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# 数学专业英语

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# 前 言

数学专业英语的学习既是基础英语学习的延伸,又是基础英语与专业知识的结合及实践。目前,国内许多高校数学类专业都开设有专业双语(英语)课程,但大都是针对某一具体课程,而针对数学类专业的基础数学专业英语教材很少,且多限于初等和古典数学内容,涉及深度不够,内容不够全面。因此,有必要进行总结并形成一本适用于数学类各专业使用的较全面、实用的教材。

本书的特色如下:(1)选材新。更多的涉及目前数学类各方向的相关题材,同时提供一定量的“新知识”。(2)内容广。不仅涉及数学专业类的语法、词汇,注重数学基础英语,附录部分提供较多的专业类网站和国外主流数学杂志,让学生更好地把握住专业问题的发展前沿。(3)使用面较宽。适用于数学类各专业方向的不同年级学生及工科学生。(4)注重应用。不仅介绍了专业的英语知识,同时引入一定量的应用实例,更好地体现专业的发展和应用。

本书在编者多年的教学实践基础上,参考中英文数学教材、英文数学专著、英文数学工具书及数学家的演讲稿等选取编写内容。通过本书学习,可以使学生了解数学专业英语的特点,掌握数学文章的文体、词法、词汇、表达等方面的知识,能够熟练阅读国外相关专业文献,培养学生利用英语交流、获取信息的能力和习惯,提高阅读、理解数学英文文章的能力。

本书使用英文系统介绍了基础数学理论中的高等数学知识,包括9个单元。1~5单元主要涉及一元函数和多元函数及其连续性、导数和偏导数、定积分和二重积分、三重积分和曲线积分、数列和级数、常系数线性微分方程和变系数线性微分方程。6~7单元主要涉及线性方程组及其解法、平面几何及空间几何、二次曲面。8~9单元主要涉及随机变量及其分布、随机变量的数字特征、大数定律和中心极限定理、参数估计和假设检验。每个单元配有单词和短语注解、练习题,并附有相关的阅读材料。此外,附录部分列出常用的数学词汇、数学符号的读法、数学学术资源网址,以便于读者了解和查询。

本书不仅可作为数学学科各专业的本科生、研究生教材使用,也适合工科及其他学科领域的学生、教师和科研人员参考和借鉴。

限于作者水平,错漏之处在所难免,欢迎广大读者批评指正!

编 者

2014年6月

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# Unit 1 Functions and Transformations

## 1.1 Sets

By a set, we mean a collection of objects. The objects that are in the set are known as elements or members of the set. If  $x$  is an element of a set  $A$ , we write  $x \in A$ . This can also be read as " $x$  is a member of the set of  $A$ " or " $x$  belongs to  $A$ " or " $x$  is in  $A$ ". The notation  $x \notin A$  means  $x$  is not an element of  $A$ .

Set  $B$  is called a subset of set  $A$ , if and only if  $x \in B$  implies  $x \in A$ . To indicate that  $B$  is a subset of  $A$ , we write  $B \subset A$ . This expression can also be read as " $B$  is contained in  $A$ " or " $A$  contains  $B$ ". In other words,  $A = B$  if and only if  $B \subset A$  and  $A \subset B$ .

The set of elements common to two sets  $A$  and  $B$  is called the intersection of  $A$  and  $B$  denoted by  $A \cap B$ .

Thus

$$x \in A \cap B \text{ if and only if } x \in A \text{ and } x \in B.$$

If the sets  $A$  and  $B$  have no elements in common, we say  $A$  and  $B$  are disjoint, and write  $A \cap B = \emptyset$ . The set  $\emptyset$  is called the empty set.

The union of sets  $A$  and  $B$ , written as  $A \cup B$ , is the set of elements that are either in  $A$  or in  $B$ . This does not exclude objects that are elements of both  $A$  and  $B$ .

Finally we introduce the set difference of two sets  $A$  and  $B$ :

$$A - B = \{x : x \in A, x \notin B\}.$$

The set of real numbers is denoted by  $\mathbf{R}$ . The set of rational numbers is denoted by  $\mathbf{Q}$ . The set of integers is denoted by  $\mathbf{Z}$ . The set of natural numbers is denoted by  $\mathbf{N}$ . It is clear that  $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R}$ .

Among the most important subsets of  $\mathbf{R}$  are the intervals. We suppose that  $a$  and  $b$  are real numbers and that  $a < b$ ,

$$\begin{aligned} (a, b) &= \{x : a < x < b\}, [a, b] = \{x : a \leq x \leq b\}, (a, b] = \{x : a < x \leq b\}, \\ [a, b) &= \{x : a \leq x < b\}, (a, +\infty) = \{x : a < x\}, [a, +\infty) = \{x : a \leq x\}, \\ (-\infty, b) &= \{x : x < b\}, (-\infty, b] = \{x : x \leq b\}. \end{aligned}$$

## 1.2 Functions

Some examples of functions from science.

A body which moves under uniform acceleration  $a$ , starting with an initial velocity  $v_0$  has a

velocity at time  $t$  given by  $v = v_0 + at$ , and travels a distance given by  $s = vt + \frac{1}{2}at^2$ .

The period  $T$  of a pendulum of length  $l$  is given by  $T = 2\pi \frac{l}{g}$ , and  $g$  is the acceleration due to gravity.

In these examples we have a quantity depending on another quantity or a relation between quantities.

A function is a relation for each  $x$ -value there is a unique  $y$ -value of the ordered pair. These means that if  $(x, y)$  and  $(x, z)$  are ordered pairs of a function then  $y = z$ .

Functions are usually denoted with lower case letters such as  $f, g, h$ .

e. g.

$$f(x) = 3x + 2, y = 2x - 1$$

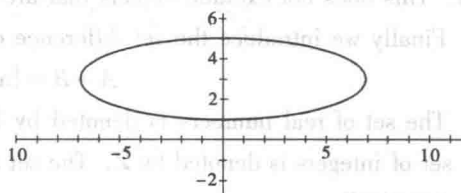
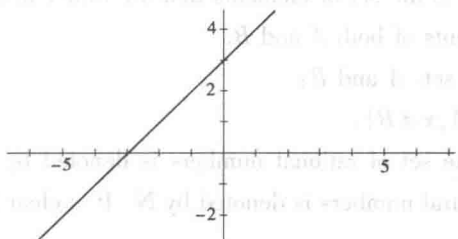
Note that the notation  $f: X \rightarrow Y$ ,  $X$  is the domain but  $Y$  is not necessarily the range. We write domain of  $f$  as  $\text{dom}f$  and range of  $f$  as  $\text{ran}f$ . We call  $y$  the dependent variable and  $x$  the independent variable.

One way to identify if a relation is a function is to draw a graph of the relation and apply the vertical line test.

If a vertical line can be drawn anywhere on the graph and it only ever intersects the graph a maximum of once, the relation is a function.

$y = x + 3$  is a function.

$$\frac{x^2}{49} + \frac{(y-3)^2}{4} = 1 \text{ is not a function.}$$



When the domain of a function is not explicitly stated, it is assumed to consist of all real numbers for which the rule has a meaning. We refer to the implied (maximal) domain of a function, because the domain is implied by the rule.

**Example 1-1** Determine the domain and the range of the function defined by the equation  $f(x) = \sqrt{1 - x^2}$ .

**Solution** Since  $\sqrt{1 - x^2}$  is defined as long as  $1 - x^2 \geq 0$ , that is  $-1 \leq x \leq 1$ , so the domain of this function is the closed interval  $[-1, 1]$ , the corresponding range is the interval  $[0, +\infty)$ .

## 1.3 Elementary Functions

### 1.3.1 Inverse Functions

If  $f$  is a one-to-one function, then for each number  $y$  in the range of  $f$  there is exactly one number  $x$  in the domain of  $f$  such that  $f(x) = y$ . Thus, a new function  $f^{-1}$  called the inverse of  $f$  may be defined by

$$f^{-1}(y) = x \text{ if } f(x) = y, \text{ for } x \in \text{ran} f, y \in \text{dom} f.$$

A function has an inverse function if and only if it is one-to-one relation.

It is not difficult to see what the relation between  $f$  and  $f^{-1}$  means geometrically. The point  $(x, y)$  is on the graph of  $f^{-1}$  if the point  $(y, x)$  is on the graph of  $f$ . Therefore to get the graph of  $f^{-1}$  from the graph of  $f$ , the graph of  $f$  is to be reflected in the line  $y = x$ .

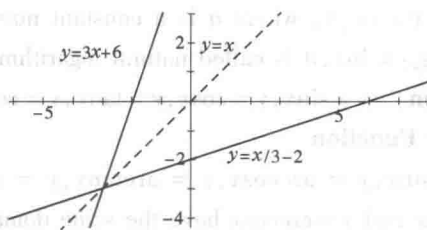
**Example 1-2** Find the inverse function of the function  $f(x) = 3x + 6$  and sketch the graph of  $y = f(x)$  and  $y = f^{-1}(x)$  on the one set of axes.

**Solution** Consider the function  $y = 3x + 6$ , we solve for  $x$  in terms of  $y$ ,

$$x = \frac{1}{3}y - 2$$

Interchanging  $x$  and  $y$  yields a customary notation, so the inverse function is

$$y = \frac{1}{3}x - 2$$



**Example 1-3** Let  $f: [-3, -5] \rightarrow \mathbb{R}, f(x) = (x+2)^2 - 1$ . Find  $f^{-1}$  and state its domain and range.

**Solution** The inverse function has rule

$$x + 2 = \pm \sqrt{y + 1}, \text{ dom} f = [-3, -5],$$

Hence

$$x = -\sqrt{y + 1} - 2$$

Interchanging  $x$  and  $y$ , hence the inverse function is

$$f^{-1}(x) = -\sqrt{x + 1} - 2, \text{ dom} f = [0, 8]$$

$$\text{i. e. } f^{-1}: [0, 8] \rightarrow [-3, -5], f^{-1}(x) = -\sqrt{x + 1} - 2.$$

### 1.3.2 Composite Functions

If  $y = f(x)$  and  $z = g(y)$ , then in the expression for  $z$  as a function of  $y$ , we can substitute the expression for  $y$  as a function of  $x$  to obtain an expression for  $z$  as a function of  $x$ . We can write  $z = g[f(x)]$  to indicate this, e. g. let consider the function  $y = 2x - 3$  and  $z = y^2 - 5$ , and then  $z = (2x - 3)^2 - 5$ , which is  $g[f(x)] = (2x - 3)^2 - 5$ .

We also write  $g \circ f$  to indicate the composed function, thus

$$g \circ f = g[f(x)]$$

Note:

(1) If we reverse the order of composition of the functions, we generally obtain a different function, e. g. we have

$$g \circ f = g[f(x)] = g(2x - 3) = (2x - 3)^2 - 5$$

and

$$f \circ g(x) = f[g(x)] = f(x^2 - 5) = 2(x^2 - 5) - 3 = 2x^2 - 13$$

(2) It is not true that there is a composite function for any given two functions, e. g.  $y = -x^2 - 1$  and  $z = \sqrt{y} - 5$ , then  $z = \sqrt{-x^2 - 1} - 5$ , there is no value for  $z$  in the domain of  $\mathbf{R}$ .

### 1.3.3 The Basic Elementary Functions

The basic elementary functions include the following functions,

**Power Function**  $y = x^a$ , where  $a$  is a constant number of  $\mathbf{R}$ .

**Exponential Function**  $y = a^x$ , where  $a$  is a constant number of  $\mathbf{R}$  and  $a > 0, a \neq 1$ .

**Logarithmic Function**  $y = \log_a x$ , where  $a$  is a constant number of  $\mathbf{R}$  and  $a > 0, a \neq 1$ .

When  $a = e$ , we write  $y = \log_e x = \ln x$ , it is called natural logarithmic function.

**Trigonometric Function**  $y = \sin x, y = \cos x, y = \tan x, y = \cot x$ .

**Inverse Trigonometric Function**

$$y = \arcsin x, y = \arccos x, y = \arctan x, y = \operatorname{arccot} x.$$

The functions  $y = \arcsin x$  and  $y = \arccos x$  have the same domain

$$-1 \leq x \leq 1.$$

The ranges are  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$  for  $y = \arcsin x$  and  $0 \leq y \leq \pi$  for  $y = \arccos x$ .

The functions  $y = \arctan x$  and  $y = \operatorname{arccot} x$  have the same domain,

$$-\infty < x < +\infty.$$

The ranges are  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$  for  $y = \arctan x$  and  $0 \leq y \leq \pi$  for  $y = \operatorname{arccot} x$ .

### 1.3.4 Elementary Functions

The functions that are made up of the basic elementary functions and constants by a finite number of the four arithmetic operations and a finite number of the composition and can be represented by a single formula are called elementary functions.

## 1.4 Linear Functions

### 1.4.1 Equation of a Straight Line

An equation like  $y = 3x + 2$  is a more convenient way of describing the relationship between two variables, it is called a linear function because it involves a linear expression  $3x + 2$  and its graph on the number plane is a straight line.

A linear function has the form  $f(x) = mx + c$ , where  $m$  and  $c$  are constants.

The constant  $m$  is the gradient of the straight line, it is the rate of change of  $y$  relative to  $x$ , demonstrating that  $y$  changes at a steady rate.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Through any two points it is only possible to draw a single straight line. Hence given any two points on the line  $(x_1, y_1)$  and  $(x_2, y_2)$ , the gradient of the line can be found as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Example 1-4** Find the equation of the line that passes through the points  $(1, 2)$  and  $(-1, 3)$ .

**Solution**

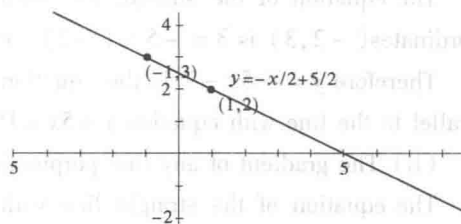
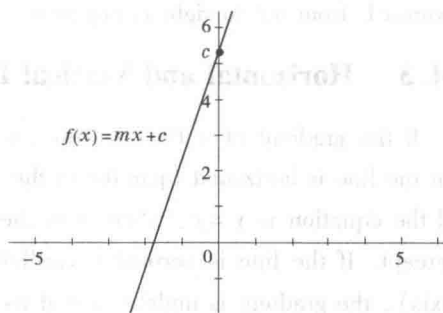
$$\text{The gradient} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{-1 - 1} = -\frac{1}{2}$$

Hence the equation is  $y = -\frac{1}{2}x + c$ .

Substituting the coordinates of the point

$$(1, 2) \text{ into the equation, } 2 = -\frac{1}{2} \times 1 + c, c = \frac{5}{2}.$$

So the equation of the line is  $y = -\frac{1}{2}x + \frac{5}{2}$ .



### 1.4.2 Sketching Graphs

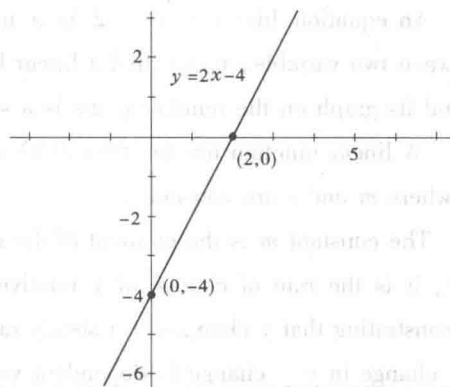
The intercept form  $ax + by = c$  is an alternative standard notation. While it is necessary to transpose the function into gradient form if you wish to find the gradient, it is often convenient to work with linear in the intercept form.

A convenient way to sketch graphs of straight lines is to use  $y$ -axis intercept (let  $x = 0$ ) and the  $x$ -axis intercept (let  $y = 0$ ). That is to plot the two axes intercepts.

**Example 1-5** Sketch the graph of  $y = 2x - 4$ , by first finding the intercepts.

**Solution** We let  $x = 0$ , then  $y$ -axis intercept is  $-4$ ; let  $y = 0$ , then the  $x$ -axis intercept is  $2$ . So the straight line goes through the two points  $(0, -4)$ ,  $(2, 0)$ .

It should be noted that the gradient of a line that slopes upwards from left to right is positive, and the gradient of a line that slopes downwards from left to right is negative.



### 1.4.3 Horizontal and Vertical Lines

If the gradient of a line is zero ( $m = 0$ ), then the line is horizontal (parallel to the  $x$ -axis) and the equation is  $y = c$ , where  $c$  is the  $y$ -axis intercept. If the line is vertical (parallel to the  $y$ -axis), the gradient is undefined and its rule is given as  $x = a$ , where  $a$  is the  $x$ -axis intercept. If the value of  $m$  is the same for two lines, then the lines are parallel. If two straight lines are perpendicular, the product of their gradients is  $-1$ , this result holds if one of the two lines is not parallel to an axis.

**Example 1-6** Find the equation of the straight line which passes through  $(-2, 3)$  and is

- Parallel to the line with equation  $y + 5x = 15$ .
- Perpendicular to the line with equation  $y + 5x = 15$ .

**Solution** The equation  $y + 5x = 15$  can be rearranged to  $y = -5x + 15$ .

Hence the gradient of the line can be seen to be  $-5$ .

- Therefore the gradient of any line parallel to this line is  $-5$ .

The equation of the straight line with this gradient and passing through the point with coordinates  $(-2, 3)$  is  $3 = -5 \times (-2) + c$ , so  $c = -7$ .

Therefore  $y = -5x - 7$  is the equation of the line which passes through  $(-2, 3)$  and is parallel to the line with equation  $y + 5x = 15$ .

- The gradient of any line perpendicular to the line with equation  $y + 5x = 15$  is  $0.2$ .

The equation of the straight line with this gradient and passing through the point with coordinates  $(-2, 3)$  is  $3 = 0.2 \times (-2) + c$ , so  $c = 3.4$ .

Therefore  $y = 0.2x + 3.4$  is the equation of the line which passes through  $(-2, 3)$  and is perpendicular to the line with equation  $y + 5x = 15$ .

### 1.4.4 Linear Models

In many practical situations a linear function can be used.

**Example 1-7** The following table shows the extension of a spring when weights are attached

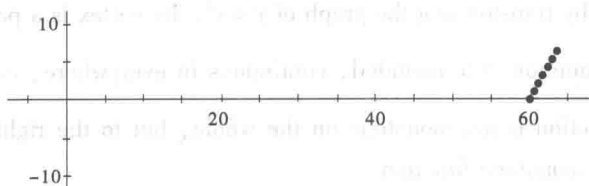
to it.

- Sketch a graph to show the relationship between  $w$  and  $x$  and find the linear function.
- What is the gradient represent?
- What will be the weight if the extension  $w = 8$  cm?

$x$ , weight(g)	60	60.5	61	61.5	62	62.5	63
$w$ , extension (cm)	0	1	2	3	4	5	6

### Solution

(a)



$$\text{The gradient} = \frac{1-0}{60.5-60} = 2$$

Hence the equation is  $w = 2x + c$ . Substituting the coordinates of the point  $(60, 0)$  into the equation,

$$0 = 2 \times 60 + c$$

$$c = -120$$

So the equation of the line is  $w = 2x - 120$ .

(b) The gradient 2 represents the elongation of spring for each gram added.

(c) When  $w = 8$  cm.

$$8 = 2x - 120$$

$$x = 64$$

Hence the weight is 64 g if the extension  $w = 8$  cm.

**Example1-8** There are two possible methods for paying gas bills.

Method A: A fixed charge of \$ 15 per quarter + 30 c per unit of gas used

Method B: A fixed charge of \$ 30 per quarter + 15 c per unit of gas used

Determine the number of units which must be used before method B becomes cheaper than method A.

**Solution** Let  $C$  = charge in \$ using method A,  $D$  = charge in \$ using method B,  $x$  = number of units of gas used.

$$\text{Now } C = 0.3x + 15; D = 0.15x + 30.$$

It can be seen from the graph that if the number of units exceeds 100, method B is cheaper.

## 1.5 Quadratic Functions

A quadratic function is defined by the general rule  $y = ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are constants and  $a \neq 0$ . It is called polynomial form. The graph is called a parabola.

By a process called completing the square all quadratics in polynomial form  $y = ax^2 + bx + c$  may be transposed into what will be called the turning point form  $y = a$

$(x - h)^2 + k$ . Graphs of the form  $y = a(x - h)^2 + k$  are formed by transforming the graph of  $y = x^2$ . Its vertex is a point with coordinates:

$\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$ , the function is unbounded, continuous in everywhere, even at  $b = c = 0$ , and non-periodic. The function is not monotone on the whole, but to the right or to the left of the vertex it behaves as a monotone function.

It is not essential to convert quadratics to turning point form in order to sketch the graph. We can find the  $x$ -axis and  $y$ -axis intercepts and the axis of symmetry from polynomial form by alternative methods and use these details to sketch the graph.

Let  $x = 0$  in the general equation  $y = ax^2 + bx + c$ , then we have  $y = c$ , i. e., the  $y$ -axis intercept is always equal to  $c$ . Let  $y = 0$ , then we have  $ax^2 + bx + c = 0$ . In order to solve such an equation, it is necessary to factorize the left-hand side and use the null factor theorem. Once the  $x$ -axis intercepts have been found, the turning point can be found by using the symmetry properties of the parabola.

**Example 1-9** Sketch the graph of  $y = x^2 + 2x$ .

**Solution**

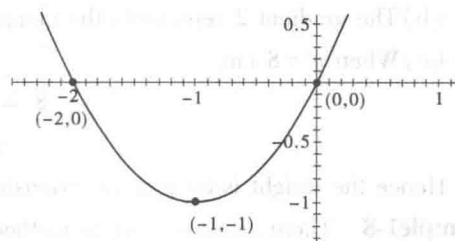
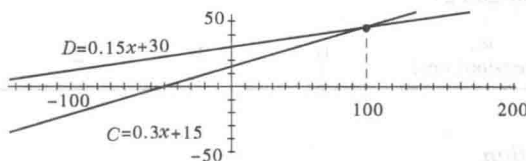
Let  $x = 0$ , Hence  $y$ -axis intercept is  $(0, 0)$ .

Let  $y = 0$  and factorize left-hand side equation,  $x^2 + 2x = 0$ , so  $x = 0$  or  $x = -2$ . Hence  $x$ -axis intercepts are  $(0, 0)$  and  $(-2, 0)$ .

Due to the symmetry of the parabola, the axis of symmetry will be the line bisecting the two  $x$ -axis intercepts. Hence axis of symmetry is the line with equation

$$x = \frac{0 + (-2)}{2} = -1, \text{ When } x = -1, y = -1.$$

So the turning point has coordinates  $(-1, -1)$ .



### 1.5.1 The Discriminant

The form and location of a quadratic parabola  $y = ax^2 + bx + c$  in a coordinate system

depends completely on two parameters: the coefficient  $a$  of  $x^2$  and discriminant  $\Delta = b^2 - 4ac$ . These properties follow from analysis of the quadratic equation roots.

(i) If the discriminant  $\Delta < 0$ , then the equation  $ax^2 + bx + c = 0$  has zero solutions and the corresponding parabola will have no  $x$ -axis intercepts.

(ii) If the discriminant  $\Delta = 0$ , then the equation  $ax^2 + bx + c = 0$  has one solution and the corresponding parabola will have one  $x$ -axis intercept. (We sometimes say the equation has two coincident solutions.)

(iii) If the discriminant  $\Delta > 0$ , then the equation  $ax^2 + bx + c = 0$  has two solutions and the corresponding parabola will have two  $x$ -axis intercepts.

### 1.5.2 Quadratic Models

**Model 1** Mary wishes to fence off a rectangular vegetable garden in her backyard. She has 40 m of fencing wire which she will use to fence three sides of the garden, with the existing timber fence forming the fourth side. Calculate the maximum area she can enclose.

**Solution** Let  $A$  = area of the rectangular garden,  $x$  = length of the garden.

So

$$\text{Width} = \frac{40 - x}{2}$$

Hence

$$\begin{aligned} A &= \left( \frac{40 - x}{2} \right) x \\ &= -\frac{1}{2}x^2 + 20x \end{aligned}$$

The maximum area is  $200 \text{ m}^2$  when  $x = 20 \text{ m}$ .

**Model 2** A cricket ball is thrown by a fielder. The path of the cricket ball is a parabola with the equation  $y = ax^2 + bx + c$ . It leaves his hand at a height of 2 m, above the ground and the wicketkeeper takes the ball 40 m away again at a height of 1 m. It is known that after the ball has gone 10 m, it is 4.75 m above the ground.

(a) Find the values of  $a$ ,  $b$ , and  $c$ .

(b) Find the maximum height of the ball above the ground.

(c) Find the height of the ball 5 m horizontally before it hits the wicketkeeper's gloves.

**Solution** (a) Because the points  $(0, 2)$ ,  $(40, 1)$ ,  $(10, 4.75)$  are on the parabola, we obtain three equations,

$$2 = c$$

$$1 = 1600a + 40b + c$$

$$4.75 = 100a + 10b + c$$

Hence  $a = -0.01$ ,  $b = 0.375$ ,  $c = 2$

(b) The equation of the parabola is  $y = -0.01x^2 + 0.375x + 2$ , so the turning point has

coordinates  $(18.75, 5.516)$ . The maximum height is 5.516 m when  $x = 18.75$  m.

(c) When  $x = 5$ ,  $y = 3.625$ . So the height of ball is 3.625 m when the ball 5 m horizontally before it hits the wicketkeeper's gloves.

## Words and Phrases

set 集,集合

subset 子集,子设备

intersection 交集,相交

disjoint 不相交的

intervals 区间

function 函数

domain 定义域

inverse 相反的,反向的

elementary 基本的,初级的,基础的

horizontal 水平线,水平面,横线,横切面

vertical 垂直线,垂直位置

gradient 梯度,坡度,斜率

vertex 最高点,顶点

parabola 抛物线

discriminant 判别式

## Exercise

1. A line has gradient 3 and passes through the points with coordinates  $(-1, 2)$  and  $(b, 4)$ . Please find the value of  $b$ .

2. A car starts from point  $A$  on a highway 10 km past the Wangaratta post office. The car travels at an average speed of 90 km/h towards picnic stop  $B$ , which is 120 km further on from  $A$ . Let  $t$  hours be the time after the car leaves point  $A$ .

(a) Find an expression for the distance  $d_1$  of the car from the post office at time  $t$  hours.

(b) Find an expression for the distance  $d_2$  of the car from point  $B$  at time  $t$  hours.

(c) On separate sets of axes sketch the graphs of  $d_1$  against  $t$  and  $d_2$  against  $t$  and state the gradient of each graph.

3. The reservoir feeding an intravenous drip contains 500 ml of a saline solution. The drip releases the solution into a patient at the rate of 2.5 mL/minute.

(a) Construct a rule which relates  $v$ , the amount of solution left in the reservoir, to time  $t$  minutes.

(b) State the possible values of  $t$  and  $v$ .

(c) Sketch the graph of the relation. Robyn and Cheryl race over 100 metres.

4. A parabola has vertex with coordinates  $(1, 3)$  and passes through the point with coordinates  $(2, 4)$ . Find the equation for the parabola.

5. A construction firm has won a contract to build cable-car pylons at various positions on the side of a mountain. Because of difficulties associated with construction in the Alpine areas, the construction firm will be paid an extra amount,  $C$  (\$), given by the formula  $C = 50h^2 + 120h$ , where  $h$  is the height in km above sea level.

- (a) Sketch the graph of  $C$  as a function of  $h$ . Comment on the possible values of  $h$ .
  - (b) Does  $C$  have a maximum value?
  - (c) What is the value of  $C$  for a pylon built at an altitude of 1 225 m?
6. Robyn runs so that it takes  $a$  seconds to run 1 metre and Cheryl runs so that it takes  $b$  seconds to run 1 metre. Cheryl wins the race by 1 second. The next day they again race over 100 metres but Cheryl gives Robyn a 5 metre start so that Robyn runs 95 metres. Cheryl wins this race by 0.4 second. Find the values of  $a$  and  $b$  and the speed at which Robyn runs.

## Reading Materials

### Math Study Skills: Active Study vs. Passive Study

**Be actively involved in managing the learning process, the mathematics and your study time.** Taking responsibility of studying, recognizing what you do and don't know, and knowing how to get your instructor to help you with what you don't know. Attend class every day and take complete notes. Instructors formulate test questions based on material and examples covered in class as well as on those in the text.

**Be an active participant in the classroom.** Get ahead in the book, try to work some of the problems before they are covered in the class, anticipate what the instructor's next step will be.

Ask questions in class! There are usually other students wanting to know the answers to the same questions you have. Go to office and ask questions, the instructor will be pleased to see that you are interested, and you will be actively helping yourself. Good study habits throughout the semester make it easier to study for tests.

Not too helpful comment, "I don't understand this section." The best you can expect in reply to such a remark is a brief review of the section, and this will likely overlook the particular thing(s) which you don't understand.

Good comment, "I don't understand why  $f(x+h)$  doesn't equal  $f(x) + f(h)$ ." This is a very specific remark that will get a very specific response and hopefully clear up your difficulty.

Good question, "How can you tell the difference between the equation of a circle and the equation of a line?"

Okay question, "How do you solve Q17?"

Better question, "Can you show me how to solve Q17?" (The Instructor can let you try to finish the problem on your own), or "This is how I tried to solve Q17. What went wrong?" The focus of attention is on your thought process.

Right after you get help with a problem, solve another similar problem by yourself.

**College Math is different from High School Math.** A college math class meets less