



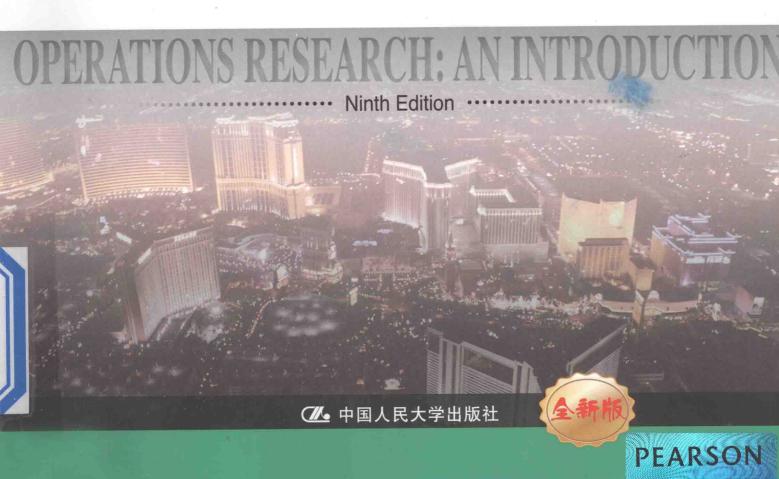
管理科学与工程经典教材

MANAGEMENT SCIENCE AND ENGINEERING CLASSICS

运筹学导论

英文版・第9版・基础篇

哈姆迪·A·塔哈 (Hamdy A. Taha) 著



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PERATIONS RESEARCH: AN INTRODUCTION

Ninth Edition

中国人民大学出版社 北京・

图书在版编目(CIP)数据

运筹学导论:第9版,基础篇:英文/塔哈著.一北京:中国人民大学出版社,2014.9管理科学与工程经典教材. ISBN 978-7-300-19874-3

Ⅰ. ①运… Ⅱ. ①塔… Ⅲ. ①运筹学-高等学校-教材-英文 Ⅳ. ①022

中国版本图书馆 CIP 数据核字(2014)第 200300号

管理科学与工程经典教材 运筹学导论(英文版・第9版・基础篇) 哈姆迪・A・塔哈 著

出版发行 中国人民大学出版社

社 址 北京中关村大街31号

邮政编码 100080

电 话 010-62511242(总编室)

010-62511770 (质管部)

010-82501766(邮购部)

010-62514148 (门市部)

010-62515195(发行公司)

010-62515275(盗版举报)

址 http:// www. crup. com. cn

http://www.ttrnet.com(人大教研网)

经 销 新华书店

XX

印 刷 三河市汇鑫印务有限公司

规 格 215 mm×275 mm 16开本

版 次 2014年9月第1版

印 张 26.25 插页 2

印 次 2014年9月第1次印刷

字 数 756 000

定 价 55.00 元

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印装差错 负责调换

随着我国加入 WTO,越来越多的国内企业参与到国际竞争中来,用国际上通用的语言思考、工作、交流的能力也越来越受到重视。这样一种能力也成为我国各类人才参与竞争的一种有效工具。国家教育机构、各类院校以及一些主要的教材出版单位一直在思考,如何顺应这一发展潮流,推动各层次人员通过学习来获取这种能力。双语教学就是这种背景下的一种尝试。

双语教学在我国主要指汉语和国际通用的英语教学。事实上,双语教学在我国教育界已经不是一个陌生的词汇了,以双语教学为主的科研课题也已列入国家"十五"规划的重点课题。但从另一方面来看,双语教学从其诞生的那天起就被包围在人们的赞成与反对声中。如今,依然是有人赞成有人反对,但不论是赞成居多还是反对占上,双语教学的规模和影响都在原有的基础上不断扩大,且呈大发展之势。一些率先进行双语教学的院校在实践中积累了经验,不断加以改进;一些待进入者也在模仿中学习,并静待时机成熟时加入这一行列。由于我国长期缺乏讲第二语言(包括英语)的环境,开展双语教学面临特殊的困难,因此,选用合适的教材就成为双语教学成功与否的一个重要问题。我们认为,双语教学从一开始就应该使用原版的各类学科的教材,而不是由本土教师自编的教材,从而可以避免中国式英语问题,保证语言的原汁原味。各院校除应执行国家颁布的教学大纲和课程标准外,还应根据双语教学的特点和需要,适当调整教学课时的设置,合理选择优秀的、合适的双语教材。

顺应这样一种大的教育发展趋势,中国人民大学出版社同众多国际知名的大出版公司,如麦格劳·希尔出版公司、培生教育出版公司等合作,面向大学本科生层次,遴选了一批国外最优秀的管理类原版教材,涉及专业基础课,人力资源管理、市场营销及国际化管理等专业方向课,并广泛听取有着丰富的双语一线教学经验的教师的建议和意见,对原版教材进行了适当的改编,删减了一些不适合我国国情和不适合教学的内容;另一方面,根据教育部对双语教学教材篇幅合理、定价低的要求,我们更是努力区别于目前市场上形形色色的各类英文版、英文影印版的大部头,将目标受众锁定在大学本科生层次。本套教材尤其突出了以下一些特点:

●保持英文原版教材的特色。本套双语教材根据国内教学实际需要,对原书进行了一定的改编,主要是删减了一些不适合教学以及不符合我国国情的内容,但在体系结构和内容特色方面都保持了原版教材的风貌。专家们的认真改编和审定,使本套教材既保持了学术上的完整性,又贴近中国实际;既方便教师教学,又方便学生理解和掌握。

- 突出管理类专业教材的实用性。本套教材既强调学术的基础性,又兼顾应用的广泛性;既侧重让学生掌握基本的理论知识、专业术语和专业表达方式,又考虑到教材和管理实践的紧密结合,有助于学生形成专业的思维能力,培养实际的管理技能。
- 体系经过精心组织。本套教材在体系架构上充分考虑到当前我国在本科教育阶段推广双语教学的进度安排,首先针对那些课程内容国际化程度较高的学科进行双语教材开发,在其专业模块内精心选择各专业教材。这种安排既有利于我国教师摸索双语教学的经验,使得双语教学贴近现实教学的需要;也有利于我们收集关于双语教学教材的建议,更好地推出后续的双语教材及教辅材料。
- •篇幅合理,价格相对较低。为适应国内双语教学内容和课时上的实际需要,本套教材进行了一定的删减和改编,使总体篇幅更为合理;而采取低定价,则充分考虑到了学生实际的购买能力,从而使本套教材得以真正走近广大读者。
- •提供强大的教学支持。依托国际大出版公司的力量,本套教材为教师提供了配套的教辅材料,如教师手册、PowerPoint 讲义、试题库等,并配有内容极为丰富的网络资源,从而使教学更为便利。

本套教材是在双语教学教材出版方面的一种尝试。我们在选书、改编及出版的过程中得到了国内许多高校的专家、教师的支持和指导,在此深表谢意。同时,为使我们后续推出的教材更适于教学,我们也真诚地期待广大读者提出宝贵的意见和建议。需要说明的是,尽管我们在改编的过程中已加以注意,但由于各教材的作者所处的政治、经济和文化背景不同,书中内容仍可能有不妥之处,望读者在阅读时注意比较和甄别。

徐二明 中国人民大学商学院

What's New in the Ninth Edition

The ninth edition continues to streamline both the text materials and the software support providing a broad focus on algorithmic and practical implementation of Operations Research techniques.

- For the first time in this book, the new Section 3.7 provides a comprehensive (math-free) framework of how the different LP algorithms (simplex, dual simplex, revised simplex, and interior point) are implemented in commercial codes (e.g., CPLEX and XPRESS) to provide the computational speed and accuracy needed to solve very large problems.
- The new Chapter 2 of volume two covers efficient heuristics/metaheuristics designed to find good approximate solutions for integer and combinatorial programming problems. The need for heuristics/metaheuristics is in recognition of the fact that the performance of the exact algorithms has been less than satisfactory from the computational standpoint.
- The new Chapter 3 of volume two is dedicated to the important traveling salesperson problem. The presentation includes a variety of applications and the development of exact and heuristic solution algorithms.
- All the algorithms in the new Chapters 2 and 3 of volume two are coded in Excel in a manner that permits convenient interactive experimentation with the models.
- All detailed AMPL models have been moved to Appendix C to complement the AMPL syntactical rules presented in the appendix. The models are cross-referenced opportunely in the book.
- Numerous new problems have been added throughout the book.
- The TORA software has been updated.
- In keeping with my commitment to maintain a reasonable count of printed pages, I found it necessary to move some material to the website, including the AMPL appendix.

^{*《}运筹学导论》(第9版)原书篇幅过大,英文版拆分为两册,即基础篇和提高篇,章节顺序作了相应调整,但网上(www.pearsonhighered.com/taha)文件保持原状,对文件序号未作更改。

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CHAPTER 1 What Is Operations Research?

1.1 INTRODUCTION

The first formal activities of Operations Research (OR) were initiated in England during World War II, when a team of British scientists set out to make scientifically based decisions regarding the best utilization of war materiel. After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.

This chapter introduces the basic terminology of OR, including mathematical modeling, feasible solutions, optimization, and iterative computations. It stresses that defining the problem correctly is the most important (and most difficult) phase of practicing OR. The chapter also emphasizes that, while mathematical modeling is a cornerstone of OR, unquantifiable factors (such as human behavior) must be accounted for in the final decision. The book presents a variety of applications using solved examples and chapter problems. In particular, Chapter 26 on the website is entirely devoted to the presentation of fully developed case analyses.

1.2 OPERATIONS RESEARCH MODELS

Imagine that you have a 5-week business commitment between Fayetteville (FYV) and Denver (DEN). You fly out of Fayetteville on Mondays and return on Wednesdays. A regular round-trip ticket costs \$400, but a 20% discount is granted if the round-trip dates span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

We can look at the situation as a decision-making problem whose solution requires answering three questions:

- 1. What are the decision alternatives?
- 2. Under what restrictions is the decision made?
- 3. What is an appropriate objective criterion for evaluating the alternatives?

Three plausible alternatives come to mind:

- 1. Buy five regular FYV-DEN-FYV for departure on Monday and return on Wednesday of the same week.
- 2. Buy one FYV-DEN, four DEN-FYV-DEN that span weekends, and one DEN-FYV.
- 3. Buy one FYV-DEN-FYV to cover Monday of the first week and Wednesday of the last week and four DEN-FYV-DEN to cover the remaining legs. All tickets in this alternative span at least one weekend.

The restriction on these options is that you should be able to leave FYV on Monday and return on Wednesday of the same week.

An obvious objective criterion for evaluating the proposed alternative is the price of the tickets. The alternative that yields the smallest cost is the best. Specifically, we have

Alternative 1 cost =
$$5 \times 400 = $2000$$

Alternative 2 cost =
$$.75 \times 400 + 4 \times (.8 \times 400) + .75 \times 400 = $1880$$

Alternative
$$3 \cos t = 5 \times (.8 \times 400) = $1600$$

Alternative 3 is the best because it is the cheapest.

Though the preceding example illustrates the three main components of an OR model-alternatives, objective criterion, and constraints—situations differ in the details of how each component is developed and how the resulting model is solved. To illustrate this point, consider forming a maximum-area rectangle out of a piece of wire of length L inches. What should be the best width and height of the rectangle?

In contrast with the tickets example, the number of alternatives in the present example is not finite; namely, the width and height of the rectangle can assume an infinite number of values because they are continuous variables. To formalize this observation, the alternatives of the problem are identified by defining the width and height algebraically as

w = width of the rectangle in inches h = height of the rectangle in inches

Based on these definitions, the restrictions of the situation can be expressed verbally as

- 1. Width of rectangle + Height of rectangle = Half the length of the wire
- 2. Width and height cannot be negative

These restrictions are translated algebraically as

1.
$$2(w+h)=L$$

2.
$$w \ge 0, h \ge 0$$

The only remaining component now is the objective of the problem; namely, maximization of the area of the rectangle. Let z be the area of the rectangle, then the complete model becomes

Maximize z = wh

subject to

$$2(w+h) = L$$
$$w, h \ge 0$$

Using differential calculus, the best solution of this model is $w = h = \frac{L}{4}$, which calls for constructing a square shape.

Based on the preceding two examples, the general OR model can be organized in the following general format:

Maximize or minimize **Objective Function** subject to

Constraints

A solution of the model is **feasible** if it satisfies all the constraints. It is **optimal** if, in addition to being feasible, it yields the best (maximum or minimum) value of the objective function. In the tickets example, the problem considers three feasible alternatives, with the third alternative yielding the optimal solution. In the rectangle problem, a feasible alternative must satisfy the condition $w + h = \frac{L}{2}$, where w and h are nonnegative variables. This definition leads to an infinite number of feasible solutions and, unlike the tickets problem, which uses simple price comparison, the optimum solution is determined by using differential calculus.

Though OR models are designed to "optimize" a specific objective criterion subject to a set of constraints, the quality of the resulting solution depends on the completeness of the model in representing the real system. Take, for example, the tickets model. If all the dominant alternatives for purchasing the tickets are not identified, then the resulting solution is optimum only relative to the choices represented in the model. To be specific, if alternative 3 is left out of the model, then the resulting "optimum" solution would call for purchasing the tickets for \$1880, which is a **suboptimal** solution. The conclusion is that "the" optimum solution of a model is best only for that model. If the model happens to represent the real system reasonably well, then its solution is optimum also for the real situation.

PROBLEM SET 1.2A1

- 1. In the tickets example, identify a fourth feasible alternative.
 - (a) Define an infeasible alternative.
 - (b) Identify a fourth feasible alternative and determine its cost.
- 2. In the rectangle problem, identify three feasible solutions, and determine which one is better.
- **3.** Determine the optimal solution of the rectangle problem (*Hint*: Use the constraint to express the objective function in terms of one variable, then use differential calculus.)
- 4. Amy, Jim, John, and Kelly are standing on the east bank of a river and wish to cross to the west side using a canoe. The canoe can hold at most two people at a time. Amy, being the most athletic, can row across the river in 1 minute. Jim, John, and Kelly would take 3, 6, and 9 minutes, respectively. If two people are in the canoe, the slower person dictates the crossing time. The objective is for all four people to be on the other side of the river in the shortest time possible.
 - (a) Identify at least two feasible plans for crossing the river (remember, the canoe is the only mode of transportation, and it cannot be shuttled empty).
 - (b) Define the criterion for evaluating the alternatives.
 - *(c) What is the smallest time for moving all four people to the other side of the river?
- *5. In a baseball game, Jim is the pitcher and Joe is the batter. Suppose that Jim can throw either a fast or a curve ball at random. If Joe correctly predicts a curve ball, he can maintain a .400 batting average, else if Jim throws a curve ball and Joe prepares for a fast ball, his batting average is kept down to .200. On the other hand, if Joe correctly predicts a fast ball, he gets a .250 batting average, else his batting average is only .125.
 - (a) Define the alternatives for this situation.
 - **(b)** Define the objective function for the problem, and discuss how it differs from the familiar optimization (maximization or minimization) of a criterion.
- **6.** During the construction of a house, six joists of 24 feet each must be trimmed to the correct length of 23 feet. The operations for cutting a joist involve the following sequence:

Operation	Time (seconds)	
Place joist on saw horses	15	
2. Measure correct length (23 feet)	no he 5 no	
3. Mark cutting line for circular saw		
4. Trim joist to correct length	20	
5. Stack trimmed joist in a designated area	20	

Three persons are involved: Two loaders must work simultaneously on operations 1, 2, and 5, and one cutter handles operations 3 and 4. There are two pairs of saw horses on which untrimmed joists are placed in preparation for cutting, and each pair can hold up to three side-by-side joists. Suggest a good schedule for trimming the six joists.

7. A (two-dimensional) pyramid is constructed in four layers: The bottom layer consists of (equally-spaced) dots 1, 2, 3 and 4, The next layer includes dots 5, 6, and 7; the following

¹Asterisk designates problems whose solution is given in Appendix B.

layer has dots 8 and 9; and the top layer has dot 10. You want to invert the pyramid (bottom layer has one dot and top layer has four) by moving the dots around.

- (a) Identify two feasible solutions.
- (b) Determine the smallest number of moves needed to invert the triangle.²
- 8. You have five chains each consisting of four solid links. You need to make a bracelet by connecting all five chains. It costs 2 cents to break a link and 3 cents to resolder it.
 - (a) Identify two feasible solutions and evaluate them.
 - (b) Determine the cheapest cost for making the bracelet.
- 9. The squares of a rectangular board of 11 rows and 9 columns are numbered sequentially 1 through 99 with a *hidden* monetary reward between 0 and 50 dollars assigned to each square. A game using the board requires the player to choose a square by selecting any two-digits and then subtracting the sum of its two digits from the selected number. The player then receives the reward assigned to the selected square. What monetary values should be assigned to the 99 squares to minimize the player's reward (regardless of how many times the game is repeated)? To make the game interesting, the assignment of \$0 to all the squares is not an option.

1.3 SOLVING THE OR MODEL

In OR, we do not have a single general technique to solve all mathematical models that can arise in practice. Instead, the type and complexity of the mathematical model dictate the nature of the solution method. For example, in Section 1.2 the solution of the tickets problem requires simple ranking of alternatives based on the total purchasing price, whereas the solution of the rectangle problem utilizes differential calculus to determine the maximum area.

The most prominent OR technique is **linear programming**. It is designed for models with linear objective and constraint functions. Other techniques include **integer programming** (in which the variables assume integer values), **dynamic programming** (in which the original model can be decomposed into smaller more manageable subproblems), **network programming** (in which the problem can be modeled as a network), and **nonlinear programming** (in which functions of the model are nonlinear). These are only a few among many available OR tools.

A peculiarity of most OR techniques is that solutions are not generally obtained in (formula-like) closed forms. Instead, they are determined by **algorithms**. An algorithm provides fixed computational rules that are applied repetitively to the problem, with each repetition (called **iteration**) moving the solution closer to the optimum. Because the computations associated with each iteration are typically tedious and voluminous, it is imperative that these algorithms be executed on the computer.

Some mathematical models may be so complex that it becomes impossible to solve them by any of the available optimization algorithms. In such cases, it may be necessary to abandon the search for the *optimal* solution and simply seek a *good* solution using **heuristics/metaheuristics** or *rules of thumb*.

²Problems 7 and 8 are adapted from Bruce Goldstein, *Cognitive Psychology: Mind, Research, and Everyday Experience*, Wadsworth Publishing, 2005.

1.4 QUEUING AND SIMULATION MODELS

Queuing and simulation deal with the study of waiting lines. They are not optimization techniques; rather, they determine measures of performance of waiting lines, such as average waiting time in queue, average waiting time for service, and utilization of service facilities.

Queuing models utilize probability and stochastic models to analyze waiting lines, and simulation estimates the measures of performance by imitating the behavior of the real system. In a way, simulation may be regarded as the next best thing to observing a real system. The main difference between queuing and simulation is that queuing models are purely mathematical and hence are subject to specific assumptions that limit their scope of application. Simulation, on the other hand, is flexible and can be used to analyze practically any queuing situation.

The use of simulation is not without drawbacks. The process of developing simulation models is costly in both time and resources. Moreover, the execution of simulation models, even on the fastest computer, is usually slow.

1.5 ART OF MODELING

The illustrative models developed in Section 1.1 are exact representations of real situations. This is a rare occurrence in OR, as the majority of applications usually involve (varying degrees of) approximations. Figure 1.1 depicts the levels of abstraction that characterize the development of an OR model. We abstract the assumed real world from the real situation by concentrating on the dominant variables that control the behavior of the real system. The model expresses in an amenable manner the mathematical functions that represent the behavior of the assumed real world.

To illustrate levels of abstraction in modeling, consider the Tyko Manufacturing Company, where a variety of plastic containers are produced. When a production order is issued to the production department, necessary raw materials are acquired

FIGURE 1.1 Levels of abstraction in model development

