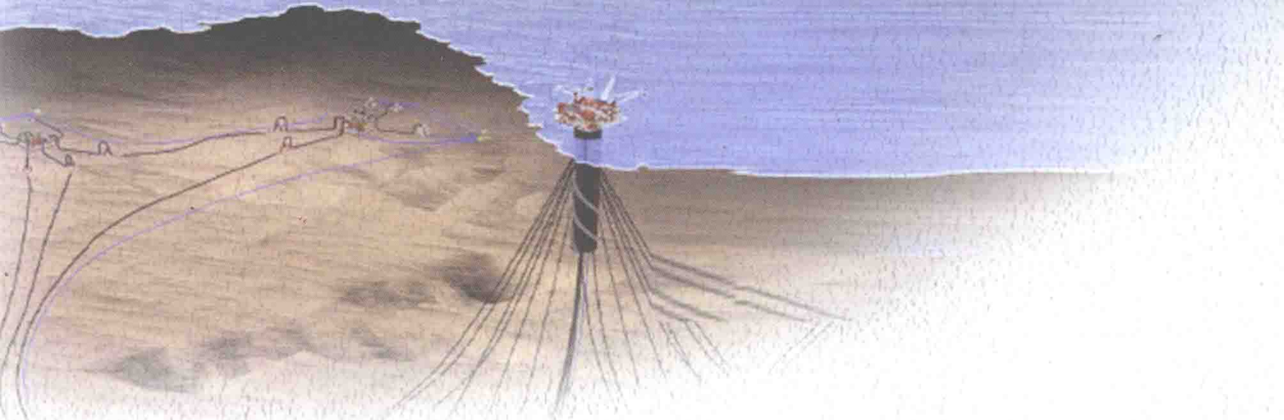

Fluid-Structure Dynamic Interaction

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ACADEMY PRESS

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Academy Press

图书在版编目(CIP)数据

流体-结构动力耦合作用=Fluid-Structure Dynamic Interaction:英文/张永良著.
—北京:学苑出版社,2009.12
ISBN 978-7-5077-3318-1

Ⅰ.①流… Ⅱ.①张… Ⅲ.①流体力学:结构力学-英文 Ⅳ.①O35 ②O342

中国版本图书馆CIP数据核字(2009)第210437号

出 版 人:孟　白
责任编辑:刘　丰
出版发行:学苑出版社
社　　址:北京市丰台区南方庄2号院1号楼
邮政编码:100079
网　　址:www.book001.com
电子信箱:xueyuan@public.bta.net.cn
销售电话:010-67674055、67675512、67678944
印 刷 厂:高碑店市鑫宏源印刷包装有限责任公司
开本尺寸:787×1092　16开本
印　　张:15.25
字　　数:450千字
版　　次:2010年1月第1版
印　　次:2010年1月第1次印刷
定　　价:50.00元

PREFACE

This book reports the investigation of fluid-structure dynamic interaction, particullarly initially-stretched cylindrical pipes conveying fluid, and examines the effects on the vibratory characteristics of flow velocities, fluid pressures and initial tensions.

The pipes examined are both slender and thick- and thin-walled. The vibration of these pipes conveying fluid is explored by using a beam and a shell theory. To assess the dynamic problems of pipes conveying fluid internally, the steady/unsteady fluid flow modelling and beam-/shell-type vibrations are combined in the models. The investigation therefore comprises: (1) developing a new finite element model for the vibration of initially-stretched pipes conveying fluid using the method of kinematic correction and a penalty function technique, to examine natural frequencies and damping ratios; (2) verifying experimentally certain new features concerning the mode coupling and node shift with flow velocities, and the effect of external vibrations on fluid flow; (3) developing a new model for the vibration of initially-tensioned thick and thin cylindrical shells conveying fluid and examining the influence of flow velocities, hydrostatic pressures, initial tensions, geometric properties and material properties on the dynamic behaviour; (4) developing a new model for the shell-vibration of pipes conveying viscous pulsating fluid flow, and the associated calculation of hydrodynamic pressure, axial and lateral displacements and flow velocities; (5) developing a new finite element model for the vibration of an initially-tensioned circular cylindrical shell conveying a viscous fluid using the Navier-Stokes equation and the nonlinear theory of elasticity.

The book aims to aid designers, researchers and postgraduate students of pipes conveying fluid in predicting their dynamic behaviour for various flow velocities, fluid pressures and initial tensions as well as varying geometric and material properties. It also aims to provide practically useful information of interactions between fluids and structures. Throughout, numerical results are carefully compared with experimental observations, and conclusions drawn as to the appropriateness and accuracy of the models used.

The author is much indebted to Professors Daniel G. Gorman, Jason M. Reese, and K. Vairavamoorthy for their valuable guidance, counsel and assistance.

Thanks are also due to many other individuals, including Prof. J. Horacek (Academy of Science of the Czech Republic) for valuable discussion and advice on fluid-structure interaction. I cannot mention here the names of all those who assistanted me in various ways. Finally, I would like to thank Ms. BIAN Ye for her considerable support.

Finally, I wish to acknowledge the support by the National Natural Science Foundation of China (50539070), Tsinghua University Initative Scientific Research Program (2009THZ 07060) and the Major State Basic Research Development Program of China (2006CB403304).

List of Principal Symbols

a	fluid wave speed, m/s
$\bar{a}$	dimensionless wave speed, defined by Equations (11.20) and (11.43), respectively
a_0	constant, defined by Equation (2.23i)
$[\mathbf{a}]$	matrix, defined by Equation (8.28a)
$\bar{A}$	dimensionless area, defined by Equations (11.20) and (11.43), respectively
A_f, A_t	cross-sectional areas of the fluid and tube, respectively, m^2
$[\mathbf{A}]$	matrix, defined by Equation (9.136)
$[\mathbf{A}_{bb}]$	integral coefficient of the pressure over the fluid-structure interface
b_0, b_1	constants defined by Equations (2.23i)
$[\mathbf{B}]$	strain-displacement matrix, defined by Equations (2.10) and (9.6), respectively
$[\mathbf{B}_a]$	strain-displacement matrix, defined by Equation (9.17)
$[\mathbf{B}_L]$, $[\mathbf{B}_{NL}]$	linear and nonlinear strain-displacement matrices, respectively
$[\mathbf{B}_t]$	strain-displacement matrix, defined by Equation (9.13)
c	structural damping coefficient
$\bar{c}$	dimensionless damping coefficient, defined by Equations (11.20) and (11.43), respectively
C_1, C_2	constants to be determined from the flow conditions
$[\mathbf{c}]$	elemental fluid-tube damping matrix
$[\mathbf{c}_f]$	elemental fluid damping matrix
$[\mathbf{c}_f^N]$, $[\mathbf{c}_f^c]$, $[\mathbf{c}_f^d]$	matrices, defined by Equation (10.14b)
$[\mathbf{c}_f^{pv}]$, $[\mathbf{c}_f^{pp}]$, $[\mathbf{c}_f^p]$	matrices, defined by Equation (10.14b)
$[\mathbf{c}_k]$	dimensionless k-th elemental damping matrix
$[\mathbf{c}_k]_g$, $[\mathbf{c}_k]_l$	k-th elemental damping matrices in the global and local co-ordinate systems, respectively
$[\mathbf{C}]$	global fluid-structure damping matrix, dimensionless assembly damping matrix

$[\mathbf{C}^i]$	assembly damping matrix at i-th time step (Δt)
$[\mathbf{C}_\mathrm{f}]$	fluid damping matrix associated with the Coriolis force, i. e. $[\mathbf{C}_\mathrm{f}] = [\mathbf{S}_1]^T[\mathbf{H}_\mathrm{bb}]^{-1}[\mathbf{S}_2]$
dA	surface area of the control volume
d_{ij}	defined by Equation (2.43)
dV	volume element of the control volume
$\bar{D}$	dimensionless internal diameter, defined by Equation (11.61)
D_e, D_i	external and internal diameters of the tube, respectively, m
D_{ij}	independent elastic constants, defined by Equation (7.6b)
$[\mathbf{D}]$	tube stress-strain matrix, defined by Equations (2.2), (7.6a, c, d) and(8.7a,b), respectively
$[\mathbf{D}_\mathrm{a}]$	stress-strain matrix, defined by Equation (9.16)
e_ω	relative error, defined by Equation (2.50)
e_{ij}	defined by Equation (2.43)
$[\mathbf{e}]$	matrix, defined by Equation (8.28a)
$\{\mathbf{e}_\mathrm{U}\}$	relative error vector, defined by Equation (2.50)
E	Young's modulus, N/m^2
EI	flexural rigidity, Nm^2
E_ex	energy of external forces on the fluid-tube element, Nm
E_in	strain energy of the tube element, Nm
E_pul	pulse period, ms
E_x, E_θ	Young's moduli in the axial and circumferential directions, respectively
E_{xx}, $E_{\theta\theta}$, E_{rr}	Young's moduli in the axial, tangential and radial directions, respectively
f	friction factor defined by Equation (5.2)
$f(\bar{t})$	dimensionless excitation force, defined by Equations (11.20) and (11.43), respectively
f_n	normal force per unit length acting on the tube wall, N/m
f_s	Darcy – Weisbach friction factor
f_t	tangential force per unit length acting on the tube wall, N/m
$\{\mathbf{f}_k\}$	vector of dimensionless k-th elemental force
$\{\mathbf{f}\}$	vector of elemental load
$\{\mathbf{f}_\mathrm{f}\}$, $\{\mathbf{f}_\mathrm{t}\}$	vectors of elemental forces exerted on the fluid and tube, respectively
$\{\mathbf{f}_k^e\}_\mathrm{g}$, $\{\mathbf{f}_k^\mathrm{ld}\}_\mathrm{g}$	vectors of the k-th elemental load and fictitious load in the global coordinate system, respectively
$\{\mathbf{f}_k^e\}_\mathrm{l}$, $\{\mathbf{f}_k^\mathrm{ld}\}_\mathrm{l}$	vectors of the k-th elemental load and fictitious load in the local coordinate system, respectively

$\{\mathbf{f}_k\}_g$, $\{\mathbf{f}_k\}_l$	vectors of the k-th elemental force in the global and local co-ordinate systems, respectively
$\{\mathbf{f}_{0t}\}$	vector of external forces, excluding fluid-structure interaction forces
F	harmonic excitation force amplitude
$\{\mathbf{F}\}$	vector of global external force exerted on the fluid-tube, dimensionless assembly forcing vector
$\{\mathbf{F}^i\}$	assembly forcing vector at i-th time step (Δt)
$\{\mathbf{F}_t^T, \mathbf{F}_b^T\}^T$	vector of the nodal external force exerted on the tube
g	gravity acceleration, m/s^2
$\bar{g}$	dimensionless gravity acceleration, defined by Equation (11.20)
G	shear modulus, N/m^2
$G(t)$	relaxation function
G^∞	final value of the relaxation function
$G_{\theta x}$, G_{xr}, $G_{r\theta}$	shear moduli which characterize the variation of the angles in the direction θ and x, x and r, and r and θ, respectively
$\{\mathbf{g}\}$	$\{\mathbf{g}\} = \{\mathbf{g}_t\} + \{\mathbf{g}_f\}$
$\{\mathbf{g}_f\}$, $\{\mathbf{g}_t\}$	vectors of body force of the fluid and tube, respectively
$[\mathbf{G}]$	matrix, defined by Equation (8.19a)
h	tube wall thickness
$h(s)$	dissipation function
$H_1(\omega)$, $H_2(\omega)$	frequency response function estimators
$[\mathbf{H}_1]$, $[\mathbf{H}_2]$	matrices, defined by Equation (8.5)
i_1, i_2	$i_1 = 1,2,3,4$ and $i_2 = 5,6,7,8$
I, I_f	area moments of inertia of the tube and fluid, respectively, m^4
$[\mathbf{I}]$	identity tensor
$[\mathbf{I}_3]$	unit matrix of order 3
j	$j = 1,2,\cdots,k$
$J_n(\lambda r)$	n-th modified Bessel functions of the first kind
k	number of characteristic roots
k_t	stiffness of the tube wall
K	shear coefficient of the tube material
K_f	fluid bulk modulus of elasticity, N/m^2
$[\mathbf{k}]$	elemental fluid-tube stiffness matrix, i. e. $[\mathbf{k}] = [\mathbf{k}_f] + [\mathbf{k}_{t(L)}] + [\mathbf{k}_{t(NL)}]$
$[\mathbf{k}_f]$, $[\mathbf{k}_t]$	elemental fluid and tube stiffness matrices, respectively, i. e. $[\mathbf{k}_t] = [\mathbf{k}_{t(L)}] + [\mathbf{k}_{t(NL)}]$

$[\mathbf{k}_k]_g$, $[\mathbf{k}_k]_l$	k-th elemental stiffness matrices in the global and local co-ordinate systems, respectively
$[\mathbf{k}_k]$	dimensionless k-th elemental stiffness matrix
$[\mathbf{K}]$	global fluid-structure stiffness matrix
$[\mathbf{K}^i]$	assembly fluid-tube stiffness matrix at i-th time step (Δt), dimensionless assembly fluid-tube stiffness matrix
$[\mathbf{K}_f]$	fluid stiffness matrix associated with the centrifugal force, i. e. $[\mathbf{K}_f] = [\mathbf{S}_1]^T[\mathbf{H}_{bb}]^{-1}[\mathbf{S}_3]$
l, L	elemental length and total length of the tube, respectively, m
m	total mass of the fluid and tube per unit length, $m = m_f + m_t$, kg/m
m_f, m_t	fluid and tube masses per unit length, respectively, kg/m
M	number of finite elements in longitudinal direction
M_{xx}, $M_{\theta\theta}$, $M_{x\theta}$	bending moments of the shell in cylindrical coordinates
M_x, M_θ, $\overline{M}_{x\theta}$	resultant bending stresses in cylindrical co-ordinates
$[\mathbf{m}]$	elemental fluid-tube mass matrix
$[\mathbf{m}_k]_g$, $[\mathbf{m}_k]_l$	k-th elemental mass matrix in the global and local co-ordinate systems, respectively
$[\mathbf{m}_f]$, $[\mathbf{m}_t]$	elemental fluid and tube mass matrices, respectively
$[\mathbf{m}_{f+t}]$	$[\mathbf{m}_{f+t}] = [\mathbf{m}_f] + [\mathbf{m}_t]$
$[\mathbf{m}_k]$	dimensionless k-th elemental mass matrix
$[\mathbf{M}]$	global fluid-structure mass matrix, dimensionless assembly fluid-tube mass matrix
$[\mathbf{M}^i]$	assembly fluid-tube mass matrix at i-th time step (Δt)
$[\mathbf{M}_f]$	fluid mass matrix associated with the inertia force, i. e. $[\mathbf{M}_f] = [\mathbf{S}_1]^T[\mathbf{H}_{bb}]^{-1}[\mathbf{S}_1]$
n	n-th circumferential wavenumber, total number of elements
$\{\mathbf{n}\}$	unit outward vector normal to the tube surface (from the tube into the fluid)
N	number of finite elements in radial direction, total number of elements
N_i	components of the shape function matrix $[\mathbf{N}]$, $i = 1, 2, \cdots, 10$
N_{xx}, $N_{\theta\theta}$, $N_{x\theta}$	stress components in cylindrical coordinates
N_{xx}^0, $N_{\theta\theta}^0$	initial uniform axial and circumferential normal resultant-stresses, respectively
$[\mathbf{N}]$	shape function matrix, defined by Equations (9.10) and (9.18 – 20), respectively
$[\mathbf{N}_f]$, $[\mathbf{N}_t]$	matrices of shape function for the shell and fluid, respectively

$[\mathbf{N}_p]$	fluid pressure shape function matrix
$[\mathbf{N}_t]$	matrix of shape function, defined by Equations (2.5) and (9.10a), respectively
$[\mathbf{N}_u],[\mathbf{N}_\varphi],[\mathbf{N}_w]$	shape function matrices, defined by Equation (2.7)
$[\mathbf{N}_{tr}]$	shape function matrix for the radial displacement, defined via $u_r=[\mathbf{N}_{tr}]\{\bar{\mathbf{u}}\}$
p	fluid pressure, N/m^2
p_s	hydrostatic pressure, N/m^2
$\bar{p}$	dimensionless pressure, defined by Equation (11.43)
$\hat{p}$	hydrodynamic pressure (perturbation pressure), N/m^2
p_x,p_θ,p_r	fluid pressures exerted on the shell surface in x-, θ- and r-directions, respectively, N/m^2
$\{\mathbf{p}\}$	vector of nodal fluid pressure
$\{\mathbf{p}_f\}$	vector of nodal hydrodynamic pressure
$\{\bar{\mathbf{p}}_t\}$, $\{\bar{\mathbf{p}}_f\}$	vectors of the tube (shell) and fluid boundary tractions at the fluid-structure interface, respectively, N/m^2
$\{\bar{\mathbf{p}}\}$	$\{\bar{\mathbf{p}}\}=\{\bar{\mathbf{p}}_t\}+\{\bar{\mathbf{p}}_f\}$, N/m^2
$\{\bar{\mathbf{p}}_t^s\}$, $\{\bar{\mathbf{p}}_t^d\}$	vectors of the prescribed tube boundary traction arising from the hydrostatic and hydrodynamic pressures, respectively, N/m^2
$[\mathbf{P}]$	matrix, defined by Equation (9.13b)
$[\mathbf{P}_1],[\mathbf{P}_2]$	matrices, defined by Equation(7.22b)
$\{\mathbf{P}_b\}$	generalized nodal radial traction vector due to hydrostatic and hydrodynamic pressures
$\{\mathbf{P}_f\}$	generalized nodal fluid pressure vector for fluid elements excluding fluid-structure boundary elements.
$\{\bar{\mathbf{q}}_t\}$	vectors of the external forces, N/m^2
Q	resultant shear force, N/m^2; flow rate, m^3/s
$\bar{Q}$	dimensionless flow rate, defined by Equation (11.20)
Q_{max}	maximum flow rate, m^3/s
$[\mathbf{Q}]$	matrix, defined by Equation (9.13b)
$[\mathbf{Q}_1],[\mathbf{Q}_2]$	matrices, defined by Equation (7.22a)
r	co-ordinate along the radial direction of the element
R	tube mean radius
R^{in},R^{ex}	internal and external radius of the tube, respectively
Re	Reynolds number, defined by Equations (11.20) and (5.1), respectively

Re(I)	real part of $I(n,\lambda,R)$
$[\mathbf{R}_1]$, $[\mathbf{R}_2]$	matrices, defined by Equation (7.22b)
s	total number of finite elements
S_0, S_1	constants, defined by Equation (2.23i)
S_t	Strouhal number, defined by Equation (11.43)
$[\mathbf{s}_1]$, $[\mathbf{s}_2]$, $[\mathbf{s}_3]$	matrices, defined by Equation (7.22b) and (8.28c)
$[\mathbf{S}_1]$, $[\mathbf{S}_2]$, $[\mathbf{S}_3]$	integral matrices over the fluid-structure interface, respectively
$[\mathbf{S}]$	matrix, defined by Equation (8.19b)
t	time, s
$\bar{t}$	dimensionless time defined by Equation (11.20)
$\{\mathbf{t}\}$	unit vector tangential to the tube surface
T	axial tension of the tube, N
T_0	initial axial tension of the tube, N
T_p	pulsation period, ms
$\bar{T}$	dimensionless axial tension defined by Equation (2.34)
T_x	axial tension, N
$\bar{T}_x$	dimensionless axial tensions, defined by Equation (7.35)
$[\mathbf{T}]$	matrix, defined by Equation (7.18b)
$[\mathbf{T}_i]$	coordinate transformation matrix of the ith element, $i=1, 2, \cdots, m$
$[\mathbf{T}_k]$	co-ordinate transformation matrix of the k-th element, $k=1,2,\cdots,n$
$\{\mathbf{T}_0\}$	vector of initial axial force, i. e., $\{\mathbf{T}_0\}=\{T_0\cos\varphi,0,T_0\sin\varphi\}^T$
u, w	displacements in the x – and y – directions, respectively, m
u, v, w	axial, tangential and radial displacements, respectively, m
u_x, u_θ, u_r	axial, tangential and radial displacements, respectively, m
$\bar{u}$	dimensionless axial displacement defined by Equation (11.20)
$\{\mathbf{u}_0\}$	kinematic boundary condition, i. e., $\{\mathbf{u}_0\}=\{0,0,y_0(t)\}^T$,
$\{\mathbf{u}_t\}$, $\{\mathbf{u}_f\}$	displacement vectors of the tube and fluid elements, respectively, i. e., $\{\mathbf{u}_t\}=\{u,\varphi,w\}^T$, $\{\mathbf{u}_f\}=\{u_f,\varphi_f,w_f\}^T$
$\{\dot{\mathbf{u}}_t\}$, $\{\dot{\mathbf{u}}_f\}$	velocity vectors of the tube and fluid elements, respectively, m/s
$\{\ddot{\mathbf{u}}_t\}$, $\{\ddot{\mathbf{u}}_f\}$	acceleration vectors of the tube and fluid elements, respectively, m/s^2
$\{\mathbf{u}_{re}\}$	displacement vector of the fluid element relative to the tube displacement $\{\mathbf{u}_t\}$, m
$\{\bar{\mathbf{u}}\}$, $\{\dot{\bar{\mathbf{u}}}\}$, $\{\ddot{\bar{\mathbf{u}}}\}$	vectors of nodal displacement, velocity and acceleration, respectively
$\{\bar{\mathbf{u}}_t\}$	nodal displacement vector of the shell, m
$\{\dot{\bar{\mathbf{u}}}_f\}$	vector of nodal velocity of the fluid, m/s
U	steady fluid flow velocity, m/s

U_0	constant mean flow velocity, m/s
U_*	time mean cross sectional average velocity, m/s
$\bar{U}$	dimensionless flow velocity, defined by Equations (2.34), (7.34), (10.22), (11.20) and (11.43), respectively
$\{\mathbf{U}\}$	global generalized fluid-shell displacement vector
$\{\dot{\mathbf{U}}\}$	global generalized fluid-shell velocity spectrum
$\{\mathbf{U}^i\}$	vector of global displacement at i-th time step (Δt)
$\{\mathbf{U}_\mathrm{b}\}$, $\{\mathbf{U}_\mathrm{t}\}$	vectors of the generalized radial nodal displacement for the shell elements adjacent to the fluid domain and the generalized nodal displacement vector excluding radial nodal displacement for shell elements adjacent to the fluid domain
$\hat{v}_x, \hat{v}_\theta, \hat{v}_r$	perturbation flow velocities in the axial, tangential and radial directions, respectively
$\{\mathbf{v}\}$	flow velocity vector, i. e. $\{\mathbf{v}\} = v_r\mathbf{i} + v_\theta\mathbf{j} + v_x\mathbf{k}$
$\{\bar{\mathbf{v}}\}$	nodal flow velocity vector, i. e. $\{\bar{\mathbf{v}}\} = \{\{\bar{\mathbf{v}}_x\}^T, \{\bar{\mathbf{v}}_\theta\}^T, \{\bar{\mathbf{v}}_r\}^T\}^T$
$\{\tilde{\mathbf{v}}\}$	perturbation velocity
$\{\mathbf{v}_r\}$, $\{\mathbf{v}_\theta\}$, $\{\mathbf{v}_x\}$	vectors of nodal velocities in radial, tangential and axial directions
$\{\mathbf{v}_\mathrm{s}\}$	steady flow velocity vector
$\{\mathbf{v}_\mathrm{re}\}$	vector field of fluid velocity relative to the moving tube, i. e., $\{\mathbf{v}_\mathrm{re}\} = \{U\cos\varphi, 0, U\sin\varphi\}^T$
w	transverse displacement, m
$w(x,t)$	transverse displacement of the tube located at a distance x from the left hand end, m
$\bar{w}$	dimensionless lateral displacement defined by Equation (11.20)
w_{am}	peak lateral displacement amplitude of the tube in the y-direction, m
w_o	external excitation displacement in y-direction, m
$W(x)$	complex modal displacement, m
$\bar{W}$	defined by Equation (2.34)
x	co-ordinate along the longitudinal direction of the element, distance between the upstream end of the tube and the node
$\bar{x}$	dimensionless co-ordinate, defined by Equation (11.1)
x, y	co-ordinate in x,y co-ordinate system, respectively
$[\mathbf{X}_1]^T[\mathbf{X}_2]$	$[\mathbf{X}_1]^T[\mathbf{X}_2] = [\mathbf{R}_1]^T[\mathbf{R}_2] + [\mathbf{Q}_1]^T[\mathbf{Q}_2]$
y_am	excitation displacement amplitude, m
$y_0(\mathrm{t})$	external excitation displacement in y-direction, m
$Y_n(\lambda r)$	nth modified Bessel functions of the second kind

Greek letters

α	defined by Equations (11.20), (11.43) and (12.2), respectively
$\alpha(\omega)$	acceptance, defined by Equation (5.3)
α_M, α_K	Rayleigh damping constants
β	dimensionless mass, defined by Equation (2.34); complex characteristic value, defined by Equations (11.20) and (11.43), respectively
β_i	defined by Equation (8.5), $(i=1,2,\cdots,9)$
χ	defined by Equations (11.20) and (11.43), respectively
$\varepsilon(t)$	strain at time t
$\varepsilon_{xx}^0, \varepsilon_{\theta\theta}^0$	initial axial and circumferential strains, respectively
$\varepsilon_{xx}, \varepsilon_{\theta\theta}, \varepsilon_{rr}$	strain components in cylindrical coordinates
$\{\boldsymbol{\varepsilon}\}$	vector of strain of the tube
$\{\boldsymbol{\varepsilon}_f\}$	fluid strain rate tensor, defined by Equation (10.3)
$\{\boldsymbol{\varepsilon}_0\}$	vector of the initial strain
$\{\boldsymbol{\varepsilon}_F^0\}$, $\{\boldsymbol{\varepsilon}_p^0\}$	vectors of strains due to the initially-tensioned forces and the initial hydrostatic pressures, respectively
$\{\boldsymbol{\varepsilon}_t\}$	shell strain tensor field
φ	angle between the tube centerline and datum line
φ_1, φ_2	defined by Equation (11.20)
φ_j, φ_{j+1}	cross-sectional rotation of j-th and $(j+1)$-th nodes, respectively
Φ	potential flow function
$\Phi_{,x}, \Phi_{,\theta}, \Phi_{,r}$	operators $\Phi_{,x} \equiv \partial\Phi/\partial x$, $\Phi_{,\theta} \equiv \partial\Phi/\partial\theta$, and $\Phi_{,r} \equiv \partial\Phi/\partial r$
ψ	defined by Equation (7.26)
γ	constant penalty parameter
$\gamma_{x\theta}, \gamma_{\theta r}, \gamma_{rx}$	shear strain components in cylindrical coordinates
$\gamma^2(\omega)$	coherence function, defined by Equation (5.4)
η	$\eta = y/h$
η_d	damping value, defined by Equation (5.6)
$\eta_{x\theta,x}$, $\eta_{\theta x,\theta}$	coefficients of mutual influence of the first kind, respectively
κ	shear coefficient, $(=6EI/KGA_t l^2)$
λ	stretch rate; $\lambda=\beta/R$
λ_j	characteristic value, $(j=1,2,\cdots,k)$
λ_p	penalty parameter
μ	small excitation parameter; dynamic viscosity coefficient

ν	Poisson's ratio; coefficient of kinematic viscosity
$\nu_{\theta x}$, ν_{xr}, $\nu_{r\theta}$	Poisson coefficients which characterize compression in the x-, r- and θ-directions for tensions in the θ-, x- and r- directions, respectively
θ	circumferential co-ordinate; angle between the tube element position and x-axis
Θ	defined by Equations (11.20) and (11.43), respectively
Θ_D	defined by Equation (8.7b)
$\Theta_N, \Theta_{N,x}, \Theta_{N,\theta}$	defined by Equation (7.2), respectively
ρ_t, ρ_f	densities of the tube and fluid, respectively, kg/m^3
$[\boldsymbol{\rho}_f]$, $[\boldsymbol{\rho}_t]$	fluid and tube inertia force-acceleration matrices, respectively
$[\boldsymbol{\rho}_{f+t}]$	$[\boldsymbol{\rho}_{f+t}] = [\boldsymbol{\rho}_f] + [\boldsymbol{\rho}_t]$
$\sigma_{xx}, \sigma_{\theta\theta}$, σ_{rr}	stress components in cylindrical coordinates
$\{\boldsymbol{\sigma}\}$, $\{\boldsymbol{\sigma}_0\}$	stress and initial stress vectors, respectively, N/m^2
ζ	modal damping ratio
$\tau_{x\theta}, \tau_{\theta r}$, τ_{rx}	shear stress components in cylindrical coordinates
τ_0	shear stress on fluid-tube interface, N/m^2
$\{\boldsymbol{\tau}\}$	viscous component tensor of the fluid, defined by Equation (10.3)
ω	frequency, Hz
$\bar{\omega}$	dimensionless frequency, defined by Equation (2.34), Hz
ω_{mn}	natural frequency at circumferential wave number n and longitudinal half wave number m
ω_n	n-th natural frequency, Hz
$\bar{\omega}_{mn}$	dimensionless natural frequency, defined by Equations (7.34) and (7.35)
ω_{0n}	n-th natural frequency at $U=0$ or $T_0 = 0.0$, Hz
ω_p	pulsation frequency, Hz
Ω	excitation frequency, Hz
Ω_{f+t}	solution domain of the system, i. e. $\Omega = \Omega_t \cup \Omega_f$
Ω_f, Ω_t	fluid and tube spatial domains, respectively
ξ	$\xi = x/l$
Γ_f, Γ_t	fluid and tube boundaries, respectively, $\Gamma = \Gamma_t \cup \Gamma_f$
Π	total potential energy, i. e., $\Pi = E_{in} + E_{ex}$, Nm
$[\boldsymbol{\Xi}]$	matrix, defined by Equation (7.6b)
Δt	time step, s
∇	$\nabla = (\partial/\partial r)\mathbf{i} + (\partial/r\partial\theta)\mathbf{j} + (\partial/\partial x)\mathbf{k}$

$\sum_{k=1}^{n}$	matrix assembly

Subscript

f, t	fluid and tube quantities, respectively
g, l	global and local coordinate systems, respectively
i, $i-1$	i-th and $(i-1)$-th time steps, respectively
j, k	j-th node and k-th element, respectively
L, NL	linear and nonlinear components, respectively
m, n	m- and n-th longitudinal and circumferential vibration modes, respectively
u, φ, w	quantities in the x- , ϕ- and y- directions, respectively
ω	frequencies of the system

Superscript

d, s	hydrodynamic and hydrostatic pressures, respectively
i	i-th time step
k, $k-1$	present and previous numbers of finite elements, respectively
in, ex	internal and external surfaces of the tube, respectively
T	transpose of matrix.
v	viscoelasticity

CONTENTS