms or images etc.

2. Classification of measurement

Measurement is usually divided into two classes by methods: So-called direct measurement means the measurement process by using measurement tools or measurement instruments to directly measure or read out measured values. As shown in Fig. 1. 1 – 1, use a meter scale to measure the height of a child directly, use a thermometer to measure temperature, use a stop-watch to measure time, and use a multi-meter to measure electric current, voltage and resistance directly etc. Direct measurement also includes single direct measurement and multi direct measurement. And indirect measurement means the process that measurement of some physical quantities can not be completed only by direct measurement, and the required measured values can be obtained by calculation using formulas after several physical quantities must be directly measured. As shown in Fig. 1. 1 – 2, for the measurement of density ρ of a cylinder, first we can directly measure height h and diameter d of the cylinder and weigh its mass M, and then substitute them into formula to calculate density ρ of a cylinder. In physical measurement, most of them belongs to indirect measurement, but indirect measurement is basic to all physical measurements.

Fig. 1. 1 – 1 Direct measurement diagram

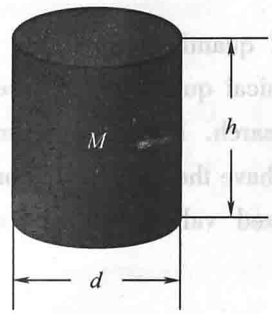

Fig. 1. 1 – 2 Indirect measurement diagram

Based on different measurement condition, measurement can also be divided into equal observations and unequal observations. Equal observations means multi repeat measurement containing five basic elements such as measurement instrument, measuring personnel, measurement method, measurement environment and the measured participating measurement that have no change. Equal observations is also called as repeatability measurement.

During multi repeat measurement, it is not possible for five elements for measurement to keep unchanged absolutely, so equal observations is an ideal concept. When equal observations is made to a fixed measured object, it is allowable for the obtained measured data to have a big or small change in a range, we can not judge which value is more nearly to the measured true value, so those values are only equally dealt with. The confidence level for the measured data is same.

Unequal observations means the measurement made that five elements participating measurement except the measured object, all or any of the other four elements changes. Unequal observations is also called as reproducibility measurement.

During measuring, because of measurement conditions changed, for example, different observers measure the measured object for different times under the different conditions using different instruments and different methods, factors affecting and determining measured results are different, and confidence level for the measured data is different. Unequal observations is often used for high accuracy measurement.

3. Basic concept of error

It is not possible to realize absolute accuracy by any measurement instrument, measurement method, measurement environment and observation of measuring operators so that it is not possible to avoid error occurrence with measurement. When analyzing various errors produced possibly in measurement, their affection shall be eliminated as far as possible, and the errors in measured results that can not be eliminated are estimated, which should be the problems covered in physical and scientific experiments, so error theory and data process methods appear.

The quantity value measured is quantitative sign of a characteristic of objective things, but they can not reflect objective existence completely accurately, only limitlessly trend to reflect a quantity value characteristic of objective things. True value, abbreviated to true value, referred to as x_0, means the quantity value of a measured object, existed objectively under a certain condition. Because measurement error exists generally, so it is not possible to get the truth value of measured object by measurement. Only optimum estimated value or called as measured value of truth value, referred to as x can be obtained by measurement.

The difference of measured value and true value is called as measurement error, expressed as

$$\Delta x = x - x_0 \tag{1.-1}$$

In which, Δx is called as absolute error of measurement.

Because absolute error only expresses measured error big or small, it is impossible to express accurate precision level of measurement that has been done by use of absolute error when the measured object with different order of magnitude is measured. For example, different physical quantities of two order of magnitudes are separately measured, if the obtained measured absolute errors are same, but their measured precisions are not same, the one with high order of magnitude, measurement precision is high, the one with low order of magnitude, measurement precision is low. Therefore, comparison is done to measured precision or the precision of measurement instrument, relative error must be used as qualification of mutual comparison.

In order to evaluate more accurately measured results good or not, people have introduced concept of relative error, which is defined as the ratio of the absolute error to the true value, and expressed by percent, that is

$$E_r = \frac{\Delta x}{x_0} \times 100\% \quad (1.1-2)$$

It is clear that relative error is a dimensionless value. The less the relative error of measured result is, which shows the measured result is closer to true value.

4. Source of error

During measuring, generally source of error can be summarized as the following reasons:

(1) Method error: It is a measurement error introduced because of the used measurement principle or measurement method itself. Main reason of this error source: Research of related knowledge about measured object is not enough, effects of different factors can not be considered completely, it is limited by objective conditions and technical level, the used measurement principle itself is approximate, or some factors that have real function in measurement are ignored, original state of measured object is damaged using contact measurement method, and static measurement method is used to measure a dynamic object etc.

(2) Instrument error: Means the error produced by effects of inherent various factors on the used measurement equipment and instrument themselves during measurement, for example, accuracy, sensitivity, minimum division value and stability good or not etc. This depends on factors such as structure of measurement device, design, performance of elements and device, performance of parts materials, manufacturing and technical level of fitting etc. And when designing and manufacturing various measurement devices, only a certain condition and real requirements are met. There is always a gap from ideal requirements, so a certain error always exists in measured value during measuring.

(3) Environmental error: Because surroundings affect measurement, error is produced in measurement. These effect factors always exist outside of measurement system, but have function on measurement system directly or indirectly, for example, temperature, humidity, atmospheric pressure, electrical field, magnetic field, mechanic vibration, acceleration, gravitation, sounding, illumination, dust, various rays or electromagnetic wave etc. In different measurements, the factors may have different degree of effects on measurement. They can affect the measurement error produced by measurement system, and sometimes can also cause change of the measured object, if it is serious, it can cause measurement equipment damaged or make measurement difficult continue. In order to distinguish environmental error and instrument error,

generally the so-called standard environment condition (reference condition) is determined artificially, or operation conditions of measurement instrument are specified on product placard and operation instruction. The measurement error produced in measurement made under reference condition is considered as the inherent error of measurement instrument (instrument error). If measurement instrument works in the environment specified without reference condition, the effects of environment factor make measurement error increase. This measurement error increment is called as additional error of instrument, which is environment error. Therefore when an instrument carries out measurement in meeting condition specified, the obtained measured value error, big or small, must not exceed the error value given on placard or in instruction. Some instruments also give the changed environment error value with environment condition varied.

(4) Subjective error: Subjective error is also called as personnel error. It is the error caused by quality condition of operators who carry out measurement, there is a class of error that is difficult to avoid, for example, observation error caused by resolution capability of sense organ, reaction retard, habit sense and operation level factor of measuring personnel. The other one is subjective error that must be avoided as far as possible: for example, measured error caused by reading, record, and calculation mistake caused by careless measuring operator, or operation fault.

The above-mentioned source of four measurement errors is summarized by four links participating measurement, that is personnel, equipment, method and condition. In specific measurement, the level of various factors affecting measurement is different, and even measurement error caused by a factor is small so that it is ignored.

5. Classification of error

In order to know measured error further systematically and summarily, classification method for measured error generally used is introduced now, where classification is made in accordance to characteristic and features of measured error. Measured error can be divided into three big classes: Accidental error, system error and gross error.

(1) Accidental error: During measuring, measured error of accidental nature is resulted from some affection of accidental factors, which is called as accidental error. Accidental error is also called as fortuitous error. It is difficult to calculate this error size and direction (positive and negative error). Even if under the same condition as far as possible to measure a specified physical quantity repeatedly, and after all obviously regular deviations are eliminated and corrected as far as possible, the measured value obtained every time is always fluctuation changed accidentally within a certain range.

Both statistical theory and experiment prove that in the most of physical measurements, when repeated measurement times is enough, accidental error obeys normal distribution law. If Δx is used to express accidental error of measured value for a physical quantity, $p(\Delta x)$ is probability the density function of accidental error. Its mathematical expression is

$$p(\Delta x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(\Delta x)^2/2\sigma^2} \tag{1.1-3}$$

Where, $\sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-x_0)^2}$ is standard error of measurement.

For an individual, accidental error is the error produced by any measurement during repeated measurement, which is irregular, can not be controlled, and is difficult to eliminate by experiment method. For total, that is the measured values obtained by many times of measurement, and accidental error obeys a certain statistical law (Normal distribution or Gaussian distribution). Therefore for accidental error, probability statistics method can be used to deal with, i. e. characteristic quantity—standard error σ is used to express. The more the measurement times n, the less the standard error σ, that is, the less the accidental error is. It is obvious that accidental error is related with measurement times. Measurement times is increased to reduce accidental error effect on measurement result.

In formula (1.1-3), measured standard error σ is a basic index to evaluate accidental error, and its values are decided by factors such as measurement tool, instrument, measurement environment, measurement personnel and measured object etc. For the same object measured, after measurement system (including the above factors) is determined, the values of standard error σ can be determined, too. When the measurement system is not same (if other standard device and other instrument is used, or measurement environment is changed, or other measurement method is used etc.), the value σ taken is different. Therefore after measurement system is determined, its standard error is a determined constant followed with it. At the moment, in the formula (1.1-3), there is only an accidental error Δx, one variation, the curve is only determined. Obviously, when σ is different, the form of corresponding curve is different, either. In Fig. 1.1-3, distribution curves of normal distribution probability density function $p(\Delta x)$ for $\sigma=0.5$, $\sigma=1$ 与 $\sigma=2$ are given separately in Fig. 1.1-3. It can be seen from the diagram, the less the value σ is, the steeper

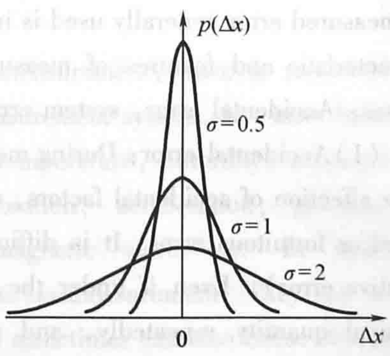

Fig. 1.1-3 Normal distribution curve of accidental error

the normal distribution curve is, and error distribution trends to concentrate.

In accordance with probability statistics theory, formula (1.1-3) is integrated to obtain

$$P = \int_{-\infty}^{+\infty} p(\Delta x)\,\mathrm{d}(\Delta x) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\Delta x)^2}{2\sigma^2}}\,\mathrm{d}(\Delta x) = 1 \qquad (1.1-4)$$

It shows that probability of accidental error of measurement set in $(-\infty, +\infty)$ interval is 1, i. e. the area under probability density distribution curve is 1. So probability density distribution function is used to calculate the probability of accidental error measured in a time set in $(-\sigma, +\sigma)$ interval, which is

$$P_\sigma = \int_{-\sigma}^{+\sigma} p(\Delta x)\,\mathrm{d}(\Delta x) = 0.683 \qquad (1.1-5)$$

In the same principle, probability of accidental error for measurement in a time set $(-2\sigma, +2\sigma)$ and $(-3\sigma, +3\sigma)$ interval are separately

$$P_{2\sigma} = \int_{-2\sigma}^{+2\sigma} p(\Delta x)\,\mathrm{d}(\Delta x) = 0.954 \qquad (1.1-6)$$

$$P_{3\sigma} = \int_{-3\sigma}^{+3\sigma} p(\Delta x)\,\mathrm{d}(\Delta x) = 0.997 \qquad (1.1-7)$$

When percentage is used to express, they are 68.3%, 95.4% and 99.7% separately, and their corresponding curves' areas are shown in Fig.1.1-4.

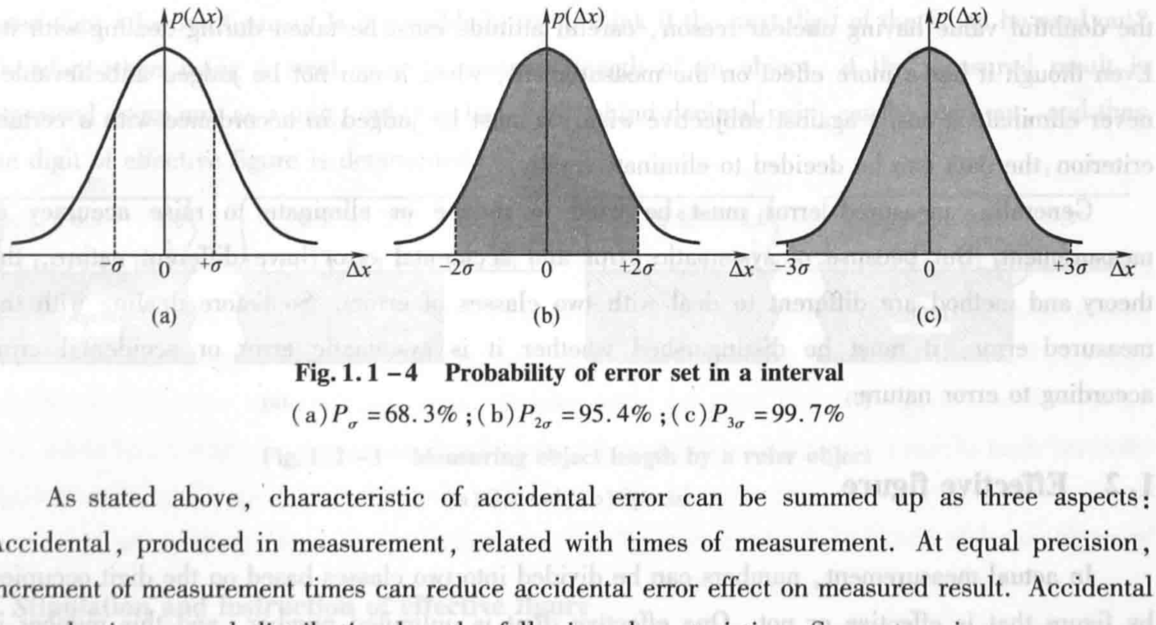

Fig. 1.1-4 Probability of error set in a interval

(a) $P_\sigma = 68.3\%$; (b) $P_{2\sigma} = 95.4\%$; (c) $P_{3\sigma} = 99.7\%$

As stated above, characteristic of accidental error can be summed up as three aspects: Accidental, produced in measurement, related with times of measurement. At equal precision, increment of measurement times can reduce accidental error effect on measured result. Accidental error obeying normal distribution has the following characteristics: Symmetry, i. e. appearance probability of positive error and negative error with equal absolute value is equal. Unimodality, i. e. probability of error appearance for small absolute value is high, and probability of error appearance for big absolute value is very low. Boundedness, probability of very big positive or negative error appearance is almost zero.

(2) Systematic error: When its size and direction have a certain rule, the measurement error is called as systematic error, also is determined error. In accordance with known change law, it can be further divided into: Fixed and unchanged systematic error is called as constant systematic error, and systematic error changed in accordance with known law is called as variable-value systematic error. Therefore systematic error can be tried to eliminate.

The determined error above is more typical systematic error according to its knowability, which is generally called as determined systematic error. For the opposite to this, if systematic error has a certain accidental nature, systematic error is called as undetermined systematic error.

(3) Gross error: Error which obviously distort measured value is called as gross error. The error is caused by wrong operation, wrong reading and wrong record etc., i. e. due to carelessness or fault, so it is also called as carelessness and fault error.

Gross error, seen from absolute value, is much greater than general systematic error value or accidental error at similar condition. Therefore the measured value with gross error has a large difference from normally measured value, so it is called as abnormal value or doubtful value.

The method to deal with gross error is to eliminate it from the measured data directly. But for the doubtful value having unclear reason, careful attitude must be taken during dealing with it. Even though it has a more effect on the measurement, when it can not be judged unbelievable, never eliminate it easily against subjective wish. It must be judged in accordance with a certain criterion, the data can be decided to eliminate finally.

Generally, measured error must be tried to reduce or eliminate to raise accuracy of measurement. But because of systematic error and accidental error have different nature, the theory and method are different to deal with two classes of errors. So before dealing with the measured error, it must be distinguished whether it is systematic error or accidental error according to error nature.

1.2 Effective figure

In actual measurement, numbers can be divided into two classes based on the digit occupied by figure that is effective or not. One effective digit is unlimited number, and this number is mostly the result of pure mathematical calculation, for example, digits $\sqrt{2}$, π, 1/3 etc., for which digits taken, as required, are all effective. The other effective digit is limited figure, and this number is mostly connected with actuality. It is not possible to determine freely its effective digit only by mathematical calculation, and the quantity value to be expressed or the accuracy possessed are expressed properly based on actuality. The effective digit for this number is limited by factors such as the precision that original data can reach, the technical level to obtain data, and the theory on which to obtain data etc. During processing the measured data, it is very important to grasp related knowledge of effective figure.

1. Basic concept of effective figure

A numerical value is composed of figures, other figures are reliable values or explicit values except that the last digit is not an explicit value or a doubtable value, and all figures forming the number including last digit is called as effective figure, i. e. accurate value plus a digit of pure doubtable number forms digit of effective figure.

As shown in Fig. 1.2 – 1(a), a steel ruler whose length is 15 cm and minimum division value is 1 mm is used to measure length of an object, and the read length is 7.26 cm. In this read data, 7.2 is read directly from the scales on the steel ruler, and the last digit 6 is estimated by a measurer, which is doubtful, because different measurers' estimated read result may be different, and it is called as pure doubtful figure. So it is can be said that a measured result has three-digit effective figures. Is it possible to try to think if the next digit of the 6 can be read out? Therefore when using a steel ruler to measure length of an object, if the measured result is expressed using mm as a unit, only the last digit behind decimal point can be read out, and then the digit of effective figure is determined.

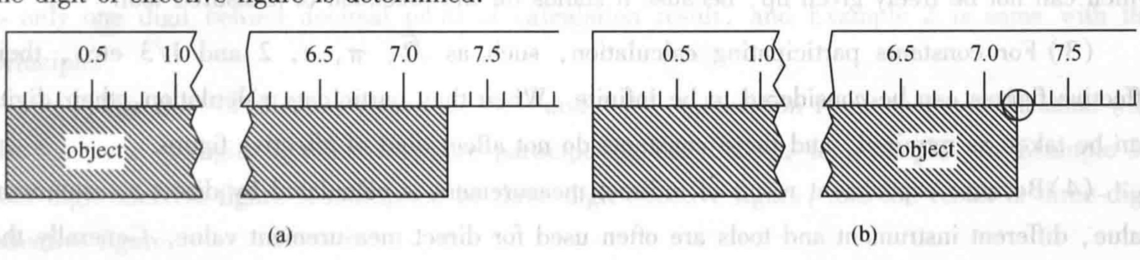

Fig. 1.2 – 1 Measuring object length by a ruler object

(a) General; (b) Special

2. Stipulation and instruction of effective figure

(1) Digit of effective figure is not related with the position of decimal point, and unit change does not affect digit of effective figure. For example, length of an object $L = 12.01$ cm =

0.120,1 m = 0.000,120,1 km, they are all four-digit effective figure. It can be seen from the example that decimal system unit change can not affect the digit of effective figure, and at the moment, generally scientific counting process can be used, for example, the length of object above can be expressed as $L = 12.01 \times 10^{-4}$ km, and it is still four-digit effective figure because power exponent of "10" is not counted in digit of effective figure.

(2) Figure "0", has special characteristic in effect figure. First look at the three expression methods, which seems different, of the above length L of an object. Where "0" appears in different positions of data. The "0" in 12.01 is obviously counted in the digit effective figure, and the "0" before "1" in the figures 0.120,1 and 0.000,120,1 is obviously not counted in the digit of effective figure, i. e. when calculating digit of effective figure, the first "0" before nonzero figure is not counted in digit of effective figure. Let's look at the measurement as shown in Fig. 1.2 – 1(b), when measuring, one end of an object just aligns with a scale line on the ruler. At the moment if the measured result can be recorded as 7.2 cm or not, the answer is not allowable. It must be counted as 7.20 cm. As viewed from mathematics, 7.2 cm and 7.20 cm are the same two values. It seems that it is not necessary to keep the "0" in the latter. But as viewed from measurement error and effective figure, both of them is not the same completely, the "2" expressed as 7.2 is already pure doubtable number, i. e. the measurement precision of measurement tool used is 0.1 cm, and the "0" expressed as 7.20 cm is a pure doubtable number, i. e. the measurement precision of measurement tool used is 0.01 cm. Therefore during recording experiment data, it is careful that when the last digit behind decimal point is "0", which can not be freely given up, because it stands for the precision of measured tool.

(3) For constants participating calculation, such as $\sqrt{2}$, π, e, 2 and 1/3 etc., their effective figures can be considered to be infinite. When they participate calculation, their digits can be taken as required, and these constants do not affect digit of effective figure.

(4) Because experiment result of indirect measurement is calculated by direct measurement value, different instrument and tools are often used for direct measurement value. Generally the obtained digit of effective figure is different for each direct measurement value, and there is a problem of acceptance and rejection of effective figure during calculation. The followings are examples to show the method how to accept and reject digit of effective figure in the process of "+" "−" "×" and "÷" calculations.

3. Basic calculation rules of effective figure

"+" and "−" calculation process is as follows: The figure with point over its head is a pure doubtable number, the principle is no matter what number, only pure doubtable number participates calculation, and the result of calculation must be pure doubtable number, but the final result only keeps a pure doubtable number. Especially pay attention to: additive calculation may make digit of effective figure increase (carry), and subtractive calculation may make digit of effective figure decrease (borrow).

$$\begin{array}{r} 146.\dot{2} \\ +\ 2.36\dot{7} \\ \hline 148.5\dot{6}\dot{7} \end{array} \Longrightarrow 148.5\dot{6}\dot{7} \Longrightarrow \underset{\text{doubtable}}{148.\dot{6}} \qquad \begin{array}{r} 30.6\dot{5} \\ -\ 4.93\dot{6} \\ \hline 25.7\dot{1}\dot{4} \end{array} \Longrightarrow 25.7\dot{1}\dot{4} \Longrightarrow \underset{\text{doubtable}}{25.7\dot{1}}$$

" × " and " ÷ " calculation process is as follows: The calculation rules is the same with the above. For division calculation, students are requested to calculate according to the principle.

$$\begin{array}{r} 6.35\dot{6} \\ \times\ 30.\dot{5} \\ \hline \dot{3}\dot{1}\dot{7}\dot{8}\dot{0} \\ 000\dot{0} \\ 1906\dot{8} \\ \hline 19\dot{3}\dot{8}\dot{5}\dot{8}\dot{0} \end{array} \Longrightarrow 19\dot{3}\dot{8}\dot{5}\dot{8}\dot{0} \Longrightarrow \underset{\text{doubtable}}{19\dot{4}}$$

For calculation of effective figure, the following laws can be obtained:

(1) The digit of pure doubtable number of " + " and " − " calculation result is same with the highest one of the digits of pure doubtable number of effective figure participating calculation, for example, in Example 1, "2" digit behind 146.2 decimal point is the highest, and therefore there is only one digit behind decimal point of calculation result, and Example 2 is same with the principle.

(2) The digit of effective figure of " × " and " ÷ " calculation result is generally same with the least one of digits of effective figure participating calculation, for example, in Example 3, four-digit effective figure is multiplied by three-digit effective figure, and the result is three-digit effective figure.

(3) The digit of effective figure of involution and evolution calculation result is same with the digit of effective figure of its bottom.

(4) For exponent, logarithm and trigonometric function etc., the digit of effective figure of calculation result can be determined by its variation. For example, an included angle measured by experiment is 19°35′, and the last pure doubtable number is 5′, converted into degree and expressed as 19.58°, so the result calculated by computer is sin19.58° = 0.335,122,7⋯. How to judge which one is pure doubtable number? Generally we use this method, i.e. a number close

to it is taken to calculate, for example, calculating sin 19.59° = 0.335,287,1…, it is found that by comparison of its calculated result, the fourth digit behind decimal point is different, so sin19.58° = 0.335,1 can be taken, in which "1" is the last digit of pure doubtable number.

As stated above, as expression of effective figure for experiment result, only one digit of pure doubtable number is in principle. But in the process of dealing with experiment data, it is determined according to actual condition for the final result that how many digits of effective figure is to keep. Related rules can be given in the section of estimation of uncertainty of measurement.

1.3 Estimation of measured result uncertainty

The purpose of measurement is to obtain the true value of the measurement, but it is difficult to determine the true value of the measurement because of existence of measurement error, for its measured result, only an approximately estimated value of a true value (i. e. so-called optimum estimated value) and an error range used to express degree of approximate. Then the concept of "uncertainty of measurement" is introduced, and the expression of uncertainty of measurement is used to quantify and evaluate measurement level or quality.

1. Mean arithmetical value is the optimum estimated value of true value

General error theory considers that for many times of measurement, mean arithmetical value is optimum estimated value of true value. Mean arithmetical value for many times of measurement is defined as: At system error eliminated, suppose a true value is x_0 physical quantity for n times of measurement, the measurement value obtained are separately x_1, x_2, x_3, …, x_n. The error of measured value for any time can be expressed as

$$\Delta x_i = x_i - x_0 \quad (i = 1, 2, 3, \cdots, n) \tag{1.3-1}$$

For formula (1.3-1) summation, we can obtain

$$\sum_{i=1}^{n} \Delta x_i = (x_1 + x_2 + x_3 + \cdots + x_n) - nx_0 \tag{1.3-2}$$

Both sides of formula (1.3-2) are divided by n to obtain

$$\frac{1}{n}\sum_{i=1}^{n} \Delta x_i = \frac{1}{n}\sum_{i=1}^{n} x_i - x_0 \tag{1.3-3}$$

In accordance to symmetry of statistic law for accidental error, when $n \to \infty$ there is

$$\lim_{n\to\infty}\sum_{i=1}^{n} \Delta x_i = 0 \tag{1.3-4}$$