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高精度紧致差分方法及其应用研究

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Research on High Accuracy Compact Finite Difference Methods and Their Applications

Candidate: Tian Zhenfu
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本论文经答辩委员会全体委员审查，确认符合上海大学博士学位论文质量要求.

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答辩委员会对论文的评语

本论文旨在建立和发展求解黏性不可压缩复杂流动数值模拟的高精度、高分辨率紧致差分格式，重点研究了CFD的高精度紧致型逼近和有效算法，建立了黏性不可压缩原始变量和涡量流函数形式的N-S方程、N-S/Boussinesq方程等求解的高精度格式与算法，模拟了驱动方腔流、自然对流换热、双扩散对流系统等问题，得到了一系列有广泛实用价值的结果. 论文选题的学术起点高，在理论和实用方面均有重要意义.

本文创新点主要体现在如下几方面：

一、提出了指数型紧致格式，据此建立了定常和非定常对流扩散问题的算法. 理论分析了方法的精度和稳定性. 数值实验表明，方法不仅精度和求解效率高，而且适合有大梯度流场的模拟；

二、提出了组合迎风型紧致格式. 数值实验表明，所创建的组合型优化紧致格式具有小色散和小耗散误差的特征；

三、针对一些二维不可压缩流动，建立了高精度数值求解方法. 计算结果证实了方法的有效性.

该论文思路清晰，表达流畅，推导严谨，数据翔实，结论正确，内容丰富，包含了多项创新性工作. 论文的工作显示出作者熟练掌握了该专业领域的国内外发展动态和趋势，具有坚实的计算数学和流体力学基础以及系统

深入的专业知识,并具有很强的独立从事科研工作的能力.

答辩过程中,论文作者论述清楚,能正确回答问题.

答辩委员会一致认为,这是一篇优秀博士学位论文.

答辩委员会表决结果

经无记名投票表决，答辩委员会全票(7票)通过田振夫同学的博士学位论文答辩，并建议授予工学博士学位.

答辩委员会主任：傅德薰

2006年8月30日

摘　要

正确数值模拟多尺度复杂流场是计算流体力学(CFD)中最为重要,也是最具有挑战性的课题之一. 近年研究表明,在提高数值模拟可靠性和有效性方面,高精度和高分辨率要求已成为技术发展中的一个决定性因素. 紧致差分格式因其网格基架点少、精度高、边界易处理且能达到与谱方法相近的分辨率等优点,日益受到人们的重视. 本文基于在应用中提出的问题和未来发展的需求,针对黏性不可压缩复杂流动的数值模拟和高阶紧致(high-order compact,HOC)差分方法,研究了对流扩散问题的指数型高阶紧致(HOC exponential type,EHOC)差分方法和"导数型"紧致差分逼近的高阶组合格式,并建立了求解原始变量和涡量流函数形式的黏性不可压缩 Navier-Stokes (N-S)方程、Navier-Stokes/Boussinesq (N-S/Boussinesq)方程等的高精度差分格式与算法,直接数值模拟了驱动方腔流、自然对流换热、双扩散对流系统等复杂流动问题,得到了若干具有理论和实际意义的成果.

全文共分九章,要点和主要工作如下:

第一章为绪论,概述了 CFD 数值方法,其中以黏性不可压缩流动问题的计算方法和计算技术为重点;特别介绍了高精度紧致型差分逼近方法和格式的研究进展;同时还概述了本文研究的主要工作内容.

第二章的主要目标是建立定常对流扩散模型问题的具有较高分辨率的无振荡 EHOC 方法. 从建立一维常系数对流扩散

方程的指数型紧致格式入手，利用余项修正技术、降维法和符号紧致算子技术等，分别提出了一维变系数、二维常系数/变系数对流扩散模型问题的EHOC格式；利用Fourier分析法，对一维常系数问题的离散逼近式进行定量分析，讨论和分析了离散式的行为特性，并与其他高阶紧致格式相比较，得出了EHOC格式的色散误差和耗散误差最小；数值结果表明，本文的EHOC方法不仅具有高分辨率和高精度，而且是稳健的.

在第三章中，运用符号紧致算子技巧，提出了非定常对流扩散方程具有系数矩阵严格对角占优的指数型高阶紧致ADI(EHOC ADI)格式. 以建立的定常一维常系数/变系数对流扩散方程的指数型四阶紧致格式为基础，分别提出了非定常二维常系数/变系数对流扩散方程的EHOC ADI方法；以常系数模型方程为例，从理论上分析了所建立格式的精度和稳定性，给出了系数矩阵严格对角占优的证明；数值实验结果表明，本文提出的EHOC ADI格式不仅精度和求解效率高，而且适用于对大梯度或边界层问题的求解，优于其他求解非定常对流扩散问题的HOC方法.

第四章的主要目的是引入组合紧致迎风(CCU)差分方法，为建立流动与传热问题的高精度差分算法打下理论基础. 利用解析特解方法构造紧致型差分逼近式，推导出可任意组合的三点或五点紧致型差分逼近式的基本格式；在此基础上，提出了三阶、四阶三点组合紧致迎风(CCU34)格式和四阶、五阶五点组合紧致迎风(CCU45)格式；采用Fourier分析法对组合紧致迎风格式的数值解的精度及行为进行了分析，并通过数值算例对组合紧致迎风格式的有效性作了进一步的验证.

在第五章中，运用在第二章和第三章为对流扩散模型问题建立的EHOC格式，提出了涡量流函数形式的不可压缩N-S方

程和 N-S/Boussinesq 方程的指数型高阶紧致型差分逼近和算法，给出了细致的算法描述；数值求解了驱动方腔流和方腔自然对流问题，验证了本文方法的可靠性和有效性；在此基础上对带后台阶封闭腔内自然对流换热问题进行了直接数值模拟，重点研究了不同 Rayleigh 数下腔体台阶下方宽度和腔体台阶上方高度变化对腔体内能量传递的影响.

第六章提出了求解黏性不可压缩 N-S 方程的高精度差分投影算法. 基于投影法和第四章提出的组合迎风紧致格式，在交错网格与非交错网格系统上分别建立了数值求解原始变量形式的二维非定常、黏性不可压缩 N-S 方程的高精度紧致投影算法，并相应地提出了求解压力梯度的显式四阶紧致格式和压力 Poisson 方程的四阶紧致格式；对两个带有周期边界条件的流动问题分别在交错网格和非交错网格上进行了数值试验，验证了方法的精度，考核了算法的有效性；在此基础上，对中高 Reynolds 数下的平面驱动方腔流进行数值模拟，确定腔内流动第一次分叉的临界 Reynolds 数的范围，描述和分析了 Reynolds 数等于 10 000 时方腔流动周期性演化的过程，并确定了周期解的周期.

第七章建立了求解原始变量形式的黏性不可压 N-S/Boussinesq 方程的四阶五阶组合紧致迎风(CCU45)差分投影算法，并直接数值模拟了倾斜闭腔内自然对流换热问题. 采用投影法和四阶五阶组合紧致迎风(CCU45)格式，构造了求解二维非定常不可压缩 N-S/B 方程的高阶紧致差分投影算法；对方腔自然对流换热问题进行了数值模拟，将所得计算结果与精细网格上的“标准解”比较，验证了本文建立的高精度紧致差分投影算法的有效性和可靠性；对重力场作用下的与倾斜角度有关的闭腔内自然对流换热问题进行了直接数值模拟研究，探讨了腔

体倾斜角度、Rayleigh 数、初值条件、边界条件和腔体尺寸对腔内流体流动状态的转变及热量传递的变化规律的影响;研究了高 Rayleigh 数情况下的闭腔内的流场、温度变化规律,描述了不同倾斜角度下腔体内流动出现周期性变化的过程.

第八章针对双扩散对流系统的数值模拟,基于第四章提出的三阶四阶组合紧致迎风(CCU34)格式,构造了温度方程、浓度方程和涡量方程的高精度紧致格式,建立了涡量流函数形式的 N-S/Boussinesq 方程的高精度紧致数值方法和算法,给出了细致的算法描述;直接数值模拟了高宽比为 2∶1 闭腔内的双扩散对流系统,将所得结果与文献中的结果进行对比,验证了所提出算法的正确性;探讨浮力比和腔体尺寸对腔内流场、温度场和浓度场的影响,直接数值模拟研究了腔体高宽比为 4∶1 的双扩散对流系统,确定流场发生周期振荡的温度浓度浮力比的临界值范围,描绘了部分浮力比下流场发生周期振荡的全过程,研究分析了流场产生周期振荡的机理及性质.

第九章为本文研究工作的主要贡献、结论和展望.

关键词 计算流体力学,不可压缩流动,对流扩散方程,Navier-Stokes 方程,紧致差分逼近,ADI 方法,数值模拟

Abstract

Numerical simulation of complex flow fields with multi-scale structures is one of the most important and challenging branches of computational fluid dynamics. From linear analysis and numerical experiments it has been discovered that the higher-order accurate method can give reliable and efficient computational results, as well as better resolution of the complex flow fields with multi-scale structures. Compact finite difference schemes, which feature higher-order accuracy and spectral-like resolution with smaller stencils and easier application of boundary conditions, has attracted more and more interest and attention.

Motivated by issues from application and requirements in the future, this dissertation is focused on the numerical simulations for viscous incompressible complex flows and the establishment and/or development of high-order compact (HOC) finite difference methods (FDMs). Firstly, the high-order compact exponential type (EHOC) FDMs for the steady and unsteady convection-diffusion problems and the combined high-order compact finite difference schemes have been studied and validated. Then, the HOC schemes and the higher accuracy algorithms for the incompressible Navier-Stokes (N-S) equations and the incompressible Navier-Stokes/

Boussinesq (N-S/Boussinesq) equations have been presented and applied to the direct numerical simulations of two dimensional incompressible driven cavity flow, natural convection heat transfer and double-diffusive convection system. Some significant results have been achieved.

The key points and main contents of this dissertation consist of nine chapters, which are described as follows:

In the first chapter, the background and development of the numerical methods for CFD are introduced, with the emphasis on numerical methods and computational technologies of viscous incompressible flow. Especially, the development of the high-order compact finite difference methods/schemes for CFD is investigated in detail. Furthermore, the work and innovation of this study are summarized.

Chapter 2 is primarily aimed at establishing EHOC FDMs with high resolution and non-oscillation property for the solution of the one- and two-dimensional steady convection-diffusion model problems. Starting with the establishment of an EHOC scheme for the one-dimensional convection-diffusion equation with constant convection coefficient, the EHOC schemes are proposed for 1D and 2D steady convection-diffusion equations. In order to derive higher order compact approximation for the model problems, the approach of remainder term modification, the method of reduction of dimension and the technology of symbolized compact approximation operator have been used. By means of the Fourier analysis, the dispersion and dissipation errors and the resolution are analyzed. It is conclude that the EHOC FDM gives higher scale resolution, compared to other high-order compact (HOC) FDMs. Numerical results show that, besides including the excellent performances of the other HOC methods in computational accuracy, efficiency and stability, the present EHOC method has the advantage of better scale resolution with smaller number of gird nodes.

In Chapter 3, the EHOC alternating direction implicit (ADI) methods, in which the Crank-Nicolson scheme is used for the time discretization and an exponential fourth-order compact difference formula for the 1D steady convection-diffusion problem is used for the spatial discretization, are presented for the solution of the 2D unsteady convection-diffusion problems. The resulting EHOC ADI scheme in each ADI solution step corresponds to a strictly diagonally dominant tridiagonal matrix equation which can be inverted by simple tridiagonal Gaussian decomposition and may also be solved by application of the one-dimensional tridiagonal Thomas algorithm with a considerable saving in computing time. The unconditionally stable character of the method was verified by means of the discrete Fourier (or von Neumann) analysis. The computational results show the present EHOC ADI method successfully combines accuracy, efficiency and robustness and requires significantly fewer number of grid nodes to accurately resolve solution gradients for the convection-dominated problem, and exhibits its superiority over the other HOC schemes in terms of accuracy and/or computational cost.

The main aim of Chapter 4 is to develop a combined compact upwind (CCU) FDM. The basic compact finite difference approximations on the three- and/or five-point stencils are derived by an approach, called as the method of analytic solution. Furthermore, a three- and fourth-order combined compact upwind (CCU34) scheme and a fourth- and fifth-order combined compact upwind (CCU45) scheme are developed. Fourier accuracy analysis shows that the CCU34 approximation has better resolving efficiency than the three-order compact upwind approximation and the fourth-order compact upwind approximation and the CCU45 approximation has better resolving efficiency than the fifth-order compact upwind approximation and the fourth-order compact upwind approximation. Subsequently, the computational results of an example further verify the accuracy and resolution.

Based on the proposed EHOC schemes for the convection-diffusion problems in Chapters 2 and 3, the EHOC difference approximations and the higher-order accurate algorithms are formulated for solving the N-S equations and the N-S/Boussinesq equations using the streamfunction-vorticity formulation in Chapter 5. The detailed descriptions for these algorithms are given. To check the validity of the present numerical methods, the computed solutions for the square driven cavity flow and the square thermally driven cavity flow are compared with the benchmark solutions in the literature. The numerical results show that the present algorithms are characterized by their simplicity, efficiency, high order accuracy and stability in computations. Furthermore, the direct numerical simulations of the natural convection heat transfer in an enclosure with a heated backward step are carried out. The effects of the geometrical size of enclosure at various Rayleigh numbers on the flow structure and heat transfer characteristics are investigated in detail.

In Chapter 6, by using the projection method and the combined compact upwind schemes proposed in Chapter 4, two classes of HOC FDMs are presented for the solution of the viscous incompressible N-S equations in primitive variables on staggered grid system and on non-staggered grid system, respectively. Correspondingly, two explicit fourth-order compact difference schemes for the pressure gradients and two fourth-order compact schemes for the pressure Poisson equation are also proposed. The computational results of two test problems show that the present methods have desired accuracy and robustness. Furthermore, the driven cavity flow is investigated by use of direct numerical simulation on a 129×129 grid based on the present HOC FDMs. It is found that the flow converges to a stationary state for Reynolds numbers ($Re \leqslant 8\,010$), while for $Re = 8\,020$ the flow becomes periodic in time which indicates a Hopf bifurcation. The periodic solution for $Re = 10\,000$ is given in detail.