David G. Luenberger
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Linear and Nonlinear Programming

Third Edition

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To Susan, Robert, Jill, and Jenna;
Daisun and Fei

PREFACE

This book is intended as a text covering the central concepts of practical optimization techniques. It is designed for either self-study by professionals or classroom work at the undergraduate or graduate level for students who have a technical background in engineering, mathematics, or science. Like the field of optimization itself, which involves many classical disciplines, the book should be useful to system analysts, operations researchers, numerical analysts, management scientists, and other specialists from the host of disciplines from which practical optimization applications are drawn. The prerequisites for convenient use of the book are relatively modest; the prime requirement being some familiarity with introductory elements of linear algebra. Certain sections and developments do assume some knowledge of more advanced concepts of linear algebra, such as eigenvector analysis, or some background in sets of real numbers, but the text is structured so that the mainstream of the development can be faithfully pursued without reliance on this more advanced background material.

Although the book covers primarily material that is now fairly standard, it is intended to reflect modern theoretical insights. These provide structure to what might otherwise be simply a collection of techniques and results, and this is valuable both as a means for learning existing material and for developing new results. One major insight of this type is the connection between the purely analytical character of an optimization problem, expressed perhaps by properties of the necessary conditions, and the behavior of algorithms used to solve a problem. This was a major theme of the first edition of this book and the second edition expands and further illustrates this relationship.

As in the second edition, the material in this book is organized into three separate parts. Part I is a self-contained introduction to linear programming, a key component of optimization theory. The presentation in this part is fairly conventional, covering the main elements of the underlying theory of linear programming, many of the most effective numerical algorithms, and many of its important special applications. Part II, which is independent of Part I, covers the theory of unconstrained optimization, including both derivations of the appropriate optimality conditions and an introduction to basic algorithms. This part of the book explores the general properties of algorithms and defines various notions of convergence. Part III extends the concepts developed in the second part to constrained optimization problems. Except for a few isolated sections, this part is also independent of Part I. It is possible to go directly into Parts II and III omitting Part I, and, in fact, the book has been used in this way in many universities. Each part of the book contains enough material to form the basis of a one-quarter course. In either classroom use or for self-study, it is important not to overlook the suggested exercises at the end of each chapter. The selections generally include exercises of a computational variety designed to test one's understanding of a particular algorithm, a theoretical variety designed to test one's understanding of a given theoretical development, or of the variety that extends the presentation of the chapter to new applications or theoretical areas. One should attempt at least four or five exercises from each chapter. In progressing through the book it would be unusual to read straight through from cover to cover. Generally, one will wish to skip around. In order to facilitate this mode, we have indicated sections of a specialized or digressive nature with an asterisk*.

There are several features of the revision represented by this third edition. In Part I a new Chapter 5 is devoted to a presentation of the theory and methods of polynomial-time algorithms for linear programming. These methods include, especially, interior point methods that have revolutionized linear programming. The first part of the book can itself serve as a modern basic text for linear programming. Part II includes an expanded treatment of necessary conditions, manifested by not only first- and second-order necessary conditions for optimality, but also by zeroth-order conditions that use no derivative information. This part continues to present the important descent methods for unconstrained problems, but there is new material on convergence analysis and on Newton's methods which is frequently used as the workhorse of interior point methods for both linear and nonlinear programming. Finally, Part III now includes the global theory of necessary conditions for constrained problems, expressed as zero-th order conditions. Also interior point methods for general nonlinear programming are explicitly discussed within the sections on penalty and barrier methods. A significant addition to Part III is an expanded presentation of duality from both the global and local perspective. Finally, Chapter 15, on primal–dual methods has additional material on interior point methods and an introduction to the relatively new field of semidefinite programming, including several examples.

We wish to thank the many students and researchers who over the years have given us comments concerning the second edition and those who encouraged us to carry out this revision.

Stanford, California D.G.L.
July 2007 Y.Y.

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Chapter 1 INTRODUCTION

1.1 OPTIMIZATION

The concept of optimization is now well rooted as a principle underlying the analysis of many complex decision or allocation problems. It offers a certain degree of philosophical elegance that is hard to dispute, and it often offers an indispensable degree of operational simplicity. Using this optimization philosophy, one approaches a complex decision problem, involving the selection of values for a number of interrelated variables, by focussing attention on a single objective designed to quantify performance and measure the quality of the decision. This one objective is maximized (or minimized, depending on the formulation) subject to the constraints that may limit the selection of decision variable values. If a suitable single aspect of a problem can be isolated and characterized by an objective, be it profit or loss in a business setting, speed or distance in a physical problem, expected return in the environment of risky investments, or social welfare in the context of government planning, optimization may provide a suitable framework for analysis.

It is, of course, a rare situation in which it is possible to fully represent all the complexities of variable interactions, constraints, and appropriate objectives when faced with a complex decision problem. Thus, as with all quantitative techniques of analysis, a particular optimization formulation should be regarded only as an approximation. Skill in modelling, to capture the essential elements of a problem, and good judgment in the interpretation of results are required to obtain meaningful conclusions. Optimization, then, should be regarded as a tool of conceptualization and analysis rather than as a principle yielding the philosophically correct solution.

Skill and good judgment, with respect to problem formulation and interpretation of results, is enhanced through concrete practical experience and a thorough understanding of relevant theory. Problem formulation itself always involves a tradeoff between the conflicting objectives of building a mathematical model sufficiently complex to accurately capture the problem description and building a model that is tractable. The expert model builder is facile with both aspects of this tradeoff. One aspiring to become such an expert must learn to identify and capture the important issues of a problem mainly through example and experience; one must learn to distinguish tractable models from nontractable ones through a study of available technique and theory and by nurturing the capability to extend existing theory to new situations.

This book is centered around a certain optimization structure—that characteristic of linear and nonlinear programming. Examples of situations leading to this structure are sprinkled throughout the book, and these examples should help to indicate how practical problems can be often fruitfully structured in this form. The book mainly, however, is concerned with the development, analysis, and comparison of algorithms for solving general subclasses of optimization problems. This is valuable not only for the algorithms themselves, which enable one to solve given problems, but also because identification of the collection of structures they most effectively solve can enhance one's ability to formulate problems.

1.2 TYPES OF PROBLEMS

The content of this book is divided into three major parts: Linear Programming, Unconstrained Problems, and Constrained Problems. The last two parts together comprise the subject of nonlinear programming.

Linear Programming

Linear programming is without doubt the most natural mechanism for formulating a vast array of problems with modest effort. A linear programming problem is characterized, as the name implies, by linear functions of the unknowns; the objective is linear in the unknowns, and the constraints are linear equalities or linear inequalities in the unknowns. One familiar with other branches of linear mathematics might suspect, initially, that linear programming formulations are popular because the mathematics is nicer, the theory is richer, and the computation simpler for linear problems than for nonlinear ones. But, in fact, these are *not* the primary reasons. In terms of mathematical and computational properties, there are much broader classes of optimization problems than linear programming problems that have elegant and potent theories and for which effective algorithms are available. It seems that the popularity of linear programming lies primarily with the formulation phase of analysis rather than the solution phase—and for good cause. For one thing, a great number of constraints and objectives that arise in practice *are* indisputably linear. Thus, for example, if one formulates a problem with a budget constraint restricting the total amount of money to be allocated among two different commodities, the budget constraint takes the form $x_1 + x_2 \leq B$, where x_i, $i = 1, 2$, is the amount allocated to activity i, and B is the budget. Similarly, if the objective is, for example, maximum weight, then it can be expressed as $w_1x_1 + w_2x_2$, where w_i, $i = 1, 2$, is the unit weight of the commodity i. The overall problem would be expressed as

$$\begin{aligned} \text{maximize} \quad & w_1x_1 + w_2x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq B \\ & x_1 \geq 0, \qquad x_2 \geq 0, \end{aligned}$$

which is an elementary linear program. The linearity of the budget constraint is extremely natural in this case and does not represent simply an approximation to a more general functional form.

Another reason that linear forms for constraints and objectives are so popular in problem formulation is that they are often the least difficult to define. Thus, even if an objective function is not purely linear by virtue of its inherent definition (as in the above example), it is often far easier to define it as being linear than to decide on some other functional form and convince others that the more complex form is the best possible choice. Linearity, therefore, by virtue of its simplicity, often is selected as the easy way out or, when seeking generality, as the only functional form that will be equally applicable (or nonapplicable) in a class of similar problems.

Of course, the theoretical and computational aspects *do* take on a somewhat special character for linear programming problems—the most significant development being the simplex method. This algorithm is developed in Chapters 2 and 3. More recent interior point methods are nonlinear in character and these are developed in Chapter 5.

Unconstrained Problems

It may seem that unconstrained optimization problems are so devoid of structural properties as to preclude their applicability as useful models of meaningful problems. Quite the contrary is true for two reasons. First, it can be argued, quite convincingly, that if the scope of a problem is broadened to the consideration of all relevant decision variables, there may then be no constraints—or put another way, constraints represent artificial delimitations of scope, and when the scope is broadened the constraints vanish. Thus, for example, it may be argued that a budget constraint is not characteristic of a meaningful problem formulation; since by borrowing at some interest rate it is always possible to obtain additional funds, and hence rather than introducing a budget constraint, a term reflecting the cost of funds should be incorporated into the objective. A similar argument applies to constraints describing the availability of other resources which at some cost (however great) could be supplemented.

The second reason that many important problems can be regarded as having no constraints is that constrained problems are sometimes easily converted to unconstrained problems. For instance, the sole effect of equality constraints is simply to limit the degrees of freedom, by essentially making some variables functions of others. These dependencies can sometimes be explicitly characterized, and a new problem having its number of variables equal to the true degree of freedom can be determined. As a simple specific example, a constraint of the form $x_1 + x_2 = B$ can be eliminated by substituting $x_2 = B - x_1$ everywhere else that x_2 appears in the problem.

Aside from representing a significant class of practical problems, the study of unconstrained problems, of course, provides a stepping stone toward the more general case of constrained problems. Many aspects of both theory and algorithms