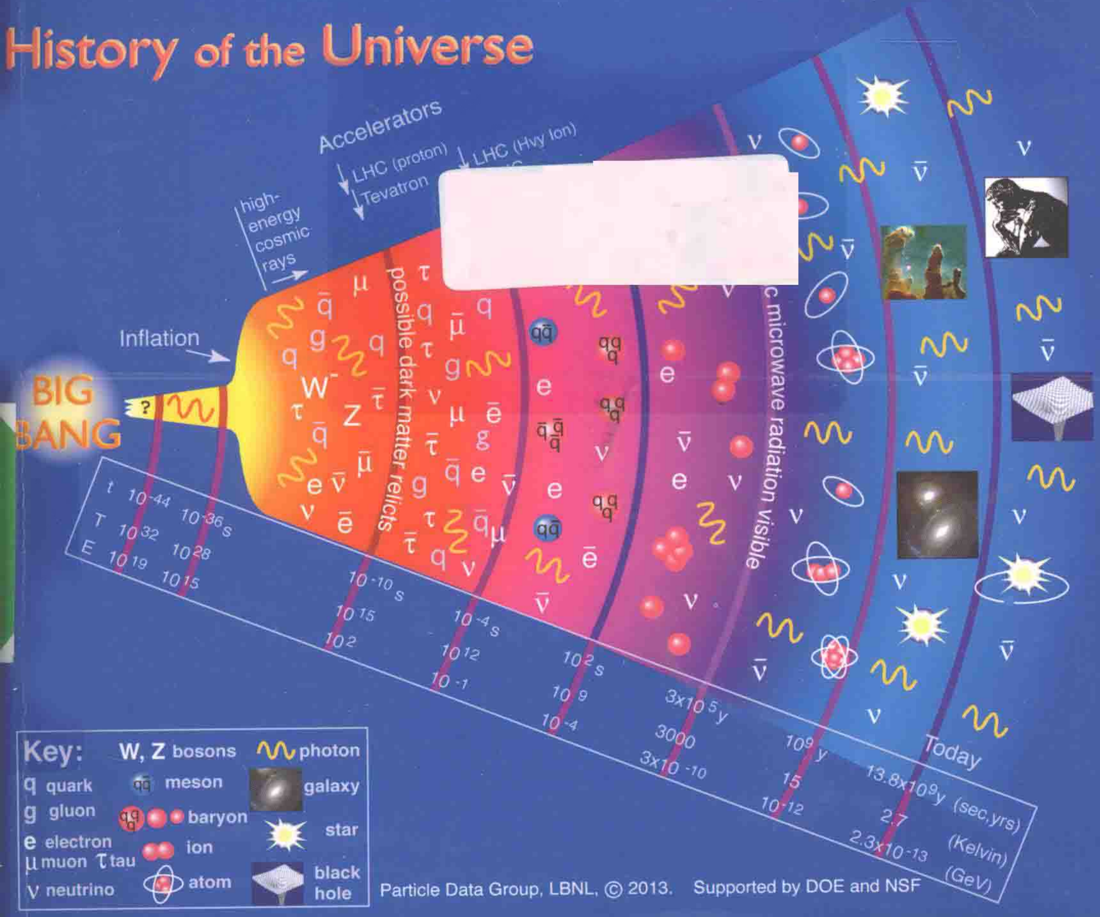
南京大学研究生精品课程

Lecture Notes in Particle Physics

The Standard Model and Beyond

标准模型及拓展

J.D. Vergados



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Lecture Notes in Particle Physics:

The Standard Model and Beyond

A' EDITION

NANJING, CHINA 2013

To the Graduate Students
Who kept the Lecture Notes

0.1 PREFACE

This book contains the material of a set of Lectures on The Standard Model and Beyond, which were delivered to the first year graduate students of the Department of Physics of Nanjing University during the Fall of 2012. It is based on my sketchy notes, which were put together and transformed into LaTex form by the following graduate students:

PhD students: Lingjun Wang, Jie Peng, Yun Zhang

M.S. students: Yong Du, Bowen Shi,

who were coordinated by Yun Zhang, who also acted as my TA for the class. I edited these notes, so I am responsible for any mistakes and omissions.

On this occasion I like to extend my appreciation to all who participated in the preparation of these lectures, to Professor Yeukkwan Edna Cheung and Dr Konstantin G. Savvidy for their hospitality and useful discussions and to Professor Zhenlin Wang, Dean of the School of Physics, for his kind invitation to visit Nanjing.

Ioannina, March 2013

Contents

Chapter 1

Introduction to Particle Physics

1.1 The natural System of Units

In basic physics we consider a system of three elementary units the length (L), the time(T) and the mass (M). From these the dimensions of the other physical quantities can be determined via definitions ($v = s/t$ etc.) and the laws of nature, (e.g. $F = ma$). This way any physical quantity dimensionally can take the form:

$$[\Pi] = L^{\lambda}T^{\mu}M^{\nu} \tag{1.1}$$

where λ, μ, ν rational numbers. So if units are defined for these, the units of all the other quantities can be obtained. Traditionally we select the meter m (m), the second (s) and the kilogram (Kg) as the units of length time and mass respectively. This system is called the International System of Units (IS). In a electromagnetism we introduced one more unit for the charge via Coulomb's law:

$$F = k_1 \frac{q_1 q_2}{r^2}, \quad k_1 = \frac{1}{4\pi\epsilon_0}, \quad \epsilon_0 = \frac{1}{\mu_0}c^2, \quad \mu_0 = 4\pi \times 10^{-7}, \tag{1.2}$$

$$\epsilon_0 = 8.85 \times 10^{-12}., \quad \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 m/F \text{ (values used in calculations)},$$

Significant digits 8.854187817 and 8.987551787 respectively (exactly)

and dimensions

$$[\kappa_1] = [\epsilon_0] = Q^{-2}L^3MT^{-2}.$$

The dimensionless quantities have the same value in all systems:

$$\alpha = \frac{e^2}{4\pi\epsilon_0}\frac{1}{\hbar c} = \frac{1}{137.035599911} \approx \frac{1}{137}, \text{ (MKSA)},$$

$$\alpha = \frac{e^2}{4\pi}\frac{1}{\hbar c} = \frac{1}{137.035599911}, \text{ (Heavyside-Lorentz)}.$$

In hindsight it might have been better had we chosen a more a unambiguous system of units like the mass m_0 of an elementary particle (e.g. the mass of the electron or the

Table 1.1: The dimensions of some physical quantities in various systems.).

quantity	symbol	M	KSA	NSI	
		λ	μ	ν	ρ
Action	$\hbar$	2	-1	1	0
velocity	v	1	-1	0	0
mass	m	0	0	1	1
length	ℓ	1	0	0	-1
time	t	0	0	1	-1
momentum	p	1	-1	1	1
Energy	E	2	-2	1	1
Fine structure constant	α	0	0	0	0
Newton's constant	G_N	3	2	-1	-6
Fermi's constant	G_F	5	-2	1	-2

proton) as the unit of mass, a given wavelength as a unit of length and the period of a simpler microscopic system as the unit of time. In particle physics we often encounter the constants ($\hbar = \frac{h}{2\pi}$, h Planck's constant and c the velocity of light. It would be nice, if we devise a system where these constants do not show up. So we use a system whereby

$$\hbar = 1, c = 1 \text{ and as unit of mass the mass } m_0 \text{ of a particle.} \tag{1.3}$$

This way:

- Length and time have the same dimension.
- The velocity is dimensionless.
- Energy has the dimension of mass.
- $E = \hbar\omega = E = \hbar/(2\pi T) \Rightarrow [T] = [E^{-1}] \Rightarrow [T] = [L] = [m_0^{-1}]$.
- For any quantity Π

$$[\Pi] = m_0^{\rho}, \quad \rho = \text{rational}.$$

For the needed conversions we will use:

$$\hbar c = 1.97 \times 10^{-13}\text{MeV - m} = 3.15 \times 10^{-15} \text{ J-m} . \tag{1.4}$$

Thus for the length we get:

$$\begin{aligned} \ell_0 &= \frac{\hbar c}{m_0 c^2} = \frac{\hbar c}{m_p c^2}\frac{m_p}{m_0} = \frac{1.97 \times 10^{-13}\text{MeV - m}}{939\text{MeV}}\frac{m_p}{m_0} \\ &\approx 2.10 \times 10^{-16}\frac{m_p}{m_0} \text{ m}, \end{aligned} \tag{1.5}$$

while for the time:

$$t_0 = \frac{\hbar}{m_0 c^2} = \frac{\hbar}{m_p c^2}\frac{m_p}{m_0} = \frac{2.10 \times 10^{-16}\ \text{m}}{3.00 \times 10^8\ \text{m/s}} = 7.00 \times 10^{-25}\frac{m_p}{m_0}\ \text{s}. \tag{1.6}$$

The left hand side is m_0^{-1}. Thus selecting as unit the proton mass $m_p = 939\ \text{GeV}/c^2$, as a basis (unit) we find:

$$m_p^{-1} = 2.10 \times 10^{-16}\ \text{m or } 1\ \text{m} = 4.76 \times 10^{15} m_p^{-1}, \tag{1.7}$$

$$m_p^{-1} = 7.00 \times 10^{-25}\ \text{s or } 1\text{s} = 1.43 \times 10^{24} m_p^{-1}. \tag{1.8}$$

Sometimes we use GeV for a unit of energy. Then

$$\text{GeV}^{-1} = 1.97 \times 10^{-16}\text{m} = 6.57 \times 10^{-25}\text{s},\ 1\text{m} = 5.06 \times 10^{15}\text{GeV}^{-1},\ 1\text{s} = 1.52 \times 10^{24}\text{GeV}^{-1}, \tag{1.9}$$

$$G_F = 1.16 \times 10^{-5}\hbar c \left(\frac{\hbar}{m_p c}\right)^2 \Rightarrow G_F = 1.16 \times 10^{-5} m_p^{-2}. \tag{1.10}$$

The converse can perhaps be found by some tricks. We will exhibit the general method which is a bit harder.
What we should do is to find a combination of $\hbar, c$ and m_p needed to express G_F in the SI system, if we know that its dimension is energy time length^3, that is

$$G_F = \text{energy} \times (\text{length})^3 = (L/T)^2 M L^3 = L^5 M T^{-2}. \tag{1.11}$$

We try

$$G_F = \hbar^p c^q m_0^r. \tag{1.12}$$

Then from table 1.1 we find:

$$[\hbar] = L^2 T^{-1} M\ ,\ [c] = L T^{-1}\ ,\ [m_0] = M. \tag{1.13}$$

Thus

$$G_F = (L^2 T^{-1} M)^p (L T^{-1})^q M^r = L^{2p+q} T^{-p-q} M^{p+r}. \tag{1.14}$$

Comparing (1.11) and (1.14) we get:

$$2p + q = 5\ ,\ -p - q = -2\ ,\ p + r = 1 \Rightarrow p = 3\ ,\ q = -1\ ,\ r = -2, \tag{1.15}$$

$$G_F = \hbar^3 c^{-1} m_0^{-2}. \tag{1.16}$$

The numerical value is obtained by comparing it with the expression of G_F given by (1.10)).
Example 1: The life time of muon μ in natural units is given by:

$$\frac{1}{\tau} = \frac{G_F^2}{192\pi^3} m_\mu^5. \tag{1.17}$$

Let us find it in seconds.

$$\frac{1}{\tau} = \frac{(G_F m_p^2)^2}{192\pi^3}\frac{m_\mu^5}{m_p^5} m_p = \frac{(1.05 \times 10^{-5})^2}{0.60 \times 10^4}\left(\frac{0.106}{0.940}\right)^5 m_p = 3.37 \times 10^{-19} m_p, \tag{1.18}$$

$$\tau = 3.10 \times 10^{18} m_p^{-1} = 3.10 \times 10^{18} \times 7.00 \times 10^{-25}\text{s} = 2.17 \times 10^{-6}\text{s}. \tag{1.19}$$

Example 2: Electromagnetism.
In the system MKSA the energy of the hydrogen atom is:

$$U = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r} = -\alpha\frac{\hbar c}{r}, \tag{1.20}$$

where α is the fine structure constant. In particle physics we choose $\epsilon_0 = 1$, so that it coincides with the Heavyside-Lorentz system. Then the unit of charge is :

$$e = \sqrt{4\pi\alpha} = 0.303. \tag{1.21}$$

So if q and q_e are some charge and and the charge of the electron respectively

$$Q = \frac{q}{q_e}\sqrt{4\pi\alpha} \text{ (natural units) }. \tag{1.22}$$

1.2 The Planck Natural system of units

Up to now the mass unit has been arbitrary. In an effort to remove this arbitrariness, we write gravitational energy as:

$$U = -G_N\frac{m^2}{r} = -\alpha_G\frac{\hbar c}{r}\ ,\ \alpha_G = \frac{G_N m^2}{\hbar c}, \tag{1.23}$$

$$m = \sqrt{\frac{\alpha_G \hbar c}{G_N}}. \tag{1.24}$$

Choosing $\alpha_G = 1$ we obtain the so called Planck mass:

$$M_P = \sqrt{\frac{\hbar c}{G_N}}. \tag{1.25}$$

This mass from a particle point of view is tremendous, almost macroscopic. Taking $\hbar c = 1.97 \times 10^{-13} \times 10^6 \times 1.610^{-19}$ Kg m^{3}s^{-2}, $G_N = 6.67 \times 10^{-11}$m^{3}Kg^{-1}s^{-2} we find

$$M_P = \left[\frac{3.15 \times 10^{-26}}{6.67 \times 10^{-11}}\right]^{1/2} \text{Kg} = 2.17 \times 10^{-8}\text{Kg} \approx 0.02 \text{ mgr}. \tag{1.26}$$

This can be written

$$M_P = \frac{2.17 \times 10^{-8}\text{Kg}}{1.67 \times 10^{-27}\text{Kg}} m_p = 1.27 \times 10^{19}\ m_p = 1.20 \times 10^{19}\text{GeV}. \tag{1.27}$$

Note: There exist other definitions of the Planck mass. If we choose $\alpha_G = \alpha = 1/137$, we find $M_P = 1.02 \times 10^{18}$.
Note also that, through the Eq. (1.25) for $M_P = 1$, in the Planck system get

$$G_N = 1\ . \tag{1.28}$$

We can verify that

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-35}\text{m} \ , \ t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44}\text{s}. \tag{1.29}$$

Summary: Boltzmann's constant in the Plank scale can be set to unity, $k_B = 1$. Thus a new unit of absolute temperature is defined by

$$T_P = \frac{M_P c^2}{k_B} = 1.42 \times 10^{32}\text{K}. \tag{1.30}$$

In the Plank system we choose ϵ_0 so that

$$\epsilon_0 = \mu_0 = 1/4\pi \Rightarrow e = \sqrt{\alpha}. \tag{1.31}$$

Thus the of charge will be:

$$q_P = \sqrt{4\pi\epsilon_0 \hbar c} = 1.88 \times 10^{-18}\text{C}, \tag{1.32}$$

while the length and time units are:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-35}\text{m} \ , \ t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44}\text{s}. \tag{1.33}$$

At energies in the plank scale, gravity will be as strong as electromagnetism. Something that happened in the Universe at the time of the order of the Plank time.
Then the school books will define the three basic units as follows:

$$\hbar = 1 \ \ , \ \ c = 1 \ \ , \ \ G_N = 1 \ . \tag{1.34}$$

Then the basic laws of nature will be written as:

$$F = -\frac{M_1 M_2}{r^2} \ \text{(Newton)} \ , F = \frac{Q_1 Q_2}{r^2} \ \text{(Coulomb)} \ \ , \ \ E = T \ \ , \ \ E = \omega. \tag{1.35}$$

1.3 Invariants in kinematics

Example: The elastic collision of two particles A and B with m_A and m_B.
We define the invariant quantity:

$$s = (p_A + p_B)^2 = (E_A + E_B)^2 - (\vec{p}_A + \vec{p}_B)^2. \tag{1.36}$$

We will evaluate s in some frames of special interest.

1. the center of momentum system $\mathbf{p}_A + \mathbf{p}_B = 0$.
   In this system the meaning of s will become simpler

   $$s = (E_A^* + E_B^*)^2 \rightarrow \sqrt{s} = E_A^* + E_B^* = \text{Center of momentum frame.}$$

2. In the laboratory frame in which, let us say, the particle B is at rest. We have

$$s = (E_A + m_B)^2 - \vec{p}_A{}^2.$$

But

$$\vec{p}_A{}^2 = E_A^2 - m_A^2$$

i.e.

$$s = 2m_B E_A + m_B^2 + m_A^2.$$

Thus

$$\sqrt{s} = E_A^* + E_B^* = \sqrt{2m_B E_A + m_A^2 + m_B^2}.$$

3. Particles of unequal mass and energies moving in opposite directions, $\mathbf{p}_A = -\mathbf{p}_B$. An example is the LINAC accelerator HERA accelerating protons and Electrons in opposite directions.

$$\begin{aligned} s &= (E_A + E_B)^2 - (p_A - p_B)^2 = E_A^2 + E_B^2 + 2E_A E_B - p_A^2 - p_B^2 + 2p_A p_B \\ &= m_A^2 + m_B^2 + 2E_A E_B \left(1 + \sqrt{1 - \frac{m_A^2}{E_A^2}} + \sqrt{1 - \frac{m_B^2}{E_B^2}}\right) \approx 4E_A E_B. \end{aligned} \quad (1.37)$$

The last relation is valid in the masses can be neglected in front of the corresponding energies.

Let us return to the fixed target experiment and express $\sqrt{s}$ in terms of kinetic energies.

$$\begin{aligned} K_{cm} = K_A^* + K_B^* &= \sqrt{2m_B E_A + m_A^2 + m_B^2} - m_A - m_B \text{ or} \\ K_{cm} &= \sqrt{2m_B K_A + (m_A + m_B)^2} - m_A - m_B. \end{aligned} \quad (1.38)$$

Thus if A $=$ electron , B $=$ positron ($m_A = m_B = m_e$) at relativistic energies the energy available in the center of momentum frame becomes:

$$K_{cm} \approx \sqrt{2m_e K_A}, \quad (1.39)$$

i.e. it increases as the square root of the energy of the beam. Thus, to obtain an energy of 2 GeV in the Center of momentum system, the beam energy should be:

$$K_A \approx \frac{K_{cm}^2}{2m_e} \approx \frac{4 \text{ GeV}^2}{0.511 \times 2 \text{ MeV}} \approx 4000 \text{ GeV!}$$

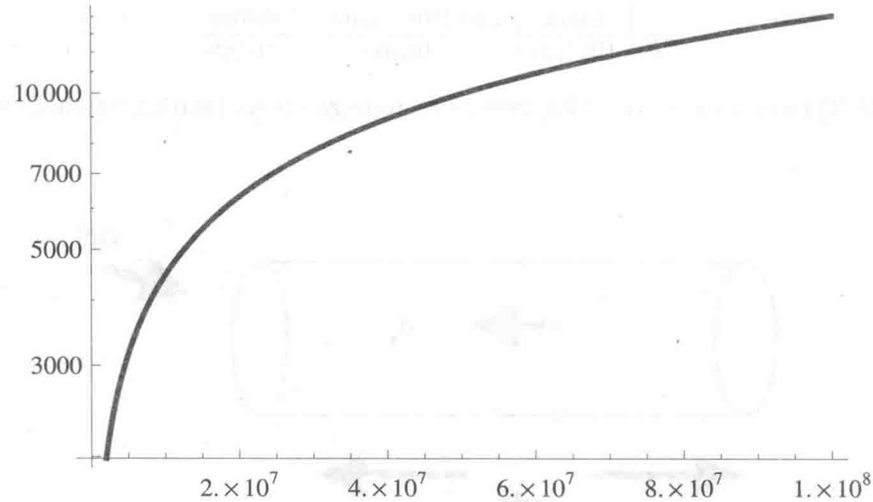

Figure 1.1: The Kinetic energy of two particles of the same mass in the center of momentum frame versus the energy of one of them in a fixed target set up. Both energies are given in units of the rest energy of each particle.

Clearly accelerating a particle this way is not a practical method to achieve high energy in the center of momentum.
In reactions the energy in the center of momentum counts, so that we do not lose most of it as kinetic energy of the fragments. Had we accelerated two particles we only needed 1 GeV each to get 2 GeV in the center of momentum.

COLLIDING BEAMS IS THE SECRET TO HIGH ENERGIES

FOR TWO BEAMS OF OPPOSITE CHARGE

ONE MACHINE AND ONE CHANNEL SUFFICE

1.4 Cross sections and Luminosities

The cross section in particle physics is expressed in units

$$1\text{b} = 1\text{barn} = 10^{-24}\text{cm}^2 = 100\ \text{fm}^2 \tag{1.40}$$

typical cross sections at energies of 100 GeV are given in table 1.2

1.4.1 Luminosity for fixed target experiments

The number of events produced during the collision is proportional to the flux of the particles in the beam (see Fig.1.2). Indeed,if n is the particle density in volume dV we will have: