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Preface

One of the main purposes for the study of complex network topology and modeling is to understand the influence of network structure on its function and consequently to find effective ways to improve network performance. In practice, directly controlling every node in a dynamical network with a huge number of nodes might be impossible but might also be unnecessary. Therefore, a pinning control strategy, that is, to achieve the goal of control by directly adding control input to a fraction of nodes selected from the network, is very important for the control of networked systems.

This monograph intends to investigate the synchronization, consensus and flocking of networked systems via pinning control strategy, which aims to control the whole dynamical network via imposing controllers on only a fraction of nodes. For a dynamical network with fixed and connected topology, the feasibility and effectiveness of different pinning control strategies are investigated. For a dynamical network with time-varying and possibly disconnected topology, consensus and flocking control of mobile multi-agent systems with limited communication capabilities are studied. Research on pinning control of networked systems can not only help better understand the mechanisms of natural collective phenomena, but also benefit the applications in mobile sensor/robot networks.

This monograph is organized as follows. Chapter 1 overviews recent research in pinning control of complex networked systems. Chapters 2 and 3 introduce synchronization of complex dynamical networks via pinning, including pinning control for complete synchronization and pinning control for cluster synchronization. Chapters 4 and 5 study consensus of multi-agent systems via pinning, including distributed pinning-controlled second-order consensus of multi-agent systems and distributed pinning-controlled consensus in a heterogeneous influence network. The pinning-controlled flocking of multi-agent systems is investigated with a virtual leader in Chap. 6 and with preserved network connectivity in Chap. 7, respectively.

We would like to thank those colleagues who have helped us on the topics of this book. They are Professor Guanrong Chen of City University of Hong Kong, Professor Zongli Lin of University of Virginia, Professor Xiang Li of Fudan University, Professor Michael Zhiqiang Chen of the University of Hong Kong, Professor Zhihai Rong of Donghua University and Doctor Wen Yang of East China University of Science and Technology. We gratefully acknowledge the support of the National Natural Science Foundation of China under Grant Nos. 61074125 and 61104140, the Research Fund for the Doctoral Program of Higher Education (RFDP) under Grant No. 20100142120023, the Natural Science Foundation of Hubei Province of China under Grant No. 2011CDB042.

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Contents

Chapter 1
Overview

Abstract This chapter overviews recent research results in pinning control of complex networked systems, which are roughly categorized as synchronization of complex dynamical networks via pinning, and consensus and flocking of multi-agent systems via pinning. Some background and motivation for pinning control of complex networked systems is provided. Theoretical and simulation analysis results regarding pinning control of networks with fixed and switching topologies are summarized. Finally, some future directions are pointed out.

Keywords Pinning control · Complex networked systems · Synchronization · Complex dynamical networks · Consensus · Flocking · Multi-agent systems

1.1 Introduction

Automatic control theory deals with influencing the behavior of dynamical systems, which is an interdisciplinary subfield of science. Many control strategies are proposed for different specific control systems, such as adaptive control, hierarchical control, intelligent control, optimal control, robust control, stochastic control, and distributed control.

With the rapid development in network theory, computer technology, and communication technology, the investigation of control theory is recently being further extended from a single dynamical system to multiple interconnected systems. In particular, the problem of coordinated control of multi-agent systems has received increasing interest in different fields including biology, physics, computer science, and control engineering, partly due to the wide applications in many control areas, including cooperative control of mobile robots and design of mobile sensor networks [1] . Coordinated control of multi-agent systems means that a large number of agents, relying only on its neighbors' local information, reach a coordinated motion, which are characterized by distributed control, local interactions and self-organization. Indeed, the spirit of coordinated control protocol is inspired by the pioneering models of Reynolds [2] and Vicsek [3] . Many versions of distributed coordinated control protocols have been proposed, including consensus, swarming, flocking, and rendezvous.

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In recent years, there has also been increasing interest in the study of complex dynamical networks, which is partially spurred by the discovery of the small-world and scale-free properties of many natural and artificial complex networks [4] . In 1998, a small-world network model was proposed by Watts and Strogatz [5] , which has the properties of a small average path length and a large clustering coefficient. The connectivity distribution of a small-world network peaks at an average value and decays exponentially. In the following year, a scale-free network model was proposed by Barabási and Albert [6] . Unlike the homogeneous distributions of small-world networks, scale-free networks have inhomogeneous degree distributions in a power-law form.

Synchronization phenomena are ubiquitous in nature. There has been significant interest in the study of synchronization from different fields (see, for example, [7,8] and the references therein). Two main lines of research on the problems of synchronization have emerged from this study. On the one hand, the two pioneering papers on synchronization in coupled systems [9] and synchronization in chaotic systems [10] have stimulated a great deal of interest in the study of complete synchronization of coupled nonlinear dynamical systems. On the other hand, there has been much interest in the study of synchronization in dynamical networks with complex topologies in the past decade [4,11] .

Control will be a necessary means for guiding or forcing the network to achieve desired synchronization if a given network of dynamical systems is not synchronizable or if the synchronized state is not the desired state. In practice, directly controlling every node in a dynamical network with huge number of nodes might be impossible but might also be unnecessary. Therefore, a pinning control strategy, that is, to achieve the goal of control by directly adding control inputs to a fraction of nodes selected from the network, is very important for the control of networked systems. A pinning control algorithm was studied to suppress spatial chaos in coupled map lattices and multi-mode laser systems [12-14] . Wang and Chen firstly studied the control of a scale-free dynamical network to its equilibrium via pinning [15] . Li, Wang, and Chen further specified the stable conditions and bridged the pinning control with virtual control propagation to explore the efficiency of selective pinning strategies [16] . Since then, many researchers have contributed to the fruitful understandings in this topic, including complete synchronization, cluster synchronization, selective strategies, controllable regions, and control methodologies for different specific scenes. Recently, some experiments were proposed to illustrate the effectiveness of the pinning control strategy [17] .

Subsequently, the idea of pinning control has also been applied to coordinated control of multi-agent systems. From the viewpoint of complex network theory, one of the main challenges with coordinated control of multi-agent systems is that the topology of the corresponding dynamical network is time-varying which depends on the states of all the agents in the network. Furthermore, connectivity of the initial network cannot guarantee connectivity of the network all the time. One way to overcome this difficulty is to assume that there is a real or virtual leader and that every agent is an informed agent which has the information of the leader so that a navigational feedback term could be added to every agent. In this way, all agents could remain cohesive and asymptotically move with the same desired velocity no matter whether the initial network is connected or not. However, this assumption is in contrast with some nature examples and may be difficult to implement in engineering applications. Therefore, by using pinning control strategy, coordinated control of multi-agent systems with only a fraction of informed agents is more practical.

In this chapter, we overview recent research results in pinning control for synchronization of complex dynamical networks, and consensus and flocking of multi-agent systems via pinning, which include the following topics:

(i) Stability conditions for synchronization of complex dynamical networks via pinning. It has been well known that the stability conditions for synchronization of complex dynamical networks via pinning depend on the intrinsic dynamics of the nodes and the topology of the network. Various methods have been investigated towards finding sufficient conditions for global or local asymptotic stability of the pinning process.

(ii) Selective strategies of pinning control. The efficiency to fulfill different specifications varies from the network considered and the pinning strategies chosen. Selective pinning strategies play an important role to achieve better efficiency for stabilizing a complex networked system.

(iii) Controllable regions. By evaluating the pinning controllable regions of a given complex dynamical network, one could decide on the effectiveness of a selective pinning strategy and select the best selective pinning strategy.

(iv) Control methodologies. In order to adapt the different specifications in realistic scenario, many control methodologies have been introduced for the pinning control of complex dynamical networks, including adaptive control, intermittent control, impulsive control, stochastic control, finite-time control, and control with time delays.

(v) Consensus and flocking of multi-agent systems with real or virtual leaders. Leaders are commonly adopted to help the agents achieve a desired common velocity or arrive at a desired destination. In switching multi-agent networks, many versions of distributed coordinated control protocols have been investigated towards finding what sufficient conditions can guarantee the stability of the multi-agent systems and what agents should be chosen as pinned candidates.

(vi) Connectivity maintenance. One effective way to achieve coordinated control is to design a protocol which could preserve connectivity of the network during the evolution. In this way, a single informed agent is enough to guide all other agents in the network.

1.2 Synchronization of Complex Dynamical Networks via Pinning

Consider a complex network consisting of N identical linearly and diffusively coupled nodes of n-dimensional dynamical system described by

$$\dot{x}_i(t) = f\big(x_i(t), t\big) + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \Gamma\big(x_j(t) - x_i(t)\big), \quad i = 1, \ldots, N, \tag{1.1}$$

where $x_i = [x_i^1, \ldots, x_i^n]^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector of the ith node, $t \in [0, +\infty)$ is continuous time, $f : \mathbf{R}^n \times [0, +\infty) \rightarrow \mathbf{R}^n$ is a continuous map, and c_{ij} denote the coupling strengths between node i and node j, $\Gamma = (\tau_{ij}) \in \mathbf{R}^{n\times n}$ is a matrix linking coupled variables, and if some pairs (i, j), $1 \leqslant i, j \leqslant n$, with $\tau_{ij} \neq 0$, then it means two coupled nodes are linked through their ith and jth state variables, respectively.

1.2.1 Stability Conditions for Complete Synchronization

Network (1.1) is said to realize complete synchronization if

$$\lim_{t\to\infty} \big\| x_i(t) - x_j(t) \big\| = 0$$

for all i and j. The problem of pinning control for synchronization of network (1.1) is to directly control a fraction of nodes in the network to achieve

$$\lim_{t\to\infty} \big\| x_i(t) - \overline{x}(t) \big\| = 0, \quad i = 1, \ldots, N,$$

where the homogeneous stationary state satisfies

$$f\big(\overline{x}(t), t\big) = 0. \tag{1.2}$$

The controlled network is described with a local negative feedback control law as follows:

$$\dot{x}_i(t) = f\big(x_i(t), t\big) + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \Gamma\big(x_j(t) - x_i(t)\big) + h_i c_i \Gamma\big(\overline{x}(t) - x_i(t)\big),$$
$$i = 1, \ldots, N, \tag{1.3}$$

where c_i is the feedback gain. If node i can obtain information from node j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. If node i is selected for feedback, then $h_i = 1$; otherwise, $h_i = 0$.

Pinning control for synchronization of complex dynamical networks was firstly investigated in [15] for the stabilization of a scale-free dynamical network. A simple criterion for the local stabilization of complex dynamical networks was proposed, which depends on the topology of the network, the feedback gain, and the node intrinsic dynamics [15] . Li, Wang, and Chen further investigated both the global and local stabilization of complex dynamical networks via the pinning control strategy [16]. Sufficient conditions were presented to guarantee the convergence of complex dynamical networks locally and globally by pinning only one node [18] . Pinning control based on the Lyapunov V-stability approach was investigated, which converts the network stability problem into the measurement of the negative definiteness of one simple matrix that characterizes the topology of the network [19]. Pinning-controlled networks were analyzed via a renormalization approach, which included two operations, edge weighting, and node reduction [20] . Based on the Lyapunov V-stability approach, a sufficient condition for the pinning control of uncertain complex networks was derived in terms of linear matrix inequalities [21] . In [22], the authors studied pinning control of complex dynamical networks with asymmetric and heterogeneous coupling. A low-dimensional condition without the pinning controllers involved for the pinning control of complex dynamical networks was proposed in [23]. By using the local pinning control algorithm, some sufficient conditions concerning the global stability of controlling a complex network with digraph topology to a homogeneous trajectory of the uncoupled system were derived in [24]. The authors proposed some low-dimensional pinning criteria for global synchronization of both directed and undirected complex networks [25] . Several pinning criteria were proposed to guarantee synchronization for the controlled dynamical network with uncertainties [26] .

Due to the finite speeds of transmission and spreading as well as traffic congestions, in reality there usually are time delays in spreading and communication. Both delay-independent and delay-dependent asymptotical stability criteria were derived for a weighted general undirected complex dynamical network with constant time delay [27] . Some generic stability criteria ensuring delay-independent stability were derived for pinning control of complex dynamical networks with heterogeneous delays in both continuous-time and discrete-time domains [28] . By adding linear and adaptive feedback controllers to a fraction of nodes, some sufficient conditions for the global synchronization were obtained without assuming the symmetry of the coupling matrix [29] . Both linear and adaptive feedback schemes were used to pin a complex delayed dynamical network to a homogeneous trajectory [30] . The delayed dynamical network was theoretically proved to be asymptotically synchronized with linear feedback control, and to be exponentially asymptotically synchronized with adaptive feedback controllers [30] . Some less conservative criteria for both continuous-time and discrete-time complex dynamical networks with time delay were obtained by solving optimal problems of a series of linear matrix inequalities [31] . Some stability conditions were attained for pinning control of time-varying polytopic directed stochastic networks [32] . Several adaptive synchronization criteria were obtained for a given complex dynamical network with both delayed and non-delayed couplings [33] . In [34], the authors proposed some local stability conditions for pinning control of fractional-order weighted complex networks. Some criteria were derived for pinning stabilization of linearly coupled stochastic neural networks [35] .

1.2.2 Stability Conditions for Cluster Synchronization

Suppose d nonempty subsets (clusters) $\{G_1, \dots, G_d\}$ form a partition of the index set $\{1, 2, \dots, N\}$, where $\bigcup_{l=1}^{d} G_l = \{1, 2, \dots, N\}$ and $G_l \neq \emptyset$. A network with N

nodes is said to realize d-cluster synchronization if

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0$$

for all i and j in the same cluster, and

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|\neq 0$$

for all i and j in different clusters.

The problem of pinning control for d-cluster synchronization is to directly control a small fraction of nodes in network (1.1) to achieve

$$\lim_{t\to\infty}\sum_{l=1}^{d}\sum_{i\in G_l}\|x_i(t)-\overline{x}_l(t)\|=0,$$

where $\overline{x}_l(t)$ is the desired state of the lth cluster G_l, which naturally is required to satisfy

$$\dot{\overline{x}}_l(t)=f\big(\overline{x}_l(t),t\big),\quad l=1,\ldots,d, \tag{1.4}$$

where $\overline{x}_l(t)$ is an equilibrium of the node system. The controlled network is described as follows:

$$\dot{x}_i(t)=f\big(x_i(t),t\big)+\sum_{j=1,j\neq i}^{N}c_{ij}a_{ij}\Gamma\big(x_j(t)-x_i(t)\big)+h_ic_i\Gamma\big(\overline{x}_{\hat{i}}(t)-x_i(t)\big),$$
$$i=1,\ldots,N, \tag{1.5}$$

where $\hat{i}$ is the subscript of the subset for which $i\in G_{\hat{i}}$. If node i is selected to be pinned, then $h_i=1$; otherwise, $h_i=0$.

Due to the specific goals in practice, many biological, social, and technological networks functionally divide into communities. Therefore, cluster synchronization of complex dynamical networks has received increasing interest and had many applications in practice. For example, two subgroups will be naturally formed in social networks when a crowd of people choose to accept or reject an opinion according to their preference. When a group of robots are to carry out a complex task, subtasks will divide the robot network into communities, and consensus should be achieved within each community. By introducing the inhibitory coupling to the coupling matrix, some sufficient conditions were presented to guarantee the cluster synchronization of linearly coupled complex networks under pinning control [36]. In [37], the authors investigated both local and global stability conditions for cluster synchronization of complex dynamical networks, in which at least those nodes with direct connections between groups in a network with community structure should be pinned. By introducing local adaptive strategies for both coupling strengths and feedback gains, it was shown that the collective dynamics of the underlying complex network can be controlled to its heterogeneous stationary states without requiring global information of the network [38] . By proposing an adaptive pinning feedback to the coupling strength, the authors of [39] derived sufficient conditions for cluster synchronization with nonlinearly coupled nonidentical dynamical systems and asymmetrical coupling matrix. In [40], some sufficient conditions were derived to guarantee global cluster synchronization by using intermittent pinning control. In [41], several sufficient conditions were proposed to guarantee cluster synchroniza tion of complex dynamical networks via the free matrix approach and stochastic analysis techniques, when at least one node in each cluster was pinned. Cluster synchronization of complex dynamical networks with community structure and nonidentical nodes was studied via pinning control and adaptive coupling strength [42] .

1.2.3 Selective Strategies of Pinning Control

Exploring network topology is crucial for understanding and predicting network dynamics. For example, using the inhomogeneous nature of scale-free networks, specifically applications, such as target immunization against epidemic spread [43] and high degree seeking algorithm in routing and searching [44] , the performance is much better than that of random methods regardless of the topological characteristic of networks. An important topic in pinning control of complex dynamical networks is to study the relationship between network topology and effective pinning strategy. In [15], the authors showed that pinning the high-degree nodes is more efficient than pinning randomly chosen ones in an unweighted symmetrical scale-free network, but there is no significant difference between a random pinning scheme and a high-degree pinning scheme in a random network. In [16], the authors further proposed the concept of virtual control and showed why a high-degree pinning scheme is more effective than a random pinning one in a scale-free network. In order to increase the synchronization performance, it was shown that the nodes adjacent to the highest degree node are good candidates to be pinned [45] . The authors of [46] showed that a max-degree pinning scheme is better than a random pinning one in a disassortative network, while the combination of the two schemes is more effective in an assortative network. In [47], the authors illustrated that pinning the small nodes is better than pinning the big nodes when the portion of pinned nodes is relatively large in an unweighted symmetrical scale-free network, but pinning the big nodes is, in fact, always better than pinning the small ones in normalized weighted scale-free networks. The investigation in [48] showed that pinning the nodes with high betweenness centrality is usually more effective than pinning the ones with high degree, partly due to the fact that betweenness centrality contains more information with the degree as well as the shortest path. The authors illustrated that for any network structure, the pinning control performance was maximized via uniform pinning of all the network nodes [49] . For a scale-free directed dynamical network, it was shown that pinning a vertex with the largest ControlRank is much more effective than pinning a vertex with the largest out-degree [50] . The authors modified the traditional degree-based strategy, and used a decrease-and-conquer approach to assign the best control strengths to the pinned highest-degree nodes, which provides better control performance than conventional methods, based on degree, betweenness centrality, and closeness [51] .

1.2.4 Controllable Regions

It is very important for a complex dynamical system to evaluate the controllable regions of the system, which can help deciding on the effectiveness of a pinning control strategy, the number of nodes to be pinned, and the location of the pinned nodes in the entire network. Based on the eigenvalue analysis, the authors obtained the controllable regions directly in the control parameter space for both the diagonal coupling and nondiagonal couplings [52] . An extension of the master stability function approach was proposed for the controllability of networks under pinning control schemes, in which the controllable regions of the system were characterized by the coupling gain, the control gain, and the number of pinned nodes [53] . Several manageable criteria were proposed, and the effects of the network topology, the location and number of pinned nodes, and the nodes' intrinsic dynamics on the global pinning-controllability were analyzed for global pinning-controllability of complex networks [54]. In [55], the stable regions of the coupled network were clarified and the eigenvalue distribution of the asymmetric coupling and control matrices were specified for asymmetric complex dynamical networks.

1.2.5 Control Methodologies

In order to fulfill different specifications varying from model to model, different control methodologies were adopted for pinning control of complex dynamical networks. Based on a centralized adaptive strategy, it is possible to control the collective dynamics of a complex network with a weak coupling strength to a desired state by pinning only one single node [18]. In [36], the centralized adaptive strategy for complete synchronization of complex dynamical networks in [18] was extended to the case of adaptive cluster synchronization. By introducing local adaptive strategy on the feedback gains, several adaptive synchronization criteria were attained for weighted complex dynamical networks [56] and general complex dynamical networks [57]. A fully decentralized adaptive pinning control scheme for cluster synchronization of undirected complex dynamical networks by introducing local adaptive strategies to both coupling strengths and feedback gains was considered in [58, 59]. An adaptive pinning control scheme was investigated for a class of uncertain complex networks against network deterioration [60] . In order to achieve finite-time synchronization, discontinuous pinning controllers were designed for coupled neural networks[61] . Without traditional assumptions on control width, a pinning periodically intermittent scheme was designed for pinning control of complex delayed dynamical networks [62] . In order to be effective on the dynamical systems which are subject to instantaneous perturbations at certain instants, pinning control of complex delayed dynamical networks was investigated by a single impulsive controller [63]. The synchronization of stochastic discrete-time networks was investigated via impulsive pinning controllers [64].

1.3 Consensus and Flocking of Multi-Agent Systems via Pinning

Consider a multi-agent system consisting of N agents of an n-dimensional dynamical system described by

$$\dot{x}_i(t) = f(x_i, u_i), \quad i = 1, \ldots, N, \tag{1.6}$$

where $x_i = [x_i^1, \ldots, x_i^n]^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector of the ith agent, f is a smooth vector field representing its dynamics, $u_i = [u_i^1, \ldots, u_i^m]^{\mathrm{T}} \in \mathbf{R}^m$ is the control input which can only use the states of its neighbors.

1.3.1 Single Virtual Leader Case

The problem of consensus with a virtual leader is to design control input u_i, $i = 1, 2, \ldots, N$, such that

$$\lim_{t \to \infty} \left\| x_i(t) - x_\gamma(t) \right\| = 0, \tag{1.7}$$

for all $i = 1, 2, \ldots, N$, where x_γ is the state of the virtual leader, which satisfies

$$\dot{x}_\gamma = f(x_\gamma). \tag{1.8}$$

If the state of each agent is composed of its position and velocity, then $x_i = [q_i, p_i]^{\mathrm{T}}$, where q_i is the position of the ith agent and p_i is the velocity of the ith agent. The problem of flocking with a virtual leader is to design control input u_i, $i = 1, 2, \ldots, N$, such that each agent can asymptotically approach the velocity of the virtual leader, the distance between any two agents is asymptotically stabilized, and collisions among the agents can be avoided.

Based on the artificial potential function method, the coordinated control of multiple autonomous agents was investigated with a virtual leader in [65]. The multi-agent consensus problem with an active leader and variable interconnection topology was studied via a neighbor-based state-estimation strategy in [66]. Consensus algorithms with a time-varying virtual leader were investigated for vehicles modeled by single integrator dynamics in [67]. Second-order consensus of multi-agent systems with a bounded control input and a fraction of informed agents was studied in [68]. Based on the contraction analysis and multiple Lyapunov functions, the consensus of multi-agent systems with general nonlinear coupling was investigated in [69]. The consensus problem for directed networks was investigated