Stochastic Finite Element Methods and Its Applications to Aircraft Engineering

随机有限单元法及其在航空工程中的应用

汪 峰／著

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This book is designed as a brief guide for engineers, researchers and senior students who deal with the analysis and modeling of structures with stochastic nature, from large engineering projects such as aircraft structures, through to small engineered components like soil and civil structures.

Stochastic Finite Element Methods and Its Applications to Aircraft Engineering provides a formidable resource covering theories and applications of SFEMs, including the fundamentals and research frontiers of SFEMs, its applications to advanced material modeling and stochastic computational engineering analysis and engineering reliability analysis.

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Dedication

This book is dedicated to my family for their continuous support and encouragement during the period of time that I pursued my Bachelor and PhD title at University of Wales, Swansea, UK. In particular, I would like to give my sincere thanks to the Beijing Aeronautical Science & Technology Research Institute (BASTRI) of COMAC for providing me an excellent working platform and career development opportunities. The more important thing is that BASTRI makes me possible to devote all my life to the rise of motherland's large commercial aircraft industry.

Preface

At the very start, I want to give my appreciate thanks to the Beijing Aeronautical Science & Technology Research Institute of the Commercial Aircraft Corporation of China and the Civil Aircraft Digital Design Simulation Technology Key Laboratory of Beijing Municipal Science & Technology Commission, for their continuous support and publication sponsorship of this book.

This book is designed as a brief guide for engineers, researchers and senior students whose deals with the analysis and modeling of structures with stochastic natures, from large engineering projects such as aircraft structures, through to small engineered components like soil and civil structures.

In recent years, the increasing development of computer technologies and the urgent needs of practical engineering could provide the impetus for vigorously developing of the stochastic finite element methods (SFEMs). Different from other books about SFEMs, this book focuses on introducing stochastic finite element methods for analyzing uncertainty propagation in practical engineering computations, especially for applications in aircraft engineering.

Stochastic Finite Element Methods and Its Applications to Aircraft Engineering provides a formidable resource covering the theories and the applications of SFEMs, which including the preliminary fundamentals and research frontiers of SFEMs, and even its application to advanced material modeling and stochastic computational engineering analysis and engineering reliability analysis. Meanwhile, necessary mathematical background is offered to make this book self-contained. It is the author's hope that the materials concluded in this book will be useful to gain a fully basic understanding for SFEMs. Thus, imagination and inherent knowledge should be combined by the readers in their endeavors.

Nowadays, the understanding of engineering systems with variable uncertainties is far from being mature; many scholars have devoted all life to this aspect of unknowns. This leads to do justice to a rapidly expanding field of SFEMs, much filtering of the contents was necessary to keep this book within reasonable pages. Even so, the readers will be acquainted with the basic concepts and nearly the full state of the art of SFEMs by this book, some valuable contents that are involved in this book will to help the readers for further research.

Without further a due, I got a bachelor degree with First Class honours of Civil Engineering from the University of Wales, Swansea, then gained the PhD title from the Civil and Computational Engineering Centre (C2EC) of University of Swansea, UK. As is known to all, the C2EC is world-renown due to its leading position in research and application of FEMs. By summarizing almost the ten years' research experiences and outcomes of SFEMs, author just want to represent the highest respect to the pioneers and older generations of FEMs with this book, at the same time to commemorate those good times that we enjoyed and shared at Swansea University.

Feng Wang

Beijing, March 21, 2014

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Chapter 1 Introduction

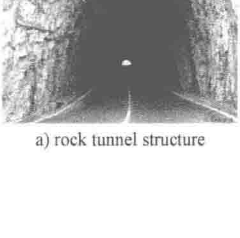
1.1 Engineering background

It is today widely recognized that computational methods permit the modeling and analysis of large-scale engineering system. Modeling an engineering system can be defined as the mathematical idealization of the physical processes governing its evolution, and this requires the definitions of input variables such as system geometries, loadings, and material properties etc., the selection of the resulted response variables such as displacements, strains, and stresses and finally the establishment of relationships between these various quantities.

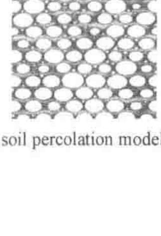
For a long time, finite element researchers and practical engineers have focused their attention on improving structural numerical models, such as beams, shells, solids elements and constitutive laws like elasticity, plasticity, damage theories etc. The conventional finite element methods and related numerical subjects are probably nowadays the most advanced and important approach for solution of these engineering problems. [1.1] But, until now, how to model structures and expected loading conditions, as well as the mechanisms for their possible deterioration with time or location is still quite a hot research topic to be solved.

Moreover, in most engineering applications, such as concrete bridges, nuclear power plants, aircraft, spacecraft and skyscrapers, the intrinsic randomness of materials or loading conditions which adheres to these structures is such that deterministic finite element models using average characteristics at best lead to approximate representation of the reality. These parameters or characteristics, if at all, can be determined in exceptional cases only. The disadvantages of using deterministic finite element models to study random medium are evident, since the deterministic analysis is only a particular realization of the random system; the values used in deterministic finite elements analysis are so called nominal values which deviate to a certain extent from the unknown true values. Meanwhile, due to the limited cognition of physical nature, it is often extremely difficult to provide sufficient information for exactly description of the medium with random characteristics; otherwise, empirical statistical quantities and as a result, the deterministic methods may lead wrong solutions; these unsatisfactory situations that arise in engineering are essentially due to the lack understanding of random medium and the immature of analysis tools for the corresponding mathematical problems.

Typical engineering projects including random medium are shown in Figs. 1.1, a) tunnel structures, b) percolation analysis of soils or rocks, c) reliability analysis or life prediction of concrete dams etc. Uncertainties in such random medium represent variability in data and are ubiquitous because of insufficient understanding of the underlying physical nature and/or inevitable measurement errors, moreover, the lack of a proper mathematical model for describing the associated stochastic fields and the immature of analysis tools for the corresponding Stochastic Partial Differential Equations (SPDEs). All in all, the understandings of engineering systems with variable uncertainties are far from being mature.



a) rock tunnel structure



b) soil percolation modeling

c) analysis of reinforced concrete dam

Figs. 1.1 Typical structures with uncertainties

Consequently, as long as random medium exists in practical engineering project, conservative safety factors are inevitably applied to the approximate mean value solution, which results in a significant increase in design, construction and operational costs. The most prominent application of safety factors, notably civil engineering, various standards exist for many types of structures in different countries, such as British Standard for steel structural design, Chinese structural standards, Japan standards for wooden structures etc. In aeronautical engineering, the airworthiness regulations specify the most likely loads to the accepted level of risk that the aircraft should resist, these are termed "limit loads" ; the aircraft must be capable of re-acting these loads multiplied by a factor called the "proof factor" without suffering substantial permanent deformation. Also, the structure must not fail below a load equal to the limit load multiplied by the "ultimate factor". For civil aircrafts the proof factor is fixed at 1.0 and the ultimate factor at 1.5, which generally provides a very high level of reliability. However, a safety factor approach cannot be used to assess the risk involved in a structural design. With the computing resources and methods to constantly enrich and computing technology level increasing enhancement, fortunately, it has been widely recognized that this should also include the quantitative consideration of the underlying uncertainties of the models and the random characteristics involved in the process of engineering design and analysis. [1.2 - 1.4]

There is also a general agreement that probabilistic methods should be strongly rooted in the basic theories of practical engineering and engineering mechanics, and hence represent the natural next step in the development of these fields. Therefore, it is of crucial importance to explore the mathematical foundation and develop effective tools for practical engineering systems that consisting of random medium. Hence in order to fully model the realistic physical phenomenon and subsequently to obtain more meaningful solutions, it is imperative to incorporate uncertainty from the beginning of the simulations and not as an afterthought. [1.5]

1.2 Engineering uncertainties

For the purposes of engineering analysis with random characteristics; it is necessary to distinguish between many kinds of uncertainties. In general, there are at least three types of uncertainty classification—statistical uncertainty, modeling uncertainty and physical uncertainty. [1.6 -1.7]

Statistical Uncertainty: statistical estimators such as the mean, variance and higher moments can be determined from available data such as experimental data or observations, and then used to calculate or estimate an appropriate probability density function and associated parameters. Generally the observations of the variable do not represent it perfectly and as a result there may be bias in the data as recorded.

Modeling Uncertainty: modeling uncertainty concerns the uncertainty in representation of physical nature, such as the limit state equations, constitutive laws etc. Modeling uncertainty is often simply due to the lack of knowledge, and effects that caused by modeling uncertainty can be reduced with increasing understanding of underlying knowledge or increased availability of data.

Physical Uncertainty: whether or not a structure or structural element fails when the loaded depends in part on the actual values of the relevant material properties that govern its strength. Examples include:

(1) variation in material properties, such as yield strength, young's modulus;

(2) variability of various loading conditions, like wind loading, static loading;

(3) variation of physical dimensions of a structural component.

Physical uncertainly might be reduced with greater availability of experimental data and observations, or, in some cases, taking material yield strength as an example, with greater effort in manufacturing quality control. However, usually it cannot be fully eliminated. This book mainly focuses on the physical uncertainties; a special case within the physical uncertainty that not discussed is the type of geometry uncertainty; Babušhka and Chleboun [1.8] investigated the condition of geometry uncertainties for a non-stochastic model; they discussed that for general boundary conditions the straight forward idea of constructing a series of problems defined on geometries converging to the correct geometry yields a model converging to a wrong solution, while their model is non-stochastic type, their observations need to be taken into account if stochastic geometries are considered. In general speaking, two kinds of the most prominent issues in the applied engineering field and theoretical research field are: unpredictable loading conditions and undetermined material properties.

1.2.1 Unpredictable loading conditions

Flying is potentially a dangerous form of transport. High speeds, a three-dimensional flight path, hostile ambient conditions at cruise altitudes, a highly inflammable fuel mass positioned close to passengers; all these are combined with the vagaries of the weather, random occurrences of other natural hazards, such as ice and bird strikes, most significantly, human frailty etc. This section will focus on the description of random loading conditions.

Unpredictable loadings conditions exist everywhere, which is a common phenomenon in nature. Figs. 1.2 reveal some appearances such as earthquake loadings, wind loadings and surrounding working temperatures etc.; the effects that induced by random loadings may produce disasters, or in a few cases, play negligible or minor roles.

a) after-effects of earthquake loadings

b) wind loading conditions

Figs. 1.2 Disasters caused by extremely loading conditions

In some cases, the unpredictable loadings could be a particular serious topic for aviation industry, as the aircraft are subjects to a number of unpredictable loadings that can seriously affect their performance; typically unpredictable loadings like soft bodies impacting on aircraft are of concern to the aviation industry since they are unexpected events that threaten the structural integrity of the aircraft, and thus the safety of the passengers. Soft bodies include projectiles such as bird and hail strikes, which are highly deformable upon impact and flow over the structure. Even a short while in these dangerous regions, is sufficient to cause serious damage to aircraft structures such as abrasions, dents and in some cases, perforations, where Fig. 1.3 shows some obvious damages that caused by unpredictable impacts. Though the analysis of flight environment like weather conditions could reduce the occurrence of intersections between flight routes and high risk regions, but the analysis of weather conditions has so far made it possible to avoid it, sometimes the passage through a hailstone becomes inevitable, and more seriously, the impact velocity and mechanical properties vary with different weather conditions or regions, which bring difficulty for the practical engineering analysis. For this particular case, some scholars have suggested that the weibull distribution, which is also used in rock fragmentation, is well suited to represent the particle size and impact velocity distributions of ice impacts. However, it is mandatory that the aircraft structures show an appropriate level of tolerance to damages caused by an external impact.