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Modular Forms and Fermat's Last Theorem

模形式与费马大定理

Springer

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Preface

This volume is the record of an instructional conference on number theory and arithmetic geometry held from August 9 through 18, 1995 at Boston University. It contains expanded versions of all of the major lectures given during the conference. We want to thank all of the speakers, all of the writers whose contributions make up this volume, and all of the "behind-the-scenes" folks whose assistance was indispensable in running the conference. We would especially like to express our appreciation to Patricia Pacelli, who coordinated most of the details of the conference while in the midst of writing her PhD thesis, to Jaap Top and Jerry Tunnell, who stepped into the breach on short notice when two of the invited speakers were unavoidably unable to attend, and to Stephen Gelbart, whose courage and enthusiasm in the face of adversity has been an inspiration to us.

Finally, the conference was only made possible through the generous support of Boston University, the Vaughn Foundation, the National Security Agency and the National Science Foundation. In particular, their generosity allowed us to invite a multitude of young mathematicians, making the BU conference one of the largest and liveliest number theory conferences ever held.

January 13, 1997

G. Cornell
J.H. Silverman
G. Stevens

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Schedule of Lectures

Wednesday, August 9, 1995

9:00–10:00	Glenn Stevens, *Overview of the proof of Fermat's Last Theorem*
10:30–11:30	Joseph Silverman, *Geometry of elliptic curves*
1:30–2:30	Jaap Top, *Modular curves*
3:00–4:00	Larry Washington, *Galois cohomology and Tate duality*

Thursday, August 10, 1995

9:00–10:00	Joseph Silverman, *Arithmetic of elliptic curves*
10:30–11:30	Jaap Top, *The Eichler-Shimura relations*
1:30–2:30	John Tate, *Finite group schemes*
3:00–4:00	Jerry Tunnell, *Modularity of* $\overline{\rho}_{E,3}$

Friday, August 11, 1995

9:00–10:00	Dick Gross, *Serre's Conjectures*
10:30–11:30	Barry Mazur, *Deformations of Galois representations: Introduction*
1:30–2:30	Hendrik Lenstra, Jr., *Explicit construction of deformation rings*
3:00–4:00	Jerry Tunnell, *On the Langlands Program*

Saturday, August 12, 1995

9:00 10:00	Jerry Tunnell, *Proof of certain cases of Artin's Conjecture*
10:30–11:30	Barry Mazur, *Deformations of Galois representations: Examples*
1:30–2:30	Dick Gross, *Ribet's Theorem*
3:00–4:00	Gerhard Frey, *Fermat's Last Theorem and elliptic curves*

Monday, August 14, 1995

9:00–10:00	Jacques Tilouine, *Hecke algebras and the Gorenstein property*
10:30–11:30	René Schoof, *The Wiles-Lenstra criterion for complete intersections*
1:30–2:30	Barry Mazur, *The tangent space and the module of Kähler differentials of the universal deformation ring*
3:00–4:00	Ken Ribet, *p-adic modular deformations of mod p modular representations*

Tuesday, August 15, 1995

9:00–10:00	René Schoof, *The Wiles-Faltings criterion for complete intersections*
10:30–11:30	Brian Conrad, *The flat deformation functor*
1:30–2:30	Larry Washington, *Computations of Galois cohomology*
3:00–4:00	Gary Cornell, *Sociology, history and the first case of Fermat*

Wednesday, August 16, 1995

9:00–10:00	Ken Ribet, *Wiles' "Main Conjecture"*
10:30–11:30	Ehud de Shalit, *Modularity of the universal deformation ring (the minimal case)*

Thursday, August 17, 1995

9:00–10:00	Alice Silverberg, *Explicit families of elliptic curves with prescribed mod n representations*
10:30–11:30	Ehud de Shalit, *Estimating Selmer groups*
1:30–2:30	Ken Ribet, *Non-minimal deformations (the "induction step")*
3:00–4:00	Michael Rosen, *Remarks on the history of Fermat's Last Theorem: 1844 to 1984*

Friday, August 18, 1995

9:00–10:00	Fred Diamond, *An extension of Wiles' results*
10:30–11:30	Karl Rubin, *Modularity of mod 5 representations*
1:30–2:30	Henri Darmon, *Consequences and applications of Wiles' theorem on modular elliptic curves*
3:00–4:00	Andrew Wiles, *Modularity of semistable elliptic curves: Overview of the proof*

Introduction

The chapters of this book are expanded versions of the lectures given at the BU conference. They are intended to introduce the many ideas and techniques used by Wiles in his proof that every (semi-stable) elliptic curve over **Q** is modular, and to explain how Wiles' result combined with Ribet's theorem implies the validity of Fermat's Last Theorem.

The first chapter contains an overview of the complete proof, and it is followed by introductory chapters surveying the basic theory of elliptic curves (Chapter II), modular functions and curves (Chapter III), Galois cohomology (Chapter IV), and finite group schemes (Chapter V). Next we turn to the representation theory which lies at the core of Wiles' proof. Chapter VI gives an introduction to automorphic representations and the Langlands-Tunnell theorem, which provides the crucial first step that a certain mod 3 representation is modular. Chapter VII describes Serre's conjectures and the known cases which give the link between modularity of elliptic curves and Fermat's Last Theorem. After this come chapters on deformations of Galois representations (Chapter VIII) and universal deformation rings (Chapter IX), followed by chapters on Hecke algebras (Chapter X) and complete intersections (Chapter XI). Chapters XII and XIV contain the heart of Wiles' proof, with a brief interlude (Chapter XIII) devoted to representability of the flat deformation functor. The final step in Wiles' proof, the so-called "3-5 shift," is discussed in Chapters XV and XVI, and Diamond's relaxation of the semi-stability condition is described in Chapter XVII. The volume concludes by looking both backward and forward in time, with two chapters (Chapters XVIII and XIX) describing some of the "pre-modular" history of Fermat's Last Theorem, and two chapters (Chapters XX and XXI) placing Wiles' theorem into a more general Diophantine context and giving some ideas of possible future applications.

As the preceding brief summary will have made clear, the proof of Wiles' theorem is extremely intricate and draws on tools from many areas of mathematics. The editors hope that this volume will help everyone, student and professional mathematician alike, who wants to study the details of what is surely one of the most memorable mathematical achievements of this century.

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