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Theory and Method for Finite Element Analysis of Bridge Structures

桥梁结构有限元分析理论及方法

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Abstract

This book makes a detailed narration about theories and methods of bar structure, beam structure, shell structure and 3D structure beyond the limit of former similar books. Besides, it also makes a brief introduction of major methods for bridge structure stability analysis and dynamic analysis and lists a range of software for finite element and its engineering examples. In this way, reader can get to know finite element method from the angle of fundamental and problems and then combine theories and methods with engineering examples to solve practical problems.

This book is a textbook mainly for overseas students, domestic undergraduates and graduates majoring in engineering type, and can also be considered as a reference book for engineering technicians.

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Preface

Finite element method (FEM) is a numerical calculation method widely applied in most scientific research and engineering analysis, which is of strict mathematics logic and clear physical conception and can solve different complicated problems. In current world, it is widely used in science, technology and engineering application where people attach great importance to it; with fast tempo of computer science and technology, it has been a key part of CAE and numerical simulation and of great significance to security and reliability of engineering structure.

With increasing prosperity of education industry in our country, a large number of overseas students feel huge differences like cultural differences in foreign countries, therefore it is necessary to update current Chinese textbook which is unsatisfactory due to limited finite element method and edit a better textbook to suit current situation accordingly.

Based on fundamental principle of finite element method, its numerical method, computer software realization and application in bridge construction etc., the book introduces element stiffness matrix derivation, overall stiffness matrix, boundary conditions disposal and non-joint load equivalence and other technologies; from plane problem to space problem; from static analysis to dynamic analysis; in the end to finite element software for calculation and its application in bridge construction; from the easy to the difficult and complicated in a step-by-step way.

This book is made up of eight chapters. The first one is an introduction mainly about fundamental, development and engineering application of finite element method. The second and the third chapters unveil the finite element analysis theory for bar system structure and beam structure; the fourth and the fifth ones make a narration about the finite element analysis theory for shell structure and 3D structure; the sixth and the seventh ones allow you to know the finite element analysis theory of structure stability and structure dynamics; the last eighth chapter introduces some common finite element computer software with applied examples in bridge engineering.

This book is a textbook mainly for overseas students, domestic undergraduates and graduates majoring in engineering type and can also be considered as a reference book for engineering technicians. Due to limited level and time, it is hard to avoid some mistakes or faults during book editing and therefore we sincerely appreciate it if you can make correction about it.

Author

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Contents

Chapter 1 Introduction

1.1 Overview

Finite element method (FEM) is the result through comprehensive development and integration of multiple disciplines such as mechanics, mathematical physics, computational science, and computer technology, etc. It is a numerical simulation and analysis method used for structural analysis of large-scale structural projects, which has been widely implemented in computer programs based on theory of structural mechanics and elasticity.

In three major research methods, including theoretical analysis, scientific experiment, and scientific computation, the scientific computation has become the most important studying method due to limitation of theoretical analysis and scientific experiment for majority of new fields. In the FEM, analysis and performance study may be made for almost all of complicated engineering structures. According to the results of analysis and study, the design is modified and optimized, the safety of engineering structures being judged and causes of engineering accident being analyzed. This method features mathematically rigorous logic and physically clear concept, and can flexibly handle complicated problems using computer-based programming. Therefore, it is widely applied in analysis of engineering structures.

The core idea of the FEM is structural discretion. The salient features of it include decentralized, analyzed in a discrete way, and then integrated, to convert the infinite freedoms into finite freedoms, that is, in the method a complicated continuum may be divided into finitely many elements which are not overlapped each other. In each element, some appropriate joints are selected as the interpolation points to find the solution of function. A linear expression consists of variables in the differential equation or the value at joint of its derivate, and the selected interpolation function. Relying on variational principle and weighted residual method, find the discrete solution of the differential equation to obtain the approximate results satisfying the engineering accuracy. Then, combine these elements to represent the original structure for realizing integral analysis of the engineering structures.

With continuous progress of electronic computer technology, FEM theory and application have been developed rapidly. The field of application has been extended to spaces and shell plates from planes of elastic mechanics; to stability, dynamics, and fluctuation from static balance; with analyzing objects to plastic, viscoelastic, viscoplastic, and composite materials etc. from elastic materials; to continuous-medium field such as fluid mechanics, heat transfer, and electromagnetism from solid mechanism. It plays an engineering role in optimization of design and combining with computer aid design (CAD) instead of only analysis and checking. In summary, based on different variational principles, FEM may be applied for every continuous-medium issue and almost all of fields. It may be expected that, with development of modern mechanics, computational mathematics, and computer technology, as a numerical analysis tool with solid theoretical foundation and extensive effectiveness of application, FEM will definitely be further developed and improved, playing a greater role in development of science and technology and the other fields.

1.2 History of FEM

FEM is an important method for numerical computation and experienced three stages including introduction (1941-1943), early development (1944-1960), and follow-up development(1961-).

In 1941, Hrennikoff was the first one who brought forward to find the solution for a question of elastic mechanics in steel frame method and structural system composed of bar may be analyzed in discrete element method (DEM). In 1943, Courant divided the section into many triangular zones when he was finding the solution of a torsional question, with setting of warping functions in each zone, finding solutions in minimum potential energy principle and in modern FEM. Also, the concept of finite element was introduced into his issued mathematical paper, *Variational Method for Balance and Vibration Problems*, which marked emergence of finite element (FE).

During early development (1944-1960), FE had original algebraic expression of FE, starting to study of element generation, and selection of element type. Also, great breakthrough was attained for study on convergence of solution. In 1955, German J. H. Argyris laid the theoretical foundation for FEM with his progress in energy principle and matrix analysis. In 1956, M. J. Turner et al. promoted to apply the idea of steel frame displacement method for plane problems of elastic mechanics during they performed analysis and design of airplane structures, dividing continuum into triangular and rectangular elements. The displacement function in elements is approximately expressed to deduce the element stiffness matrix and to establish the element stiffness equation of displacement of joint vs. force of joint, which was developed as matrix displacement method. In 1960, R . W . Clough introduced this method into civil engineering from aerospace structures and named it as finite element method (FEM) in his issued paper, *Finite Element Method for Analysis of Plane Stress*, which marked that the FEM early development was finished.

With development of computer technology and software technology, FE has been developed rapidly and improved since 1961. Over this period, FEM has been further studied, with design contents including theories in mathematical and mechanical field, element generation principle, and selection of shape function, numerical computation method and error analysis, convergence and stability study, non-linear problems and large-deformation problems etc. Meanwhile, FEM, in combination with computer-based software, forms commercialized software so that it may be widely promoted. The first generation of FE program was issued by ED Wilson from Berkeley University. The second generation of linear program was famous SAP (structural analysis program) and the non-linear program being NONSAP. Afterwards, a series of analysis software for large-scale structure were launched, including ANSYS, MIDAS, Dr. Bridge software, and famous non-linear solver i. e. ADINA (Automatic Dynamic Incremental Nonlinear Analysis), etc. It was not only applied in aerospace field, but also in civil engineering, material processing and/or machining, mechanical manufacturing, electronic and electrical, national defense and military, railway, shipbuilding, automotive, petrochemical, energy and the other fields of research. Also, it penetrated to fluid mechanics and the other aspects from conventional structural mechanics and solid mechanics.

According to the historical development of international FE software, the FE analysis method presents trend of development:

(1) Seamlessly Integrated with CAD Software

Today, one of development trends for FE analysis software is that it is integrated with commonly used CAD software, that is, after styling and design have been finished for the components and parts with CAD software, the model may be directly transferred into FE software for mesh generation, and analysis and computation, greatly improving design level and efficiency.

(2) Stronger Mesh Processing Capacity

The basic procedure of FE solution mainly includes analyzing discretion of object, FE solution, and post-processing results of computation. Because the mesh quality after structural discretion has direct influence on the time of solution and correctness of the solution results, the software has been improved for mesh generation over these years. However, some aspects need to be improved. In the next years, focus may be paid on improving 3D solid model for automatic hexahedron mesh generation and on increasing self-adaption capacity of mesh generation according to solution results.

(3) Developed for Solution of Non-Linear Problems instead of only for Linear Problems

With development of science and technology, the linear theory has been unable to meet the design requirements. A plenty of engineering issues such as material breakage and failure, crack expansion etc., can hardly meet the requirements only relying on the linear theory, which must require for non-linear solution. For example, sheet forming requires that the structural large displacement, large strain (geometrically non-linear) and plasticity (non-linearity of material), and structural time dependence, etc.

(4) Program User-oriented Openness

With increased commercialization, the software functions, usability, etc. has been improved considerably. However, because there are different requirements by the users, it is impossible to meet the requirements of all users. Therefore, an open environment must be provided for the users, allowing the users to expand the software as needed.

For several decades, FEM technology has become an important method in today's scientific and engineering analysis, being foundation of computer-based simulation, and main means of technical design. Greater attention will be paid on it in today's changing society.

1.3 Engineering Application of FE Method

Since 1940s, FE method has been developed and innovated for more than 60 years, of which fields of application are increasingly expanded, from initial rod member structure problems to elastic mechanics, viscoelastic mechanics, and plastic mechanics problems, from plane application to space applications, from static mechanics to dynamically mechanical stability analysis, from linear cases to non-linear cases, from solid mechanics to fluid mechanics, aerodynamics, thermo engineering, and electromagnetics etc. Today, FE method has become an indispensible tool for scientific research and engineering structural analysis.

FE includes linear elastic FE method and non-linear elastic FE method, where the former is the fundamental of the latter. Combination of both may realize systematical analysis of engineering structure.

1.3.1 Linear Elastic FE

Linear elastic FE takes ideal elastic structure as object of research. The considered deformation is established with assumption to small deformation. In such cases, the material stress is linearly related to strain, in line with broad Hooke's law which shows that the stress is linearly related to strain. Therefore, a linear elastic case may be finally considered to obtain solution to linear equations. In general, it may include linear elastostatics and linear elastodynamics analysis.

1.3.2 Non-linear Elastic FE

FE non-linear cases include:

(1) Non-linearity of Material

If the non-linearity of a system is caused by non-linear relationship of material stress vs. strain, it is called non-linearity of material, for example, material elasticoplasticity, slack, and creep strain etc.

(2) Non-linearity of Geometry

If change in structure may cause significant variation in force applied on the system so that it is unable to be analyzed in method of linear system, it is called non-linearity of geometry, for example, large deformation and large flexibility etc. of a structure.

(3) Non-linearity of Boundary Conditions

In engineering, many structures transfer loads relying on contacting extrusion and friction. The contacting boundary is highly non-linear. In addition, gearing, stamping and forming, and rubber vibration isolator are examples of non-linearity of boundary conditions. Therefore, when one structure contacts another structure or foreign boundary, non-linear analysis needs to be made for the boundary conditions normally.

Compared to linear elastic FE, in general a non-linear equation needs to be iteratively solved. It cannot be solved with superposition principle. And, for non-linear problems, it is impossible to find consistent solution and even sometime there is no solution. Therefore, non-linear equations are solved more complicated than linear elastic problems, with higher expenses, and with more unpredictability.

1.3.3 Analysis Procedure of FE Method

(1) Discretion of Structure

The ideology of FE method is decentralized, analyzed in a discrete way, and then integrated. Therefore, for FE analysis of structure, first make it discrete, that is, the whole structure is divided into finite units, connected with joints between units and between element and boundary, according to the requirements for energy efficiency, different precisions for different solutions to be found.

In case of structural discretion, attention must be paid to the followings:

①Selection of element type includes element shape, number of joints, number of degrees of freedom for joint etc. Normally, an element may be triangular, rectangular, and quadrangular. Geometrically, a triangular element is highly flexible, more approximate to boundary. It is necessary to avoid a narrow-shaped triangle. A rectangular element has second-order interpolation function of element, of which elements are under stress and strain being linear functions, with higher accuracy but worse boundary approximation, and low flexibility of geometrical division. A quadrangular element concurrently has advantages of both triangular and rectangular one, having high flexibility and high accuracy. However, the internal angle of an element cannot be too small or too large. Otherwise, the accuracy of result obtained may be possibly affected.

②An element shall be divided on a regular basis and at proper spacing for easy computation and automatic generation of network and good to future networking at less spacing.

③An element shall be made of the same material. At a point where the material has sharp change in thickness and elastic modulus, it shall be considered as the edge of the element, not allowing that the element crosses the point with sharp change. Basically, it is required that neither superposition nor spacing can be found between units.

(2) Carrying out Analysis for Element

The element analysis means that the discrete elements are considered as an object of research, to study the relationship of joint displacement vs. joint force, including:

①Determining Element Displacement Mode

For displacement-based FE method, the element displacement mode means that the displacement of an arbitrary point in the element is computed by joint displacement, that is, the element displacement is expressed as a function to joint displacement. Reasonable assumption of displacement function will have direct influence on computing accuracy, efficiency, and reliability of FE analysis.

②Analyzing Element Characteristics

After establishment of element displacement, the relationship of the element rod end force vs. rod end displacement is set up to attain an element stiffness matrix in virtual displacement principle and minimum potential energy principle based on relationship among stress, strain, and displacement. In this step, the load applied on the element must be equivalent to the load on the joint. Actually, element analysis is a process establishing the element stiffness matrix and equivalent joint load matrix.

(3) Integral Analysis

After the stiffness equation of each element has been determined, the elements may be assembled as an integral structure for analysis, establishing the equations for joint balance of the integral structure, i. e. integral stiffness equation. Then, the structural boundary condition is introduced to solve the equations, to obtain the joint displacement and further internal force and deformation of each element.

1.3.4 Engineering Applications of FEM

According to analysis process and solution nature of FE, its engineering applications include:

(1) Balanced application independent on time, i. e. steady application.

(2) Instantaneous application dependent on time.

(3) Characteristic value application as the promotion of the balanced application in solid mechanics and fluid mechanics.

In structural engineering, usually the beams and plate shells are structurally analyzed in FEM and complicate structures are analyzed for 2D and 3D stress, studying spreading characteristics of stress wave, dynamic response of structures to non-cyclic loads, analyzing structural stability, and studying the inherent frequency and vibration mode etc.

In foundation works, the FEM is used to study placement, excavation, side slope stability, interaction between soil and structure, to analyze stress of dams, tunnels, and drill holes etc. and to study the dynamic interaction between soil and structure, spreading of stress wave in soil and rock.

In water conservancy projects, the FEM is used to study potential flow and viscous flow of fluid, steady and unsteady seepage of water reservoir and porous medium, hydraulic structures and dams, stable seepage of fluid in soil and rock, spreading of wave in fluid, and diffuse of pollution etc.

Since FEM incorporated with computer software, it has been further developed. Especially, its application integrated with CAD system dramatically and substantially makes design level improved, which can not only optimize design and reduce material consumption and costs, but also find some potential engineering problems to avoid occurrence of accidents.