Herbert Amann
Joachim Escher

Analysis I

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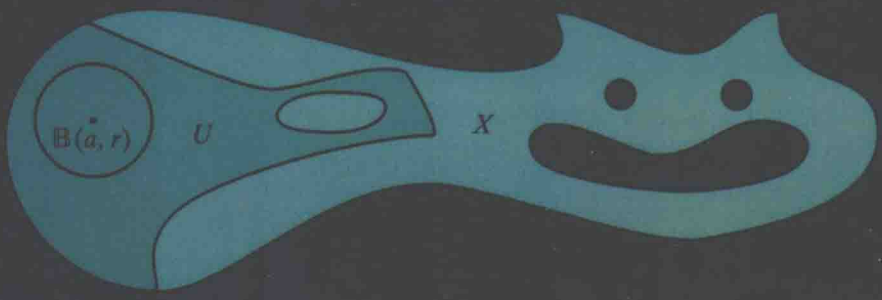

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Herbert Amann
Joachim Escher

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Preface

Logical thinking, the analysis of complex relationships, the recognition of underlying simple structures which are common to a multitude of problems — these are the skills which are needed to do mathematics, and their development is the main goal of mathematics education.

Of course, these skills cannot be learned 'in a vacuum'. Only a continuous struggle with concrete problems and a striving for deep understanding leads to success. A good measure of abstraction is needed to allow one to concentrate on the essential, without being distracted by appearances and irrelevancies.

The present book strives for clarity and transparency. Right from the beginning, it requires from the reader a willingness to deal with abstract concepts, as well as a considerable measure of self-initiative. For these efforts, the reader will be richly rewarded in his or her mathematical thinking abilities, and will possess the foundation needed for a deeper penetration into mathematics and its applications.

This book is the first volume of a three volume introduction to analysis. It developed from courses that the authors have taught over the last twenty six years at the Universities of Bochum, Kiel, Zurich, Basel and Kassel. Since we hope that this book will be used also for self-study and supplementary reading, we have included far more material than can be covered in a three semester sequence. This allows us to provide a wide overview of the subject and to present the many beautiful and important applications of the theory. We also demonstrate that mathematics possesses, not only elegance and inner beauty, but also provides efficient methods for the solution of concrete problems.

Analysis itself begins in Chapter II. In the first chapter we discuss quite thoroughly the construction of number systems and present the fundamentals of linear algebra. This chapter is particularly suited for self-study and provides practice in the logical deduction of theorems from simple hypotheses. Here, the key is to focus on the essential in a given situation, and to avoid making unjustified assumptions. An experienced instructor can easily choose suitable material from this chapter to make up a course, or can use this foundational material as its need arises in the study of later sections.

In this book, we have tried to lay a solid foundation for analysis on which the reader will be able to build in later forays into modern mathematics. Thus most

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concepts and definitions are presented, right from the beginning, in their general form — the form which is used in later investigations and in applications. This way the reader needs to learn each concept only once, and then with this basis, can progress directly to more advanced mathematics.

We refrain from providing here a detailed description of the contents of the three volumes and instead refer the reader to the introductions to each chapter, and to the detailed table of contents. We also wish to direct the reader's attention to the numerous exercises which appear at the end of each section. Doing these exercises is an absolute necessity for a thorough understanding of the material, and serves also as an effective check on the reader's mathematical progress.

In the writing of this first volume, we have profited from the constructive criticism of numerous colleagues and students. In particular, we would like to thank Peter Gabriel, Patrick Guidotti, Stephan Maier, Sandro Merino, Frank Weber, Bea Wollenmann, Bruno Scarpellini and, not the least, our students, who, by their positive reactions and later successes, encouraged our particular method of teaching analysis.

From Peter Gabriel we received support 'beyond the call of duty'. He wrote the appendix 'Introduction to Mathematical Logic' and unselfishly allowed it to be included in this book. For this we owe him special thanks.

As usual, a large part of the work necessary for the success of this book was done 'behind the scenes'. Of inestimable value are the contributions of our 'typesetting perfectionist' who spent innumerable hours in front of the computer screen and participated in many intense discussions about grammatical subtleties. The typesetting and layout of this book are entirely due to her, and she has earned our warmest thanks.

We also wish to thank Andreas who supplied us with latest versions of TEX[1] and stood ready to help with software and hardware problems.

Finally, we thank Thomas Hintermann for the encouragement to make our lectures accessible to a larger audience, and both Thomas Hintermann and Birkhäuser Verlag for a very pleasant collaboration.

Zurich and Kassel, June 1998 H. Amann and J. Escher

[1]The text was typeset using LATEX. For the graphs, CorelDRAW! and Maple were also used.

Preface to the second edition

In this new edition we have eliminated the errors and imprecise language that have been brought to our attention by attentive readers. Particularly valuable were the comments and suggestions of our colleagues H. Crauel and A. Ilchmann. All have our heartfelt thanks.

Zurich and Hannover, March 2002 H. Amann and J. Escher

Preface to the English translation

It is our pleasure to thank Gary Brookfield for his work in translating this book into English. As well as being able to preserve the 'spirit' of the German text, he also helped improve the mathematical content by pointing out inaccuracies in the original version and suggesting simpler and more lucid proofs in some places.

Zurich and Hannover, May 2004 H. Amann und J. Escher

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Chapter I

Foundations

Most of this first chapter is about numbers — natural numbers, integers, real numbers and complex numbers. Without a clear understanding of these numbers, a deep investigation of mathematics is not possible. This makes a thorough discussion of number systems absolutely necessary.

To that end we have chosen to present a constructive formulation of these number systems. Starting with the Peano axioms for the natural numbers, we construct successively the integers, the rational numbers, the real numbers and finally, the complex numbers. At each step, we are guided by a desire to solve certain 'naturally' occurring equations. These constructions are relatively long and require considerable stamina from the reader, but those readers who persevere will be rewarded with considerable practice in mathematical thinking.

Even before we can talk about the natural numbers, the simplest of all number systems, we must consider some of the fundamentals of set theory. Here the main goal is to develop a precise mathematical language. The axiomatic foundations of logic and set theory are beyond the scope of this book.

The reader may well be familiar with some of the material in Sections 1–4. Even so, we have deliberately avoided appealing to the reader's intuitions and previous experience, and have instead chosen a relatively abstract framework for our presentation. In particular, we have been strict about avoiding any concepts that are not already precisely defined, and using claims that are not previously proved. It is important that, right from the beginning, students learn to work with definitions and derive theorems from them without introducing spurious additional assumptions.

The transition from the simplest number system, the natural numbers, to the most complicated number system, the complex numbers, is paralleled by a corresponding increasing complexity in the algebra needed. Therefore, in Sections 7–8 we discuss fairly thoroughly the most important concepts of algebra. Here again we have chosen an abstract approach with the goal that beginning students become

familiar with certain mathematical structures which appear in later chapters of this book and, in fact, throughout mathematics.

A deeper understanding of these concepts is the goal of (linear) algebra and, in the corresponding literature, the reader will find many other applications. The goal of algebra is to derive rules which hold in systems satisfying certain small sets of axioms. The discovery that these axioms hold in complex problems of analysis will enable us to recognize underlying unity in diverse situations and to maintain an overview of an otherwise unwieldy area of mathematics. In addition, the reader should see early on that mathematics is a whole — it is not made up of disjoint research areas, isolated from each other.

Since the beginner usually studies linear algebra in parallel with an introduction to analysis, we have restricted our discussion of algebra to the essentials. In the choice of the concepts to present we have been guided by the needs of later chapters. This is particularly true about the material in Section 12, namely vector spaces and algebras. These we will meet frequently, for example, in the form of function algebras, as we penetrate further into analysis.

The somewhat 'dry' material of this first chapter is made more palatable by the inclusion of many applications. Since, as already mentioned, we want to train the reader to use only what has previously been proved, we are limited at first to very simple 'internal' examples. In later sections this becomes less of a restriction, as, for example, the discussion of the interpolation problems in Section 12 shows.

We remind the reader that this book is intended to be used either as a textbook for a course on analysis, or for self study. For this reason, in this first chapter, we are more thorough and cover more material than is possible in lectures. We encourage the reader to work through these 'foundations' with diligence. In the first reading, the proofs of Theorems 5.3, 9.1, 9.2 and 10.4 can be skipped. At a later time, when the reader is more comfortable with proofs, these gaps should filled.

1 Fundamentals of Logic

To make complicated mathematical relationships clear it is convenient to use the notation of symbolic logic. Symbolic logic is about **statements** which one can meaningfully claim to be true or false. That is, each statement has the *truth value* 'true' (T) or 'false' (F). There are no other possibilities, and no statement can be both true and false.

Examples of statements are 'It is raining', 'There are clouds in the sky', and 'All readers of this book find it to be excellent'. On the other hand, 'This sentence is false' is not a statement. Indeed, if the sentence were true, then it says that it is false, and if it is false, it follows that the sentence is true.

Any statement A has a **negation** $\neg A$ ('not A') defined by $\neg A$ is true if A is false, and $\neg A$ is false if A is true. We can represent this relationship in a *truth table*:

A	T	F
$\neg A$	F	T

Of course, in normal language 'not A' can be expressed in many ways. For example, if A is the statement 'There are clouds in the sky', then $\neg A$ could be expressed as 'There are no clouds in the sky'. The negation of the statement 'All readers of this book find it to be excellent' is 'There is at least one reader of this book who finds that it is not excellent' (but not 'No readers of this book find it to be excellent').

Two statements, A and B, can be combined using **conjunction** $\wedge$ and **disjunction** $\vee$ to make new statements. The statement $A \wedge B$ ('A and B') is true if both A and B are true, and is false in all other cases. The statement $A \vee B$ ('A or B') is false when both A and B are false, and is true in all other cases. The following truth table makes the definitions clear:

A	B	$A \wedge B$	$A \vee B$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Note that the 'or' of disjunction has the meaning 'and/or', that is, 'A or B' is true if A is true, if B is true, or if both A and B are true.

If $E(x)$ is an expression which becomes a statement when x is replaced by an object (member, thing) of a specified class (collection, universe) of objects, then E is a **property**. The sentence 'x has property E' means '$E(x)$ is true'. If x belongs to a class X, that is, x is an **element** of X, then we write $x \in X$, otherwise[1] $x \notin X$.

[1] It is usual when abbreviating statements with symbols (such as $\in$, $=$, etc.) to denote their negations using the corresponding slashed symbol ($\notin$, $\neq$, etc.).

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