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Bernard Chalmond

Modeling and Inverse Problems in Image Analysis

图像分析中的模型和逆问题

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by

Bernard Chalmond

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To my family

Foreword

In the last decade of the past century we witnessed an exceptional participation of mathematicians in the development of digital image processing as a science. These contributions have found a natural place at the low level of processing, with the advent of mathematical morphology, differential equations, Markov random fields, and wavelet theory. They are also reflected in the increasingly important role modeling has played in solving complex problems.

Although modeling is often a hidden stage of the solution, the benefits of correctly modeling an image processing problem are huge. Modeling is the very place where "sensitivity" steals a lead over "geometry", to put it in Pascal's words. Correct modeling introduces information that cannot be expressed by data or deduced by equations, but reflects the subtle dependency or causality between the ingredients. Modeling has more to do with pots and pans than recipes and spices, to draw out the culinary metaphor. Bernard Chalmond's work is mainly dedicated to modeling issues. It does not fully cover this field, since it is mostly concerned with two types of model: Bayesian models issued from probability theory and energy-based models derived from physics and mechanics. Within the scope of these models, the book deeply explores the various consequences of the choice of a model; it compares their hypotheses, discusses their merits, explores their validity, and suggests possible fields of application.

Chalmond's work falls into three parts. The first deals with the processing of spline functions, interpolation, classification, and auto-associative models. Although splines are usually presented with their mechanical interpretation, Bernard Chalmond provides the Bayesian interpretation as well. The last chapter of this part, devoted to auto-associative models and pursuit problems, provides an original view of this field. The second part is concerned with inverse problems considered as Markovian energy optimization. Consistent attention is paid to the choice of potentials, and optimization is classically presented either as a deterministic or as a stochastic issue. The chapter on parameter estimation is greatly appreciated, since this aspect of Markovian modeling is too often neglected in textbooks. In the last part, Bernard Chalmond takes the time to subtly dissect some image processing problems in order to develop consistent modeling. These particular problems are concerned with denoising, deblurring, detecting noisy lines, estimating parameters, etc. They have been specifically chosen to cover a wide variety of situations encountered in robot vision or image processing. They can easily be adapted to the reader's own requirements.

This book fulfills a need in the field of computer science research and education. It is not intended for professional mathematicians, but it undoubtedly deals with applied mathematics. Most of the expectations of the topic are fulfilled: precision, exactness, completeness, and excellent references to the original historical works. However, for the sake of readability, many demonstrations are omitted. It is not a book on practical image processing, of which so many abound, although all that it teaches is directly concerned with image analysis and image restoration. It is the perfect resource for any advanced scientist concerned with a better understanding of the theoretical models underlying the methods that have efficiently solved numerous issues in robot vision and picture processing.

Henri Maître

Paris, France

Acknowledgments

This book describes a methodology and a practical approach for the effective building of models in image analysis. The practical knowledge applied was obtained through projects carried out in an industrial context, and benefited from the collaboration of

R. Azencott, F. Coldefy, J.M. Dinten,
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Notation and Symbols

Part I

AAC	auto-associative composite model.
$B(t)$	B-spline function.
$\mathbf{B}$	matrix of B-splines.
$\mathbf{B}^{\tau}$	matrix of B-splines along the τ axis.
$\mathrm{Cov}(X, Y)$	covariance.
$\mathcal{C}^m$	functions that are m times continuously differentiable.
$D^m g$, $\frac{d^m g}{dt^m}$	mth derivative of $g(t)$.
$D^{i,j} g$, $\frac{\partial^{i+j} g}{\partial t^i \partial u^j}$	partial derivative of $g(t, u)$.
Δ	Laplacian.
$\Delta^m g$	mth-order difference operator.
$\mathbf{E}[X]$	mathematical expectation of the random variable X.
$\mathcal{E}_\ell$	space of finite elements e of diameter ℓ.
$g(t)$	curve.
$g(t, u)$	surface.
$g \mid_{\mathrm{e}}$	restriction of the function g to e.
$\mathrm{G}(x) = 0$	equation of a manifold.
$\mathcal{G}$	two-dimensional grid.
$\mathcal{G}^{\mathrm{theo}}$, $\mathcal{G}^{\mathrm{emp}}$	generalization error.
H^m	mth-order Sobolev space.

$I(a, \mathcal{X})$	index of the AAC of a for the cloud $\mathcal{X}$.
$\dot{I}(a, \mathcal{X})$	index of the PCA of a for the cloud $\mathcal{X}$.
$I_K(a, \mathcal{X})$	Kullback index of a for the cloud $\mathcal{X}$.
Id	identity matrix or application.
$LG(\mu, V)$	Laplace–Gauss distribution of expectation μ and variance V.
L	linear differential operator.
L^2	square integrable function space.
$L(\theta)$	likelihood of θ.
M', ${}^tM$	transpose matrix.
∇f	gradient of f.
$\mathcal{O}(X, W)$	operator for degradation of X by W.
$P(x)$	probability distribution.
P^a	projection operator with parameter a.
PCA	principal component analysis.
PPR	projection pursuit regression.
R	real numbers.
S^b	smoothing operator with parameter b.
$\mathcal{S}(\mathcal{T})$	space of cubic splines with knots $\mathcal{T}$.
$\mathcal{S}_N(\mathcal{T})$	space of natural cubic splines with knots $\mathcal{T}$.
σ^2	variance of a noise, of residuals, etc.
$T = \langle a, X \rangle$	coordinate of X on the a-axis.
$U(x)$	prior energy (or internal energy).
$U(y \mid x)$	fidelity energy y (or external energy).
$\mathrm{Var}(X)$	variance or variance–covariance matrix.
W	noise, residual, etc.
X, x	random variable X and its occurrence x.
$\mathcal{X}$	cloud of points in $\mathbb{R}^n$.
Ξ_k	symmetry.
Y, y	vector or field of observations.
Z	relative integers.
1_A, $\mathbf{1}[A]$	indicator function.
$\mid A \mid$	cardinality of A if A is a set, or absolute value if A is a number.
$A \doteq B$	by definition B is written as A.
$< s, t >$	s and t are neighbors.
$\langle X, Y \rangle$	scalar product.
$\left(x_i^j\right)^2$	a raised index and an exponent can be present at the same time, and should be distinguishable according to the context.
$\vec{y}$, $\underline{y}$, $\mathbf{y}$	to indicate that y is a vector.
$Y \mid X = x$	probabilistic conditioning.
$\widehat{\theta}$	estimation of θ, or the argument of the minimum of a function.

Part II

c	clique.
C	set of cliques of a Markov random field.
E	set of states.
E_s	set of states at site s.
Γ	cost function.
H_i	local energy difference.
$\mathbf{H} = (H_1, \ldots, H_q)'$	vector of local energy differences.
ICM	iterated conditional mode algorithm.
$\mathcal{L}(\theta)$, $\mathcal{P}_\theta(x)$	pseudolikelihood of θ.
N_s	set of neighborhood sites to s.
$N_s(x)$	values in the neighborhood N_s of s.
p_{yx}	probability of transition of y to x of a Markov chain.
$\mathcal{P}$	transition matrix of a Markov chain.
$\mathcal{P}_\theta(x)$	pseudolikelihood of θ for the field x.
Π	discrete probability distribution.
s	site.
S	set of sites.
T_n	simulated annealing temperature.
$\underset{\sim}{\mathbf{U}}(y)$	energy $\mathbf{U}(x)$ limited to a subset y of x.
$V_c(x)$	potential of the clique c.
$\breve{x}_s$	a field x deprived of the value x_s.
y^O	a specific observation of Y.
$\{Z_n\}$	Markov chain.
$\mathcal{Z}$	normalization constant of a Gibbs distribution.
$\lceil a \rceil$	integer part of a.

Part III

C, C_i, $C_{\ell,i}$	set of cliques of a Markov random field.
$C(t)$	curve.
Δf	Laplacian of f.
$\mathcal{F}$	radiographic film.
ϕ	smoothing function.
$\varphi(.)$	scattered flux.
$\{\phi_i\}$	vector basis.
GC	generalized cylinder.
$G(s_1, s_2)$, $g(s_1, s_2)$	surface.

$G(s_1, s_2, \ell)$	stained transfer function at s of level ℓ.
$\mathcal{G}$, $\mathcal{G}_\theta$	projection of a GC.
$\vec{\mathcal{G}}$	continuous deformation field.
Γ_s	indicator of curvature line at s.
$H(\ell)$	transfer function for the gray level ℓ.
$H_i(.,.)$, $H_{i,j}(.,.)$	local energy difference.
$\mathcal{H}$, $\mathbb{H}$	convolution kernel.
I, I_0, $\mathcal{I}$	an image.
$K^{(n)}$	kernel of $n-$step transition probability.
λ, Λ	energy of a photon.
Λ_s	parameters of a confidence region.
NDT	nondestructive testing.
$\mathcal{N}_s(\ell)$	set of neighboring sites to s with value ℓ.
$\aleph$	outline.
PSF	point spread function.
R_s	confidence region in s.
R_d^ℓ	ray between source ℓ and detector d.
$\mathcal{R}_x(.,.)$	Radon transform.
S, S^p, S^b	set of sites.
$\mathcal{S}$	spatial position.
SNR	signal-to-noise ratio.
STF	stained transfer function.
TF	transfer function.
T_s	degradation by a stain at site s.
$T(s,a)$	intersection between an object and a straight line (s,a).
T_ρ, T_τ, T_ς	rigid transformations.
VBL	valley bottom line.
X^p, x^p	field in gray levels.
X^b, x^b	edge field or segmentation field.
X^ℓ, x^ℓ	VBL field.
$\{Z_n\}$	Markov chain.
$Z(t)$	Brownian process.

Contents