ALM 29

Advanced Lectures in Mathematics

Selected Expository Works of Shing-Tung Yau with Commentary

(Vol. II)

丘成桐综述文章选集

附评论（第 II 卷）

Editors: Lizhen Ji · Peter Li · Kefeng Liu · Richard Schoen

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With Steve Nadis as a consulting editor on language for commentary

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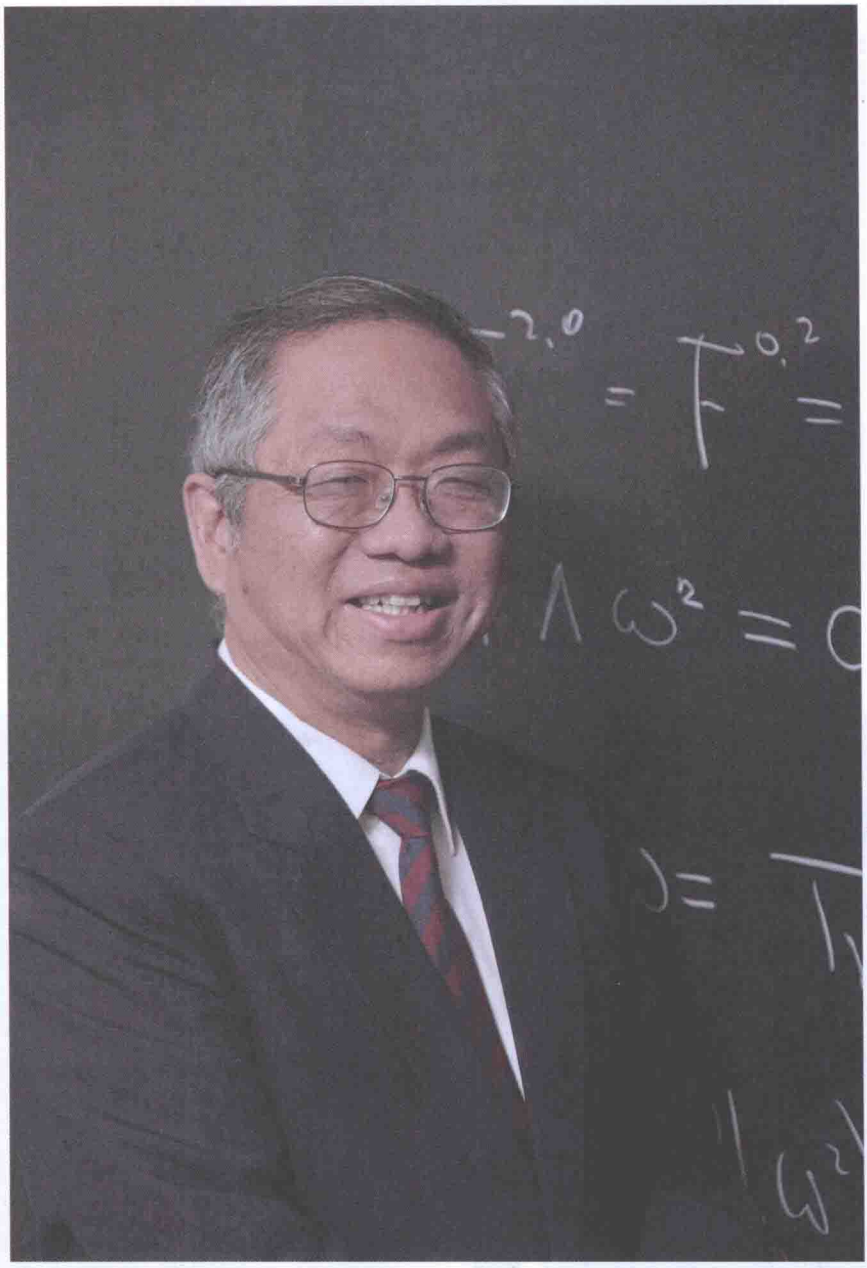

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At Lehigh University's 141st commencement on May 18, 2009

At the church of Chung Chi College on April 3, 2005

CURRICULUM VITAE

Shing-Tung Yau (丘成桐)

Last updated on Jun 12, 2014

ADDRESS: William Caspar Graustein Professor of Mathematics
Department of Mathematics
Harvard University
Cambridge, MA 02138, U.S.A.

BORN: Shantou, Guangdong, China, April 4, 1949

EDUCATION: Ph.D., Mathematics, University of California, Berkeley, 1971

HONORARY DEGREE:

May 11, 2014	Doctor of Humane Letters, University of Illinois at Chicago
Jun 17, 2011	Doctor of Mathematics, University of Waterloo
Jun 05, 2010	Doctor of Science, Cheng Kung University
May 18, 2009	Doctor of Science, Lehigh University
Nov 2005	Doctor of Science, Taiwan University
May 22, 2005	Doctor of Science, Polytechnic University in Brooklyn
Nov 2004	Doctor of Science, Hong Kong University of Science and Technology
Jul 2004	Doctor of Science, Central University (Jhongli)
Mar 2003	Doctor of Science, Zhejiang University
Oct 16, 2002	Doctor of Science, Macau University
Jun 24, 2000	Doctor of Science, Tsing Hua University (Hsinchu)
Jun 21, 1997	Doctor of Science, Chiao Tung University (Hsinchu)
Sept 1987	Master of Arts, Harvard University
Dec 1980	Doctor of Science, The Chinese University of Hong Kong

RESEARCH INTERESTS:

Differential geometry, differential equations, and general relativity

POSITIONS:

2014–Present	Director of the Center for Mathematical Sciences and Applications, Harvard University
2013–Present	Professor of Physics, Harvard University
2008–2012	Chair of Department of Mathematics, Harvard University
2003–Present	Distinguished Professor-at-Large, The Chinese University of Hong Kong
2000–Present	William Caspar Graustein Professor of Mathematics, Harvard University
2009–Present	Director of the Mathematical Sciences Center, Tsinghua University
2002–Present	Director of the Centre of Mathematical Sciences, Zhejiang University
1996–Present	Director of the Morningside Center of Mathematics at Chinese Academy of Sciences
1994–Present	Director of The Institute of Mathematical Sciences, The Chinese University of Hong Kong
1987–Present	Professor of Mathematics, Harvard University
1997–2000	Higgins Professor of Mathematics (Chair Professor), Harvard University
1994–2003	Adjunct Professor of Mathematics, The Chinese University of Hong Kong
1984–1987	Chancellor Associate Chair and Professor of Mathematics, University of California, San Diego
1979–1984	Professor of Mathematics, Institute for Advanced Study, Princeton University
1974–1979	Professor of Mathematics, Stanford University
1972–1973	Assistant Professor of Mathematics, State University of New York, Stony Brook
1971–1972	Member of the School of Mathematics, Institute for Advanced Studies, Princeton University

VISITING PROFESSOR:

Apr–May 2007	Visiting University of California, Los Angeles
Jan–Mar 2007	Visiting University of California, Irvine
Jan–Jun 2002	Gordon Moore Visiting Professor, Department of Mathematics, California Institute of Technology
1999	Samuel Eilenberg Visiting Professor, Department of Mathematics, Columbia University
1996	John Harvard Fellow, Issac Newton Institute for Mathematical Sciences and Faculty of Mathematics, University of Cambridge, U.K.
1991–1992	Special Chair, Department of Mathematics, Tsing Hua University (Hsinchu)

1991–1992	Wilson T. S. Wang Distinguished Visiting Professor, Department of Mathematics, The Chinese University of Hong Kong
Sept 1990	Distinguished Visiting Professor, Department of Mathematics, State University of New York, Stony Brook
1989	Sherman Fairchild Distinguished Visiting Scholar, Department of Mathematics, California Institute of Technology
Fall 1986	Sid Richardson Centennial Chair in Mathematics and Visiting Professor, Department of Mathematics, University of Texas at Austin

ADDITIONAL AFFILIATIONS:

2009–present	Honorary Professor, Wuhan University
2009–present	Honorary Professor, Hunan Normal University
2009–present	Honorary Professor, North University of China
2009–present	Honorary Professor, Northwest University
2006–present	Honorary Professor, Huazhong University of Science and Technology
2002–present	Honorary Professor, Zhejiang University
1999–present	Honorary Professor, University of Science and Technology of China
1998–present	Honorary Professor, Peking University
1993–present	Honorary Professor, Nankai University
1987–present	Honorary Professor, Tsinghua University
1987–present	Honorary Professor, Hangzhou University
1983–present	Honorary Professor, Fudan University
1983–present	Honorary Professor, Chinese Academy of Sciences

AWARDS and FELLOWSHIPS:

2010	The Wolf Prize
2010	AAEOY Distinguished Science & Technology Award
2003	2003 International Scientific and Technological Cooperation Award
1997	National Medal of Science, U.S.A
1994	Crafoord Prize, The Royal Swedish Academy of Sciences
1991	Humboldt Research Award, The Alexander von Humboldt Foundation, Germany
1985	John D. and Catherine T. MacArthur Fellowship
1984	One of Americas 100 brightest scientists under 40, Science Digest
1982	Fields Medal, International Congress of Mathematicians
1981	The John J. Carty Award for the Advancement of Science, National Academy of Sciences, U.S.A.

1981	The Oswald Veblen Prize in Geometry, American Mathematical Society
1980	John Simon Guggenheim Fellowship
1979	California Scientist of the Year, California Science Center
1975–1976	Alfred P. Sloan Fellow

MEMBERSHIPS:

Jan 2013	Fellow, American Mathematical Society
2009	Member of the Selection Committee for Distinguished Research Achievement Award (DRAA) of the University of Hong Kong
2008	Foreign Member, Indian National Academy of Science
2005	Foreign Member, National Academy of Lincei, Italy
Jul 2005	Member of the Overseas Expert Consultant Committee of Overseas Chinese Affairs Office of State Council
2003	Foreign Member, Russian Academy of Sciences
1995	Foreign Member, Chinese Academy of Sciences
1993	Member, National Academy of Sciences, U.S.A.
1993	Fellow, American Association for the Advancement of Science
1990–1992	Member-at-Large, Council of the American Mathematical Society
1989	Member, Scientific Advisory Council, Mathematical Sciences Research Institute
1989	Member, Board of Mathematical Sciences, National Academy of Science, U.S.A.
1985	Member, American Physical Society
1985	Member, Society for Industrial and Applied Mathematics, U.S.A.
1984	Academician, Academia Sinica, Taiwan, China
1983	Fellow, New York Academy of Sciences
1982	Member, American Academy of Arts and Sciences
1980	Honorary Member, Academic Committee, Institute of Mathematics, Chinese Academy of Sciences, Beijing, China
1971	Member, American Mathematical Society

EDITORSHIPS:

Present	Editorial Boards, Editor, *American Journal of Mathematics*
2007–Present	Editors-in-Chief, *Communications in Number Theory and Physics*
2005–Present	Editors-in-Chief, *Pure and Applied Mathematics Quarterly*
2004–Present	Editor-in-Chief, *Dynamics of Partial Differential Equations*
2001–Present	Honorary Editor, *Communications in Information and Systems*
1997–	Editorial Boards, Editor, *Journal of Mathematical Physics*

1997–Present	Editor-in-Chief, *Advances in Theoretical Mathematics and Physics*
1997–Present	Editor-in-Chief, *Asian Journal of Mathematics*
1994–Present	Editor-in-Chief, *Methods and Application of Analysis*
1994–	Editorial Boards, Editor, *Advances in Mathematics*
1993–Present	Editorial Boards, Editor, *Mathematical Research Letters*
1993–Present	Editorial Boards, Advisor, *Communications in Analysis and Geometry*
1993–Present	Editorial Boards, Advisor, *Methods and Applications of Analysis*
1982–1999	Editor, *Communication in Mathematical Physics*
1981–1988	Editor, *Inventiones Mathematicae*
1980–Present	Editor-in-Chief, *Journal of Differential Geometry*

INVITED LECTURESHIPS:

Oct 5, 2012	The Edmund R. Michalik Distinguished Lecture Series, University of Pittsburgh, U.S.A.
Jan 17, 2012	2012 McKnight-Zame Distinguished Lecture at University of Miami, U.S.A.
Jan 20–21, 2011	The Fields Institute Distinguished Lecture Series at University of Toronto, Canada
Jul 13, 2010	Distinguished Applied Math. Lecture at Chiao Tung University (Hsinchu), China
Jul 11, 2010	2010 NCTS International Conference on "Several Complex Variables and Complex Geometry" at the Center for Theoretical Sciences, Taiwan, China
Jun 2010	Workshop "Geometric Analysis and General Relativity" at The Banff International Research Station for Mathematical Innovation and Discovery (BIRS), Canada
Nov 10, 2009	Invariants in Algebraic Geometry at The University of Tokyo, Japan
Sept 7–8, 2009	Keynote Speaker for the 13th IMA Conference on The Mathematics of Surfaces at York, England, U.K.
Jun 30, 2009	Distinguished Lecture Series Conference at Taida Institute for Mathematical Sciences, China
Jun 18, 2008	ICTP Trieste, Italy
May 30, 2008	Scientist Forum at Yunnan, China
May 21, 2008	80th Hirzebruch Conference, Bar Ilan University, Tel Aviv, Israel
Mar 11, 2008	IPMU Opening Symposium, Japan
Dec 26, 2007	Indian Mathemathics Society, India
Dec 17–22, 2007	International Congress of Chinese Mathematicians, China
Aug 27–31, 2007	Geometrie differentielle, Physique mathematique, Mathematiques and societe, IHES, France
May 2007	Distinguished Lecture Series (DLS) 2006-2007 at UCLA, U.S.A.

Jan 2006	Brauer Lectures, The University of North Carolina at Chapel Hill, U.S.A.
2005	String 2005, Fields Institute, Toronto, Canada
May–Jun 2005	Andrejewski Lectures, Gottingen, Germany
Sept–Dec 2004	Eilenberg Lectures, Department of Mathematics, Columbia University, U.S.A.
May 2004	Bloomberg Lecture, University of Texas at Austin, U.S.A.
2003	Distinguished Lecture Series, Department of Mathematics, University of California at LA, U.S.A.
Oct 2003	Andre Aisenstadt Chair Lecture Series, Department of Mathematics, Université de Montréal, Canada
1999	Hans Rademacher Lecture, Department of Mathematics, University of Pennsylvania, U.S.A.
1999	Stefan Bergman Lecture, Department of Mathematics, Stanford University, U.S.A.
1998	Run Run Shaw Distinguished Lecture, The Chinese University of Hong Kong, China
1997	Rufus Bowen Lecture, Department of Mathematics University of California, Berkeley, U.S.A.
1988	American Mathematical Society Colloquium, U.S.A.
1983	James K. Whittemore Lecture, Department of Mathematics, Yale University, U.S.A.
1982	Alexander Ziwet Lecture, Department of Mathematics, University of Michigan, Ann Arbor, U.S.A.
1982	International Mathematical Union Lecture, Zurich, Switzerland
1981	The 33rd British Mathematical Colloquium, London Mathematical Society, U.K.
1979	Milton Brockett Porter Lecture, Department of Mathematics, Rice University, U.S.A.
1978	International Congress of Mathematicians, Plenary Speaker, Helsinki, Finland

Preface

In the early spring of 2013, Lizhen Ji asked me to write comments about my collected or selected works. I was too busy at the time to take on such a task. At one point, however, I gave in to his request and decided to write comments about my survey articles. Upon tallying them up, I was surprised to see that I had written far more survey articles than I had remembered.

Since I was a child, I have always been interested in history. Hence when I started to write these commentaries, I tried to stick to the facts to the best of my memory. I also consulted friends who participated in these events and looked at letters and emails that I had kept over the past forty years.

This does not mean that there are absolutely no mistakes in the statements. Nevertheless, I believe that these accounts can be interesting – and maybe even important – for students who'd like to know something about how the various papers were written and what my friends and I thought about the approaches we took.

In the course of putting together this collection, I received strong support from Lizhen Ji, Hao Xu, Kefeng Liu, Shiu-Yuen Cheng, and Hung-Hsi Wu. I am also very grateful to the publishers led by Liping Wang, Yushan Deng, and others. My friend Steve Nadis agreed to be the consulting editor for this project. I am extremely thankful for all of their help, without which this project likely would not have materialized.

Shing-Tung Yau
June 30, 2014

Preface

Shing-Tung Yau, His Mathematics and Writings

1 Why selected works

There has been a long tradition of publishing collected or selected works of distinguished mathematicians. There are several good reasons for doing this, and it has served many purposes. Probably the most obvious one is that collected and selected works provide an easy access to papers that are scattered in different journals, some of which are not easily accessible to many people. Otherwise, few people, if any, will take the time and trouble to dig up all the papers of their admired mathematicians–especially not those papers that are far away from their interests, of their admired mathematicians and read them. On the other hand, reading papers of a master dealing with different subjects or areas conveys the underlying unity and hence a big picture of mathematics, and it also allows one to gain a historical perspective (or to enter the history). In other words, collected and selected works are more than the simple sum of individual papers.

Indeed, as Abel said famously, we learn "by studying the masters, not their pupils." Even though the world is becoming smaller, few people have many chances to interact with masters who are alive. Of course, the next best way to learn from masters is to read and study their collected works.

Naturally, publishing collected or selected works is also an honor to the authors of these papers. It should be mentioned that collected works of some people can bring honor of the genre of collected works.

Now, with the wide and easy use of e-papers and e-books, most papers in journals can be obtained easily online, and a mere reprinting of papers is probably not as valuable as before. Of course, the value of selected works still stands. For example, holding and reading a beautifully printed book is definitely different from viewing papers online or on e-book readers. But they should also provide something else. Several additional things seem to be reasonable: descriptions of how ideas in the paper were formed and time and place the papers were written, relations between papers with the advantage of hindsight, and developments of subjects after the papers were published, and visions for the future. In other words, they should explain the circumstances of the birth of papers and proper, impacts of the papers, and fitting these papers in the grand scheme of mathematics.

These additional things are especially important to beginners, non-experts and even some experts. Most people often concentrate on the best known theorems and most important papers of great mathematicians, but even masters struggled and stumbled sometimes on their mathematical trips. How they found good problems and their ways in their careers, made progress and reached peaks is best described by their own papers, recollections and commentaries, but not textbooks where everything is polished and presented in a streamlined matter, without mentioning that textbooks and research books might not cover some gems in the original papers that are not directly related to the themes of the books. But many people, especially younger ones, often prefer to read polished textbooks. Of course, reading mathematics papers can be difficult (more difficult than reading textbooks), and proper arrangement of related papers and additional guides from the masters are certainly valuable and helpful. Such collected or selected works of distinguished mathematicians often tell good stories of the authors and their mathematics, and browsing through them can be enjoyable and beneficial to people who are not interested in some specific results in the papers.

In these works of expository writings of Shing-Tung Yau, all these things are printed together with his survey papers and papers on open problems. One reason for restricting these volumes to expository papers of Yau is practical. Yau has been very creative and prolific. The collected works including all his papers (both research and expository papers) up to now will occupy too many volumes. Besides, he is also still very active and productive, and the time for collected works may not be ripe yet.

2 Why expository writing

Probably some explanation is needed for publishing these volumes of expository writings of Yau now. Briefly, it is the right time for Yau to share his perspectives and his vision on the broad area of geometric analysis, and his expository writings provide a unique means to this end. They will render a valuable service to the mathematics community.

Colloquium talks have been a common means of communication between mathematicians from different subjects, after they were made successful and popular by Klein and Hilbert in Göttingen about 100 years ago. More expository talks such as *Basic notion seminars* and *What is ...* have also sprung up in many places. They provide effective ways for people to learn and enjoy some beautiful pieces of mathematics, which are outside their fields of speciality. Though there are many books and papers dealing with all kinds of subjects in mathematics, one difficulty is that there are too many of them. It is difficult for people to find the right books and papers, and people may lack the motivation to read mathematics outside their specialities, especially when they involve difficult and technical material. Many people choose to study mathematics not for fame or fortune, but for the beauty and enjoyment of the discipline. To really appreciate the beauty and power of mathematics, one has to roll up one's sleeves and do the work. But not many people can work in many different subjects in mathematics. In the history