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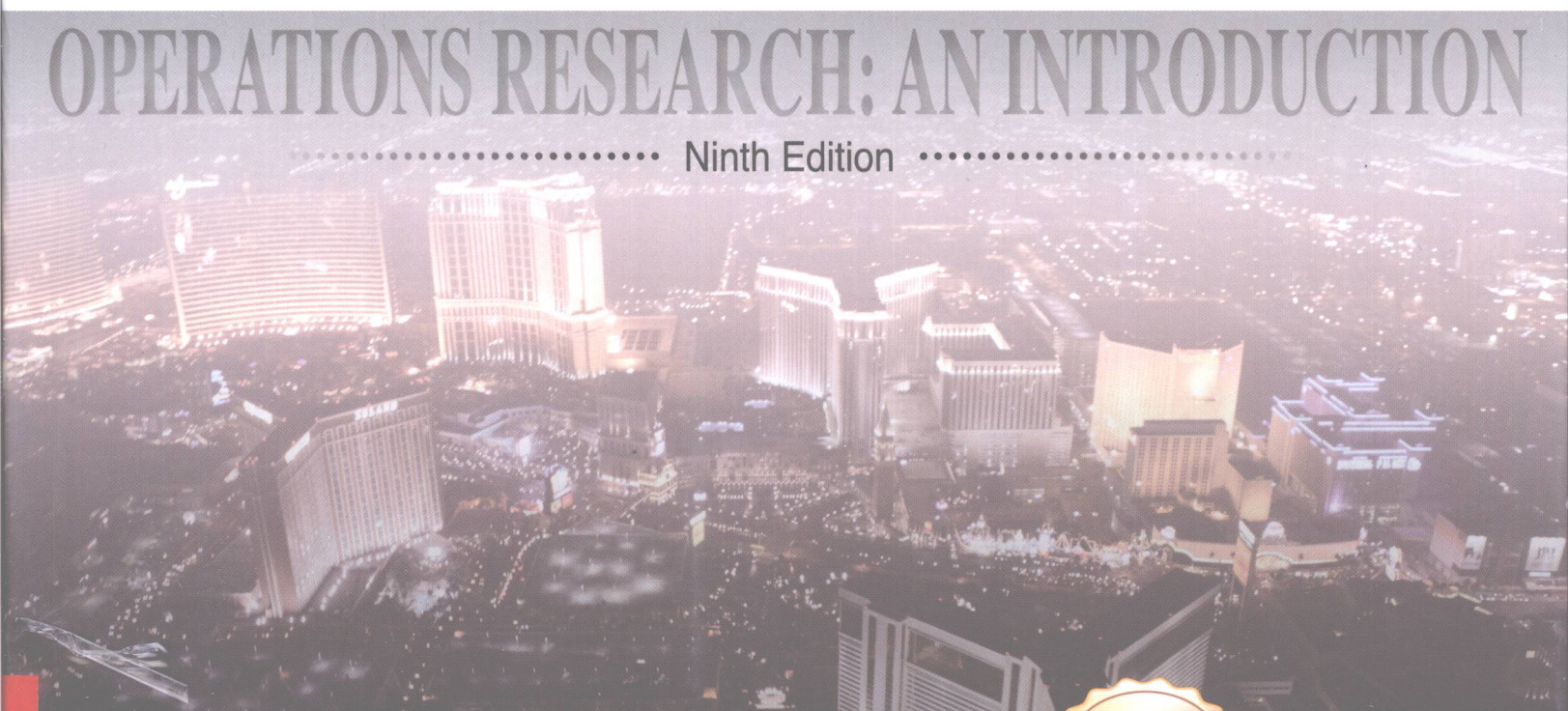
运筹学导论

英文版 · 第9版 · 提高篇

哈姆迪·A·塔哈 (Hamdy A. Taha) 著

OPERATIONS RESEARCH: AN INTRODUCTION

..... Ninth Edition



中国人民大学出版社



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OPERATIONS RESEARCH: AN INTRODUCTION

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总 序

随着我国加入 WTO，越来越多的国内企业参与到国际竞争中来，用国际上通用的语言思考、工作、交流的能力也越来越受到重视。这样一种能力也成为我国各类人才参与竞争的一种有效工具。国家教育机构、各类院校以及一些主要的教材出版单位一直在思考，如何顺应这一发展潮流，推动各层次人员通过学习来获取这种能力。双语教学就是这种背景下的一种尝试。

双语教学在我国主要指汉语和国际通用的英语教学。事实上，双语教学在我国教育界已经不是一个陌生的词汇了，以双语教学为主的科研课题也已列入国家“十五”规划的重点课题。但从另一方面来看，双语教学从其诞生的那天起就被包围在人们的赞成与反对声中。如今，依然是有人赞成有人反对，但不论是赞成居多还是反对占上，双语教学的规模和影响都在原有的基础上不断扩大，且呈大发展之势。一些率先进行双语教学的院校在实践中积累了经验，不断加以改进；一些待进入者也在模仿中学习，并静待时机成熟时加入这一行列。由于我国长期缺乏讲第二语言（包括英语）的环境，开展双语教学面临特殊的困难，因此，选用合适的教材就成为双语教学成功与否的一个重要问题。我们认为，双语教学从一开始就应该使用原版的各类学科的教材，而不是由本土教师自编的教材，从而可以避免中国式英语问题，保证语言的原汁原味。各院校除应执行国家颁布的教学大纲和课程标准外，还应根据双语教学的特点和需要，适当调整教学课时的设置，合理选择优秀的、合适的双语教材。

顺应这样一种大的教育发展趋势，中国人民大学出版社同众多国际知名的大出版公司，如麦格劳·希尔出版公司、培生教育出版公司等合作，面向大学本科生层次，遴选了一批国外最优秀的管理类原版教材，涉及专业基础课，人力资源管理、市场营销及国际化管理等专业方向课，并广泛听取有着丰富的双语一线教学经验的教师的建议和意见，对原版教材进行了适当的改编，删减了一些不适合我国国情和不适合教学的内容；另一方面，根据教育部对双语教学教材篇幅合理、定价低的要求，我们更是努力区别于目前市场上形形色色的各类英文版、英文影印版的大部头，将目标受众锁定在大学本科生层次。本套教材尤其突出了以下一些特点：

- 保持英文原版教材的特色。本套双语教材根据国内教学实际需要，对原书进行了一定的改编，主要是删减了一些不适合教学以及不符合我国国情的内容，但在体系结构和内容特色方面都保持了原版教材的风貌。专家们的认真改编和审定，使本套教材既保持了学术上的完整性，又贴近中国实际；既方便教师教学，又方便学生理解和掌握。

● 突出管理类专业教材的实用性。本套教材既强调学术的基础性，又兼顾应用的广泛性；既侧重让学生掌握基本的理论知识、专业术语和专业表达方式，又考虑到教材和管理实践的紧密结合，有助于学生形成专业的思维能力，培养实际的管理技能。

● 体系经过精心组织。本套教材在体系架构上充分考虑到当前我国在本科教育阶段推广双语教学的进度安排，首先针对那些课程内容国际化程度较高的学科进行双语教材开发，在其专业模块内精心选择各专业教材。这种安排既有利于我国教师摸索双语教学的经验，使得双语教学贴近现实教学的需要；也有利于我们收集关于双语教学教材的建议，更好地推出后续的双语教材及教辅材料。

● 篇幅合理，价格相对较低。为适应国内双语教学内容和课时上的实际需要，本套教材进行了一定的删减和改编，使总体篇幅更为合理；而采取低定价，则充分考虑到了学生实际的购买能力，从而使本套教材得以真正走近广大读者。

● 提供强大的教学支持。依托国际大出版公司的力量，本套教材为教师提供了配套的教辅材料，如教师手册、PowerPoint 讲义、试题库等，并配有内容极为丰富的网络资源，从而使教学更为便利。

本套教材是在双语教学教材出版方面的一种尝试。我们在选书、改编及出版的过程中得到了国内许多高校的专家、教师的支持和指导，在此深表谢意。同时，为使后续推出的教材更适于教学，我们也真诚地期待广大读者提出宝贵的意见和建议。需要说明的是，尽管我们在改编的过程中已加以注意，但由于各教材的作者所处的政治、经济和文化背景不同，书中内容仍可能有不妥之处，望读者在阅读时注意比较和甄别。

徐二明

中国人民大学商学院

What's New in the Ninth Edition *

The ninth edition continues to streamline both the text materials and the software support providing a broad focus on algorithmic and practical implementation of Operations Research techniques.

- For the first time in this book, the new Section 3.7 of volume one provides a comprehensive (math-free) framework of how the different LP algorithms (simplex, dual simplex, revised simplex, and interior point) are implemented in commercial codes (e.g., CPLEX and XPRESS) to provide the computational speed and accuracy needed to solve very large problems.
- The new Chapter 2 covers efficient heuristics/metaheuristics designed to find good approximate solutions for integer and combinatorial programming problems. The need for heuristics/metaheuristics is in recognition of the fact that the performance of the exact algorithms has been less than satisfactory from the computational standpoint.
- The new Chapter 3 is dedicated to the important traveling salesperson problem. The presentation includes a variety of applications and the development of exact and heuristic solution algorithms.
- All the algorithms in the new Chapters 2 and 3 are coded in Excel in a manner that permits convenient interactive experimentation with the models.
- All detailed AMPL models have been moved to Appendix C to complement the AMPL syntactical rules presented in the appendix. The models are cross-referenced opportunely in the book.
- Numerous new problems have been added throughout the book.
- The TORA software has been updated.
- In keeping with my commitment to maintain a reasonable count of printed pages, I found it necessary to move some material to the website, including the AMPL appendix.

* 《运筹学导论》（第9版）原书篇幅过大，英文版拆分为两册，即基础篇和提高篇，章节顺序作了相应调整，但网上（www.pearsonhighered.com/taha）文件保持原状，对文件序号未作更改。

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CHAPTER 1

Advanced Linear Programming

Real-Life Application—Optimal Ship Routing and Personnel Assignment for Naval Recruitment in Thailand

Thailand Navy recruits are drafted four times a year. A draftee reports to 1 of 34 local centers and is then transported by bus to one of four navy branch bases. From there, recruits are transported to the main naval base by ship. The docking facilities at the branch bases may restrict the type of ship that can visit each base. Branch bases have limited capacities but, as a whole, the four bases have sufficient capacity to accommodate all the draftees. During the summer of 1983, a total of 2929 draftees were transported from the drafting centers to the four branch bases and eventually to the main base. The problem deals with determining the optimal schedule for transporting the draftees, first from the drafting centers to the branch bases and then from the branch bases to the main base. The study uses a combination of linear and integer programming. The details are given in Case 5, Chapter 26, on the website.

1.1 SIMPLEX METHOD FUNDAMENTALS

In linear programming, the feasible solution space forms a **convex set** if the line segment joining any two *distinct* feasible points also falls in the set. An **extreme point** of the convex set is a feasible point that cannot lie on a line segment joining any two *distinct* feasible points in the set. Actually, extreme points are the same as corner points, as used in Chapters 2, 3, and 4 of volume one.

Figure 1.1 illustrates two sets. Set (a) is convex (with six extreme points), and set (b) is not.

The graphical LP solution given in Section 2.3 of volume one demonstrates that the optimum solution is always associated with a feasible extreme (corner) point of the solution space. This result makes sense intuitively, because every feasible point in the LP solution space can be determined as a function of its feasible extreme points. For

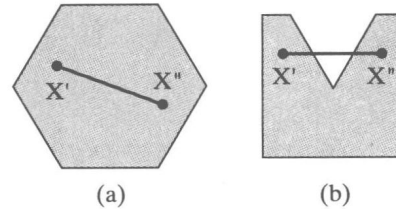


FIGURE 1.1
Examples of a convex and a nonconvex set

example, in convex set (a) of Figure 1.1, a **convex combination** of the extreme points, $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_5,$ and \mathbf{X}_6 , identifies any feasible point \mathbf{X} as

$$\begin{aligned} \mathbf{X} &= \alpha_1\mathbf{X}_1 + \alpha_2\mathbf{X}_2 + \alpha_3\mathbf{X}_3 + \alpha_4\mathbf{X}_4 + \alpha_5\mathbf{X}_5 + \alpha_6\mathbf{X}_6 \\ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 &= 1 \\ \alpha_i &\geq 0, i = 1, 2, \dots, 6 \end{aligned}$$

This observation shows that a finite number of extreme points completely define the infinite number of points in the solution space. This result is the crux of the simplex method.

Example 1.1-1

Show that the following set is convex:

$$C = \{(x_1, x_2) | x_1 \leq 2, x_2 \leq 3, x_1 \geq 0, x_2 \geq 0\}$$

Let $\mathbf{X}_1 = \{x'_1, x'_2\}$ and $\mathbf{X}_2 = \{x''_1, x''_2\}$ be any two distinct points in C . If C is convex, then $\mathbf{X} = (x_1, x_2) = \alpha_1\mathbf{X}_1 + \alpha_2\mathbf{X}_2, \alpha_1 + \alpha_2 = 1, \alpha_1, \alpha_2 \geq 0$, must also be in C . To show that this is true, we need to show that all the constraints of C are satisfied by the line segment \mathbf{X} —that is,

$$\left. \begin{aligned} x_1 &= \alpha_1x'_1 + \alpha_2x''_1 \leq \alpha_1(2) + \alpha_2(2) = 2 \\ x_2 &= \alpha_1x'_2 + \alpha_2x''_2 \leq \alpha_1(3) + \alpha_2(3) = 3 \end{aligned} \right\} \Rightarrow x_1 \leq 2, x_2 \leq 3$$

Additionally, the nonnegativity conditions are satisfied because α_1 and α_2 are nonnegative.

PROBLEM SET 1.1A

1. Show that the set $Q = \{x_1, x_2 | x_1 - x_2 \leq 3, x_1 \geq 0, x_2 \geq 0\}$ is convex. Is the non-negativity condition essential for the proof?
- *2. Show that the set $Q = \{x_1, x_2 | x_1 \geq 1 \text{ or } x_2 \geq 2\}$ is not convex.
3. Determine graphically the extreme points of the following convex set:

$$Q = \{x_1, x_2 | x_1 + x_2 \leq 2, x_1 \geq 0, x_2 \geq 0\}$$

Show that the entire feasible solution space can be determined as a convex combination of its extreme points. Hence conclude that any convex (bounded) solution space is totally defined once its extreme points are known.

4. In the solution space in Figure 1.2 (drawn to scale), express the interior point $(3, 1)$ as a convex combination of the extreme points $A, B, C,$ and D by determining the weights associated with each extreme point.

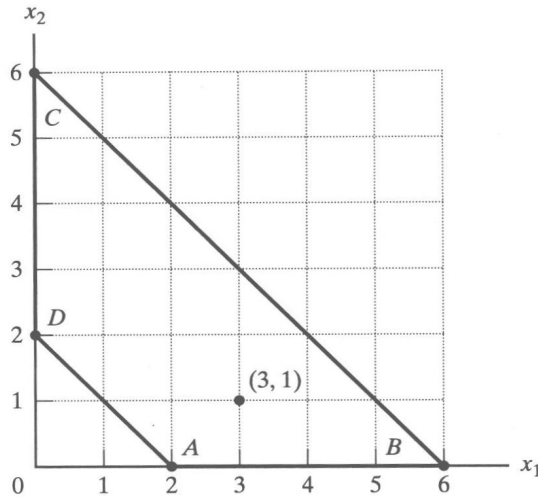


FIGURE 1.2
Solution space for Problem 4, Set 7.1a

1.1.1 From Extreme Points to Basic Solutions

It is convenient to express the general LP problem in equation form (see Section 3.1 of volume one) using matrix notation.¹ Define \mathbf{X} as an n -vector representing the variables, \mathbf{A} as an $(m \times n)$ -matrix representing the constraint coefficients, \mathbf{b} as a column vector representing the right-hand side, and \mathbf{C} as an n -vector representing the objective-function coefficients. The LP is then written as

$$\text{Maximize or minimize } z = \mathbf{CX}$$

subject to

$$\mathbf{AX} = \mathbf{b}$$

$$\mathbf{X} \geq \mathbf{0}$$

Using the format, the rightmost m elements of \mathbf{X} represent the starting basic variables. Hence, the rightmost m columns of \mathbf{A} always form an identity matrix \mathbf{I} .

A **basic solution** of $\mathbf{AX} = \mathbf{b}$ is determined by setting $n - m$ variables equal to zero, and then solving the resulting m equations in the remaining m unknowns, *provided that the resulting solution is unique*. Given this definition, the theory of linear programming establishes the following result between the geometric definition of extreme points and the algebraic definition of basic solutions:

$$\text{Extreme points of } \{\mathbf{X} \mid \mathbf{AX} = \mathbf{b}\} \Leftrightarrow \text{Basic solutions of } \mathbf{AX} = \mathbf{b}$$

The relationship means that the extreme points of the LP solution space are defined by the basic solutions of $\mathbf{AX} = \mathbf{b}$, and vice versa. Thus, the basic solutions of $\mathbf{AX} = \mathbf{b}$ provide all the information needed to determine the optimum solution of the LP problem. Furthermore, the nonnegativity restriction, $\mathbf{X} \geq \mathbf{0}$, limit the search for the optimum to the *feasible* basic solutions only.

¹A review of matrix algebra is given in Appendix D on the website.

To formalize the definition of a basic solution, the system $\mathbf{AX} = \mathbf{b}$ is written in vector form as

$$\sum_{j=1}^n \mathbf{P}_j x_j = \mathbf{b}$$

The vector \mathbf{P}_j is the j th column of \mathbf{A} . A subset of m vectors forms a **basis**, \mathbf{B} , if, and only if, the selected m vectors are **linearly independent**. In this case, the matrix \mathbf{B} is **nonsingular**. Defining \mathbf{X}_B as an m -vector of the basic variables, then

$$\mathbf{BX}_B = \mathbf{b}$$

Using the inverse \mathbf{B}^{-1} , the associated basic solution is

$$\mathbf{X}_B = \mathbf{B}^{-1}\mathbf{b}$$

If $\mathbf{B}^{-1}\mathbf{b} \geq \mathbf{0}$, then \mathbf{X}_B is feasible. The remaining $n - m$ variables are **nonbasic** at zero level.

The previous result shows that in a system of m equations and n unknowns, the *maximum* number of (feasible and infeasible) basic solutions is $\binom{n}{m} = \frac{n!}{m!(n-m)!}$.

Example 1.1-2

Determine all the basic feasible and infeasible solutions of the following system of equations.

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

The following table summarizes the results. The inverse of \mathbf{B} is determined by one of the methods in Section D.2.7 on the website.

\mathbf{B}	$\mathbf{BX}_B = \mathbf{b}$	Solution	Type
$(\mathbf{P}_1, \mathbf{P}_2)$	$\begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$	$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{4} & -\frac{1}{8} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{7}{4} \\ \frac{3}{4} \end{pmatrix}$	Feasible
$(\mathbf{P}_1, \mathbf{P}_3)$	(Not a basis because \mathbf{P}_1 and \mathbf{P}_3 are dependent)		
$(\mathbf{P}_2, \mathbf{P}_3)$	$\begin{pmatrix} 3 & -1 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$	$\begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{3}{8} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ -\frac{7}{4} \end{pmatrix}$	Infeasible

We can also investigate the problem by expressing it in vector form as follows:

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} x_1 + \begin{pmatrix} 3 \\ -2 \end{pmatrix} x_2 + \begin{pmatrix} -1 \\ -2 \end{pmatrix} x_3 = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

The two-dimensional vectors $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$, and \mathbf{b} can be represented generically as $(a_1, a_2)^T$. Figure 1.3 graphs these vectors on the (a_1, a_2) -plane. For example, for $\mathbf{b} = (4, 2)^T$, $a_1 = 4$, and $a_2 = 2$.

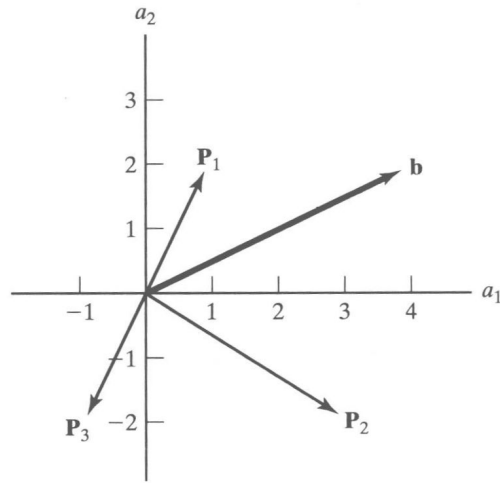


FIGURE 1.3
Vector representation of LP solution space

Because we are dealing with two equations ($m = 2$), a basis includes exactly two vectors, selected from among \mathbf{P}_1 , \mathbf{P}_2 , and \mathbf{P}_3 . The matrices $(\mathbf{P}_1, \mathbf{P}_2)$ and $(\mathbf{P}_2, \mathbf{P}_3)$ form bases because their associated vectors are independent. On the other hand, the vectors of the matrix $(\mathbf{P}_1, \mathbf{P}_3)$ are dependent, and hence the matrix is not a basis.

Algebraically, a (square) matrix forms a basis if its determinant is not zero (see Section D.2.5 on the website). The following computations show that the combinations $(\mathbf{P}_1, \mathbf{P}_2)$ and $(\mathbf{P}_2, \mathbf{P}_3)$ are bases, and the combination $(\mathbf{P}_1, \mathbf{P}_3)$ is not.

$$\det(\mathbf{P}_1, \mathbf{P}_2) = \det\begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix} = (1 \times -2) - (2 \times 3) = -8 \neq 0$$

$$\det(\mathbf{P}_2, \mathbf{P}_3) = \det\begin{pmatrix} 3 & -1 \\ -2 & -2 \end{pmatrix} = (3 \times -2) - (-1 \times -2) = -8 \neq 0$$

$$\det(\mathbf{P}_1, \mathbf{P}_3) = \det\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} = (1 \times -2) - (-1 \times 2) = 0$$

PROBLEM SET 1.1B

- In the following sets of equations, (a) and (b) have unique (basic) solutions, (c) has an infinite number of solutions, and (d) has no solution. Show how these results can be verified using graphical vector representation. From this exercise, state the general conditions for vector dependence/independence that lead to unique solution, infinity of solutions, and no solution.

(a) $x_1 + 3x_2 = 2$ (b) $2x_1 + 3x_2 = 1$

$3x_1 + x_2 = 3$ $2x_1 - x_2 = 2$

(c) $2x_1 + 6x_2 = 4$ (d) $2x_1 - 4x_2 = 2$

$x_1 + 3x_2 = 2$ $-x_1 + 2x_2 = 1$

- Use vectors to determine graphically the type of solution for each of the sets of equations below: unique solution, an infinite number of solutions, or no solution. For