

新一代信息科学与技术

# Quantum Control of Multi-Wave Mixing

多波混频量子控制

(英文版)

Yanpeng Zhang  
Feng Wen  
Min Xiao



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## Quantum Control of Multi-Wave Mixing

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## Preface

It is widely agreed that the optical properties of materials change drastically in systems where the superpositions of quantum states are coherently excited. Many interesting scientific discoveries and technical applications have been made with nonlinear optical effects in several kinds of nonlinear materials. Examples of such effects include the modification of absorptive properties resulting in electromagnetically induced transparency (EIT) and lasing without population inversion, as well as the modification of dispersive properties to give a resonantly enhanced index of refraction accompanied by vanishing absorption. There are already several excellent general textbooks covering various aspects of nonlinear optics, including “*Nonlinear Optics*” by R. W. Boyd, “*Nonlinear Optics*” by Y. R. Shen, “*Quantum Electronics*” by A. Yariv, “*Nonlinear Fiber Optics*” by G. P. Agrawal, “*Photons and Nonlinear Optics*” by D. N. Klyshko, and so on. Although these textbooks have provided solid foundations for readers to understand various nonlinear optical processes, some comprehensive and deep knowledge on a certain nonlinear optical effect is essential for studying and researching. With the intention of giving the special knowledge and recent progress about multi-wave mixing (MWM) effect, the authors have published two earlier monographs, “Multi-Wave Mixing Processes” and “Coherent Control of Four-Wave Mixing” in 2009 and 2011, respectively, which covers the experimental and theoretical studies of several topics related to MWM processes previously done in authors’ groups. The topics covered in the two monographs include difference-frequency femtosecond and sum-frequency attosecond beats of four-wave mixing (FWM) processes; heterodyne detections of FWM, six-wave mixing (SWM), and eight-wave mixing (EWM) processes; the Raman, Raman–Rayleigh, Rayleigh–Brillouin, and coexisting Raman–Rayleigh–Brillouin-enhanced polarization beats; high-order correlation functions of different noisy fields on the femto- and attosecond polarization beats, and heterodyne/homodyne detections of the ultrafast third-order polarization beats.

This new monograph is built on the previous works and extends them significantly. The intention is to present all the additional and new works done in recent years in authors’ groups. Also, many added latest results, extended detailed calculations, and more deep discussions are cited from the already published papers, which can help readers better appreciate the interesting nonlinear optical phenomena.

Besides showing more results on controls and interactions between MWM processes in hot atomic media, this monograph also presents and discusses several novel types of spatial solitons and two-photon fluorescence in the FWM process, which are completely new phenomena in multi-level atomic systems.

Chapter 1 gives the outline of this monograph and introduce some basic concepts frequently used in the later chapters, such as the suppression and enhancement conditions of MWM, two- and three-photon fluorescence, MWM in ring optical cavity, photonic band gap, and MWM with Rydberg blockade, and so on. Chapter 2 gives the extended results on the EIT-assisted MWM which includes not only the coexistence of FWM, SWM, and EWM signals in multi-EIT windows, but also the interplay and interference among the multiple MWM processes. Chapter 3 gives the results on the ac-Stark effect and Autler–Townes (AT) splitting in the MWM process, which can be accurately explained by multi-dressed state theory. Chapter 4 presents the switching methods between enhancement and suppression of MWM in both frequency and spatial domains, and the evolution of the dressed effects from pure enhancement into pure suppression when the probe detuning as well as the powers of the dressing and probe fields are changed. Chapter 5 shows the polarization-controlled MWM processes in multiple Zeeman sublevels system, in which the generated MWM signals can be modified and controlled, and the dark-state effects can be controlled to evolve from pure enhancement into pure suppression in the MWM processes via the polarization states of the laser beams. Chapter 6 presents the demonstration of the modification and control of the two-photon fluorescence process and three-photon correlation in MWM by manipulating the dark-state or EIT windows. On the other hand, the vacuum-induced Rabi splitting and optical bistability in a coupled atom-cavity system is also included. Chapter 7 shows the forming of electromagnetically induced grating (EIG) and electromagnetically induced lattice (EIL) in FWM, which can lead to spatial shift and splitting of laser beams as well as several novel types of spatial solitons of FWM signals, such as gap, dipole, vortex, and surface solitons with the generated FWM beams in different regions in experimental parametric space. Chapter 8 presents the observations of the prototype investigation of all-optical switches, routers, and space demultiplexer by the FWM process.

The authors believe that the current monograph treats some special topics of quantum controls of MWM and can be useful to researchers who are interested in the related fields. Several features presented here are distinctly different from and more advanced than the previously reported works. For example, the authors have shown the two-photon ac-Stark effect and AT splitting of MWM can be used to determine the energy-level shift of the atom. Also, theoretical calculations are in good agreement with the experimentally measured results in demonstrating the two phenomena in MWM processes. Efficient spatial–temporal interference between FWM and SWM signals generated in a four-level atomic system has been carefully investigated, which exhibits controllable interactions between two different (third- and fifth-) order nonlinear optical processes. Evolutions of the enhancement and suppression of MWM signals under various dressing schemes are experimentally investigated, and can be explained in detail with dressing state

theory. Such controllable high-order nonlinear optical processes can be used for designing new devices for all-optical communication and quantum information processing.

The authors also experimentally compare the intensity spectra of probe transmission, FWM, and fluorescence signals under dressing effects, in which the two-photon fluorescence signal with ultranarrow line width much less than the Doppler-free EIT window is obtained. Moreover, the three photons of the two coexisting FWM signals and the probe signal in a double-lambda-type system are experimentally found to be strongly correlated, or anticorrelated with each other. Such ultranarrow linewidth fluorescence, strongly correlated or anticorrelated photons and controllable cavity mode splitting as well as the optical bistability process could have potential applications in optical communication and quantum information processing.

The authors also experimentally demonstrate that by arranging the strong pump and coupling laser beams in specially designed spatial configurations (to satisfy phase-matching conditions for efficient MWM processes), the generated MWM signals can be spatially shifted and split controllably by the cross-phase modulation (XPM) in the Kerr nonlinear medium. Therefore, the periodic splitting of the metastable energy level and periodic refractive index of the medium is experimentally obtained by the spatially periodic interfered pattern of dressing fields. Moreover, in the propagation of FWM or probe beams, when the spatial diffraction is balanced by XPM, the spatial beam profiles of the beams can become stable to form spatial optical solitons. For different geometrical configuration and experimental parameters (such as laser powers, frequency detunings, and temperature), novel gap, vortex, dipole, and surface soliton have been shown to experimentally appear in the multi-level atomic system in vapor cells. These studies have opened the door for achieving all-optical controlled spatial switch, routing, and soliton communications with an ultrashort response time.

This monograph serves as a reference book intended for scientists, researchers, graduate students, and advanced undergraduates in nonlinear optics and related fields.

We take this opportunity to thank all the researchers and collaborators who have worked on the research projects as described in this book.

*Yanpeng Zhang, Feng Wen, and Min Xiao*

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## 1

## Introduction

The subjects of this book center mainly around three topics. The first topic (Chapters 2–5) covers the quantum interference of coexistent four-wave mixing (FWM) processes because the generated FWM signal can be selectively enhanced or suppressed via electromagnetically induced transparency (EIT) windows and induced atomic coherence. Both enhancement and suppression of dressed FWM can be observed directly by scanning the dressing field instead of the probe field. With specially designed spatial patterns and phase-matching conditions for laser beams, coexisting FWM and six-wave mixing (SWM) processes can also be generated very efficiently. Also, three dual-dressed schemes (nested, sequential, and parallel) of coexisting FWM have been studied. Frequency, spatial, and temporal interferences that occur between two different wave mixing processes for the relative phase between different multi-wave mixing (MWM) processes is modulated. In such cases, the FWM and SWM signals are modulated with phase difference. By regulating the laser beam, the constructive and destructive interference can be selected, and then ac-Stark effect and Autler–Townes (AT) splitting are observed. Furthermore, by manipulating the polarization states of the laser beams, the MWM processes can also be modified and controlled. The second topic (Chapter 6) relates to nonclassical properties of the MWM process; the correlation or anticorrelation between two coexisting FWM signals; and comparison among the depths of the probe transmission, FWM, and the two-photon fluorescence signals in the same nonlinear process. Especially, one can switch from bright to dark states in the FWM and fluorescence channels with the relative phase modulated from 0 to  $-\pi$ . The relationship of vacuum Rabi splitting (VRS) and optical bistability (OB) of cavity MWM signals and methods to control VRS and OB are also included. The third topic (Chapters 7 and 8) relates to the interplays in frequency and spatial domains of MWM processes induced by atomic coherence in multi-level atomic systems. The generated two-dimensional surface solitons, multi-component dipole, and vortex vector solitons of the MWM signal accompanying the formation of electromagnetically induced lattice (EIL) are presented in Chapter 7. Finally, an application of spatial displacements and splitting of the probe and generated FWM beams, that is, all-optical spatial routing, switching, and demultiplexer are shown and investigated in Chapter 8. Experimental results are presented and compared with the theoretical calculations

throughout the book. In this book, the emphasis is on the work done by the authors' groups in the past few years. Some of the work presented in this book are built on our previous books "Multi-wave Mixing Processes" and "Coherent Control of Four-Wave Mixing," published by Higher Education Press and Springer 2009 and 2011, respectively, where we have mainly discussed the coexistence and interactions between efficient MWM processes enhanced by atomic coherence in multi-level atomic systems in the former, and in the latter the control in frequency and spatial domains of FWM processes induced by atomic coherence in multi-level atomic systems. Before starting the main topics of this book, some basic physical concepts and mathematical techniques, which are useful and needed in the later chapters, are briefly introduced and discussed in this introduction chapter.

## 1.1

### Suppression and Enhancement Conditions of the FWM Process

In this section, the dressed state theory with suppression and enhancement of FWM, dark-state theory in high-order nonlinear processes is introduced. Specifically, in Section 1.1.1, we discuss different dressing states including singly dressing and doubly dressing states. And the AT splitting for FWM by scanning the detuning of the probe field induced by the dressing effect will also be introduced. In Section 1.1.2, the same phenomenon is investigated using the quantum interference method. Also, we discuss that the high-order nonlinear processes can be effectively controlled by the dark state. In Section 1.1.3, we focus on suppression and enhancement of the FWM process by scanning the detuning of the dressing field. In addition, the interaction between dressing fields is introduced in different dressing schemes.

#### 1.1.1

##### Dressed State Theory

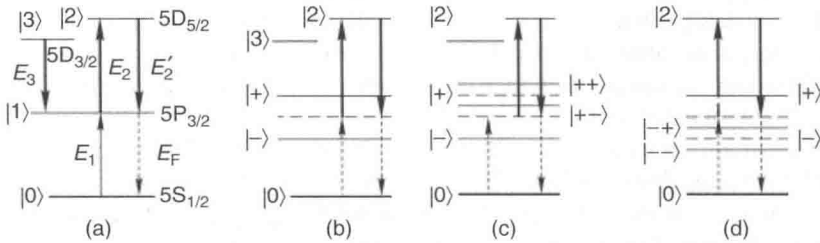
The most successful applications of resonant systems is in the two-photon rather than single-photon transition process because EIT [1, 2] forming in a two-photon interference process can reduce the linear absorption of a probe beam with a strong coupling beam resonant with the up-level transition. For example, enhanced MWM processes due to two-photon Raman resonances [3, 4] have been experimentally demonstrated in several multi-level atomic systems [5–7]. The keys in such enhanced nonlinear optical processes include the enhanced nonlinear susceptibilities due to induced atomic coherence and slowed laser beam propagation in the atomic medium [8–10], as well as greatly reduced linear absorption of the generated optical fields due to EIT [11]. By changing the strength [12], detuning [13], polarization [7], and phase [14] of the dressing fields, the EIT window and nonlinear susceptibilities

can be effectively modulated. Therefore, the generated MWM signal can be enhanced or suppressed selectively. On the other hand, AT splitting of the MWM [15–17] signal can be controlled effectively at the same time.

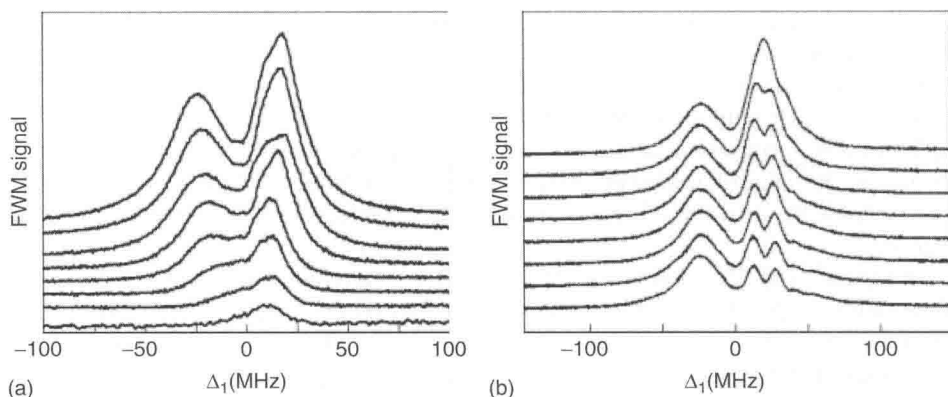
In order to describe more precisely what is the physics meaning of the AT splitting and suppression or enhancement of the wave mixing process mentioned, we introduce the dressing state theory first and use it to briefly describe the relations between the AT splitting and the suppression or enhancement of MWM.

In Figure 1.1(a), the external dressing field  $E_3$  and self-dressing field  $E_2$  ( $E'_2$ ) dress the energy level  $|1\rangle$  simultaneously. First,  $E_2$  ( $E'_2$ ) splits the state  $|1\rangle$  to create the primary dressed states  $|\pm\rangle$  written as  $|\pm\rangle = \sin\theta_1|1\rangle + \cos\theta_1|2\rangle$ . We can obtain the eigenvalues of  $|+\rangle$  and  $|-\rangle$  as  $\lambda_+ = (\Delta_2 + \sqrt{\Delta_2^2 + 4|G_2|^2})/2$  and  $\lambda_- = (\Delta_2 - \sqrt{\Delta_2^2 + 4|G_2|^2})/2$  (measured from level  $|1\rangle$ ), respectively, as shown in Figure 1.1(b). Next,  $E_3$  further splits  $|+\rangle$  or  $|-\rangle$  to create the secondary dressed states  $|+\pm\rangle$  or  $|-\pm\rangle$  determined by the detuning of  $E_3$ , as shown in Figure 1.1(c) and (d). For instance, if  $E_3$  couples with the upper dressed state  $|+\rangle$ , then secondary dressed states are given as  $|+\pm\rangle = \sin\theta_2|+\rangle + \cos\theta_2|3\rangle$  (Figure 1.1(c)), where  $\sin\theta_1 = -a_1/a_2$ ,  $\cos\theta_1 = G_2^b/a_2$ ,  $\sin\theta_2 = -a_3/a_4$ ,  $\cos\theta_2 = G_3/a_4$ ,  $a_1 = \Delta_2 - \lambda_{\pm}$ ,  $a_2 = \sqrt{a_1^2 + |G_2^b|^2}$ ,  $a_3 = \Delta_3 - \lambda_+ - \lambda_{+\pm}$ ,  $a_4 = \sqrt{a_3^2 + |G_3|^2}$ , and  $G_2^b = G_2 + G'_2$ . The eigenvalues of  $|++\rangle$  and  $|+-\rangle$  are  $\lambda_{++} = (\Delta'_3 + \sqrt{\Delta_3'^2 + 4|G_3|^2})/2$  and  $\lambda_{+-} = (\Delta'_3 - \sqrt{\Delta_3'^2 + 4|G_3|^2})/2$  (measured from level  $|+\rangle$ ), respectively, where  $\Delta'_3 = \Delta_3 - \lambda_+$ .

By scanning the detuning of the probe field ( $\Delta_1$ ), the primary AT splitting for the FWM signal (Figure 1.2(a)), or primary and secondary AT splitting (Figure 1.2(b)) can be obtained. In Figure 1.2(a), the AT splitting that results from the field  $E_2$  ( $E'_2$ ) has two bright states (two peaks) and one dark state (the dip at  $\Delta_1 = 0$ ), and the splitting separation  $\Delta_{\lambda 1}$  ( $\Delta_{\lambda 1} = \lambda_+ - \lambda_-$ ) gets larger with the power of  $E_2$  ( $E'_2$ ) increasing. In addition, if  $E_3$  also couples  $|-\rangle$ , Figure 1.2(b) presents the primary and secondary AT splitting for different coupling field power  $P_3$ . It is obvious that the secondary AT splitting  $\Delta_{\lambda 2}$  ( $\Delta_{\lambda 2} = \lambda_{++} - \lambda_{+-}$ ) (Figure 1.2(b)) gets larger with increasing power of  $E_3$ .



**Figure 1.1** The diagrams of (a) a Y-type four-level, (b) the singly-dressed state, and (c) and (d) the doubly-dressed state.



**Figure 1.2** (a) Measured primary AT splitting and (b) primary and secondary AT splitting of FWM signals.

### 1.1.2

#### Dark-State Theory in MWM Processes

In the following, we discuss in detail the dark-state theory in three-level, Y-type four-level, and K-type five-level. The evolution of the FWM signal is fitted well with the theoretical calculation [5, 14].

Well known in quantum optics, the phenomenon of coherent dark states [18] is based on a superposition of long-lived system eigenstates that decouple from the light field. The dark state can be used to make a resonant, opaque medium transparent by means of quantum interference. Associated with the induced transparency is a dramatic modification of the refractive properties; therefore, it has found numerous applications. Prominent examples are EIT and lasing without inversion [19, 20], subrecoil laser cooling [21], and ultrasensitive magnetometers [22]. The possibility of coherently controlling the propagation of quantum light pulses via dark-state polaritons opens up interesting applications involving the generation of nonclassical states in atomic ensembles (squeezed or entangled states), reversible quantum memories for light waves [23–25], and high-resolution spectroscopy [26–28]. Furthermore, the combination of the present technique with studies on few-photon nonlinear optics [29–31] can be used, in principle, for processing of quantum information stored in collective excitations of matter. In the following, we discuss how high-order nonlinear processes can be effectively controlled by the dark state.

First, let us consider how a simple FWM process in a  $\Xi$ -type three-level could be manipulated and analyzed with the dark-state theory developed in this section. As shown in Figure 1.3, in the interaction picture, the Hamilton of a coupled atom-field system can be written as  $H = \hbar(G_1|1\rangle\langle 0| \exp(-i\Delta_1 t/\hbar) + G_2|1\rangle\langle 2| \exp(-i\Delta_2 t/\hbar) + \text{HC})$ , and the wave function of an atom in bare state is  $|\psi\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle + c_2(t)|2\rangle$ . If the coupled system of the laser field and atom is