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編新
識常
各科
問答

改版本 ②

1933

上海新華書局發行

三角法問答

I. 式之變形

(一) 銳角三角式之變形

及數值計算問題

(A) 摘要

$$\left. \begin{aligned} \sin A \operatorname{cosec} A &= 1 \dots (1) \\ \cos A \sec A &= 1 \dots (2) \\ \tan A \cot A &= 1 \dots (3) \end{aligned} \right\} (A) \text{ 又 } \left\{ \begin{aligned} \operatorname{cosec} A &= \frac{1}{\sin A} \\ \sec A &= \frac{1}{\cos A} \\ \cot A &= \frac{1}{\tan A} \end{aligned} \right.$$

$$\left. \begin{aligned} \tan A &= \frac{\sin A}{\cos A} \dots\dots\dots (4) \\ \cot A &= \frac{\cos A}{\sin A} \dots\dots\dots (5) \end{aligned} \right\} (B) \text{ 又 } \left\{ \begin{aligned} \tan A \cos A &= \sin A \\ \cot A \sin A &= \cos A \end{aligned} \right.$$

$$\left. \begin{aligned} \sin^2 A + \cos^2 A &= 1 \dots\dots\dots (6) \\ 1 + \tan^2 A &= \sec^2 A \dots\dots\dots (7) \\ 1 + \cot^2 A &= \operatorname{cosec}^2 A \dots\dots\dots (8) \end{aligned} \right\} (C)$$

(B) 問題反解法

(1) $(\cos A + \sin A)^2 + (\cos A - \sin A)^2$ 試簡單之

解： 原式 $= \cos^2 A + 2\cos A \sin A + \sin^2 A + \cos^2 A$
 $- 2\cos A \sin A + \sin^2 A$
 $= 2(\cos^2 A + \sin^2 A) = 2$

(2) 試變 $\sin^4 A + \cos^4 A$ 爲 $1 - 2\sin^2 A \cos^2 A$

解： $\sin^4 A + \cos^4 A = \sin^4 A + 2\sin^2 A \cos^2 A + \cos^4 A$
 $- 2\sin^2 A \cos^2 A$
 $= (\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A$
 $= 1 - 2\sin^2 A \cos^2 A$

(3) 試簡單 $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

解： 原式 $= \frac{1-\sin\theta}{1-\sin\theta} + \frac{1+\sin\theta}{1-\sin\theta}$
 $= \frac{2}{1-\sin\theta} = \frac{2}{\cos^2\theta} = 2\sec^2\theta$

(4) $\sec^2 \alpha + \operatorname{cosec}^2 \alpha$ 試簡單之

解： 原式 $= \frac{1}{\cos^2 \alpha} + \frac{1}{\sin^2 \alpha} = \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos^2 \alpha \sin^2 \alpha}$
 $= \frac{1}{\cos^2 \alpha \sin^2 \alpha} = \sec^2 \alpha \operatorname{cosec}^2 \alpha$

(5) 試將 $(\tan A + \sec A)^2$ 以 $\sin A$ 項表之

$$\begin{aligned}
 \text{解： } (\tan A + \sec A)^2 &= \left(\frac{\sin A}{\cos A} + \frac{1}{\cos A} \right)^2 = \left(\frac{\sin A + 1}{\cos A} \right)^2 \\
 &= \frac{(1 + \sin A)^2}{\cos^2 A} = \frac{(1 + \sin A)^2}{1 - \sin^2 A} \\
 &= \frac{(1 + \sin A)^2}{(1 + \sin A)(1 - \sin A)} = \frac{1 + \sin A}{1 - \sin A}
 \end{aligned}$$

(6) $\frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta}$ 試簡單之

$$\text{解： 原式} = \frac{\tan \alpha + \tan \beta}{\frac{1}{\tan \alpha} + \frac{1}{\tan \beta}} = \frac{\tan \alpha + \tan \beta}{\frac{\tan \alpha + \tan \beta}{\tan \alpha \tan \beta}} = \tan \alpha \tan \beta$$

(7) 試証 $(1 + \sin A + \cos A)^2 = 2(1 + \sin A)(1 + \cos A)$

$$\begin{aligned}
 \text{解： 左邊} &= [(1 + \sin A) + \cos A]^2 \\
 &= (1 + \sin A)^2 + 2(1 + \sin A)\cos A + \cos^2 A \\
 &= (1 + \sin A)^2 + 2(1 + \sin A) + 1 - \sin^2 A \\
 &= (1 + \sin A)(1 + \sin A + 2\cos A + 1 - \sin A) \\
 &= 2(1 + \sin A)(1 + \cos A)
 \end{aligned}$$

(8) $\tan \theta \frac{1 - \sin \theta}{1 + \cos \theta} = \cot \theta \frac{1 - \cos \theta}{1 + \sin \theta}$ 試證明之

$$\text{解： 左邊} = \cot \theta \tan^2 \theta \frac{1 - \sin \theta}{1 + \cos \theta}$$

$$= \cot \theta \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1 - \sin \theta}{1 + \cos \theta}$$

$$= \cot \theta \frac{1 - \cos^2 \theta}{1 - \sin^2 \theta} \cdot \frac{1 - \sin \theta}{1 + \cos \theta} = \cot \theta \cdot \frac{1 - \cos \theta}{1 + \sin \theta}$$

$$(\text{別解1}) \quad \frac{1 - \sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{1 + \sin \theta} \cdot \frac{1 + \sin \theta}{1 - \cos \theta} \cdot \frac{1 - \sin \theta}{1 + \cos \theta}$$

$$= \frac{1 - \cos \theta}{1 + \sin \theta} \cdot \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{1 - \cos \theta}{1 + \sin \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \frac{1 - \cos \theta}{1 + \sin \theta} \cot^2 \theta$$

$$\therefore \text{左邊} = \tan \theta \cot^2 \theta \frac{1 - \cos \theta}{1 + \sin \theta} = \cot \theta \frac{1 - \cos \theta}{1 + \sin \theta}$$

(別解2) 本式去分母

$$\tan \theta (1 - \sin \theta) (1 + \sin \theta)$$

$$= \cot \theta (1 - \cos \theta) (1 + \cos \theta) \cdots \cdots (1)$$

$$\text{然} \quad \tan \theta (1 - \sin \theta) (1 + \sin \theta) = \tan \theta (1 - \sin \theta)^2$$

$$= \tan \theta \cos^2 \theta$$

$$= \frac{\sin \theta}{\cos \theta} \cos^2 \theta = \sin \theta \cos \theta$$

$$\cot \theta (1 - \cos \theta) (1 + \cos \theta) = \cot \theta (1 - \cos^2 \theta)$$

$$= \cot \theta \sin^2 \theta$$

$$= \frac{\cos \theta}{\sin \theta} \sin^2 \theta = \sin \theta \cos \theta$$

故(1)式左邊之值等於右邊之值

$$\therefore \tan \theta (1 - \sin \theta)(1 + \sin \theta) = \cot \theta$$

$$(1 - \cos \theta)(1 + \cos \theta)$$

$$\therefore \tan \theta \frac{1 - \sin \theta}{1 + \cos \theta} = \cot \theta \frac{1 - \cos \theta}{1 + \sin \theta}$$

(9) 試證明下式

$$\sec A + \tan A = \frac{1}{\sec A - \tan A}$$

解：由公式 $1 + \tan^2 A = \sec^2 A$

$$\sec^2 A - \tan^2 A = 1$$

$$\therefore (\sec A + \tan A)(\sec A - \tan A) = 1$$

$$\therefore \sec A + \tan A = \frac{1}{\sec A - \tan A}$$

數值計算問題

(1) $\tan \theta = \frac{m^2 + 2mn}{2mn + 2n^2}$ 時 $\operatorname{cosec} \theta$ 之值如何試求之

解： $\tan \theta = \frac{m^2 + 2mn}{2mn + 2n^2}$ 故 $\cot \theta = \frac{2mn + 2n^2}{m^2 + 2mn}$

$$\begin{aligned}\therefore \operatorname{cosec}^2 \theta &= 1 + \cot^2 \theta = 1 + \left(\frac{2mn + 2n^2}{m^2 + 2mn} \right)^2 \\ &= \frac{(m^2 + 2mn)^2 + (2mn + 2n^2)^2}{(m^2 + 2mn)^2}\end{aligned}$$

$$\begin{aligned}\text{然分子} &= m^4 + 4mn + 4m^2n^2 + 4n^2(m+n)^2 \\ &= m^4 + 4m^2n(m+n) + 4n^2(m+n)^2 \\ &= [m^2 + 2n(m+n)]^2 = (m + 2mn + 2n^2)^2\end{aligned}$$

$$\therefore \operatorname{cosec}^2 \theta = \frac{(m^2 + 2mn + 2n^2)^2}{(m^2 + 2mn)^2}$$

$$\therefore \operatorname{cosec} \theta = \pm \frac{m^2 + 2mn + 2n^2}{m^2 + 2mn}$$

但 θ 爲銳角則將負號棄之

$$\text{答 } \operatorname{cosec} \theta = \frac{m + 2mn + 2n^2}{m^2 + 2mn}$$

(2) $\tan \theta = \frac{a}{b}$ 時，試求 $a \cos \theta + b \sin \theta$ 之值，

解：由公式 $1 + \tan^2 \theta = \sec^2 \theta = \frac{1}{\cos^2 \theta}$

$$\cos^2 \theta = \frac{1}{1 + \tan^2 \theta} = \frac{1}{1 + \frac{a^2}{b^2}} = \frac{b^2}{a^2 + b^2}$$

$$\therefore \cos \theta = \frac{b}{\sqrt{a^2 + b^2}} \quad (\text{因 } \theta \text{ 爲銳角故棄負號})$$

$$\text{次由公式 } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin \theta = \tan \theta \cdot \cos \theta = \frac{a}{b} \cdot \frac{b}{\sqrt{a^2 + b^2}} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\therefore \text{原式} = \frac{ab}{\sqrt{a^2 + b^2}} + \frac{ab}{\sqrt{a^2 + b^2}} = \frac{a}{\sqrt{a^2 + b^2}}$$

(3) $\operatorname{cosec} \alpha = 3$ 時，試求 $\sec \alpha - \tan \alpha$ 之值！

$$\text{解： } \sec \alpha - \tan \alpha = \frac{1}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 - \sin \alpha}{\cos \alpha}$$

$$\text{然 } \sin \alpha = \frac{1}{\operatorname{cosec} \alpha} = \frac{1}{3}$$

$$\therefore \cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\therefore \cos \alpha = \frac{2\sqrt{2}}{3} \quad (\text{但 } \alpha \text{ 爲銳角})$$

$$\therefore \sec \alpha - \tan \alpha = \frac{1 - \frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

(4) $\cot A = \frac{p}{q}$ 時 則 $\frac{p \cos A - q \sin A}{p \cos A + q \sin A}$ 之值如何?

解:
$$\frac{p \cos A - q \sin A}{p \cos A + q \sin A} = \frac{p \cot A - q}{p \cot A + q}$$

$$= \frac{p \cdot \frac{p}{q} - q}{p \cdot \frac{p}{q} + q} = \frac{p^2 - q^2}{p^2 + q^2}$$

(別解) 由公式 $\cot A = \frac{\cos A}{\sin A}$

$$\frac{\cos A}{\sin A} = \frac{p}{q} \quad \therefore \frac{\cos A}{p} = \frac{\sin A}{q} = k$$

$$\therefore \cos A = pk \quad \sin A = qk$$

以此代入本式

$$\frac{p \cos A - q \sin A}{p \cos A + q \sin A} = \frac{p^2 k - q^2 k}{p^2 k + q^2 k} = \frac{p^2 - q^2}{p^2 + q^2}$$

(二) 餘角三角式之變形

(A) 摘要

$$\begin{cases} \sin(90^\circ - A) = \cos A \\ \cos(90^\circ - A) = \sin A \\ \tan(90^\circ - A) = \cot A \end{cases} \quad \begin{cases} \sec(90^\circ - A) = \csc A \\ \csc(90^\circ - A) = \sec A \\ \cot(90^\circ - A) = \tan A \end{cases}$$

$$\left\{ \begin{array}{l} \sin 0^\circ = \cos 90^\circ = 0 \\ \cos 0^\circ = \sin 90^\circ = 1 \\ \tan 0^\circ = \cot 90^\circ = 0 \\ \cot 0^\circ = \tan 90^\circ = \pm \infty \end{array} \right. \quad \left\{ \begin{array}{l} \sin 30^\circ = \cos 60^\circ = \frac{1}{2} \\ \cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2} \\ \tan 30^\circ = \cot 60^\circ = \frac{1}{\sqrt{3}} \\ \cot 30^\circ = \tan 60^\circ = \sqrt{3} \end{array} \right.$$

$$\left\{ \begin{array}{l} \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} \\ \tan 45^\circ = \cot 45^\circ = 1 \end{array} \right. \quad \sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

(B) 問題及解法

(1) 下式試簡單之

$$\cot(90^\circ - A) \cot A \cos(90^\circ - A) \tan(90^\circ - A)$$

$$\text{解： } \cot(90^\circ - A) = \tan A, \quad \cos(90^\circ - A) = \sin A$$

$$\tan(90^\circ - A) = \cot A$$

$$\therefore \text{原式} = \tan A \cot A \sin A \cot A$$

$$\text{然 } \tan A \cot A = 1$$

$$\therefore \text{原式} = \sin A \frac{\cos A}{\sin A} = \cos A$$

(2) $\sin^2(45^\circ + A) + \sin^2(45^\circ - A) = 1$ 試證明之

$$\text{解： 因 } (45^\circ + A) + (45^\circ - A) = 90^\circ$$

$$\text{故 } 45^\circ + A \text{ 與 } 45^\circ - A \text{ 互為餘角，}$$

$$\sin(45^\circ - A) = \cos(45^\circ + A)$$

$$\therefore \sin^2(45^\circ + A) + \sin^2(45^\circ - A)$$

$$= \sin^2(45^\circ + A) + \cos^2(45^\circ + A) = 1.$$

(三) 一般角三角函數式之變形 及數值計算問題

問題及解法

$$(1) \quad \frac{\sin(180^\circ - \alpha)}{\tan(180^\circ + \alpha)} \times \frac{\cot(90^\circ - \alpha)}{\tan(90^\circ + \alpha)} \\ \times \frac{\cos(360^\circ - \alpha)}{\sin(-\alpha)} \text{ 試簡單之! }$$

$$\text{解: } \sin(180^\circ - \alpha) = \sin \alpha \quad \tan(180^\circ + \alpha) = \tan \alpha$$

$$\cot(90^\circ - \alpha) = \tan \alpha \quad \tan(90^\circ + \alpha) = -\cot \alpha$$

$$\cos(360^\circ - \alpha) = \cos \alpha \quad \sin(-\alpha) = -\sin \alpha$$

$$\begin{aligned} \text{故原式} &= \frac{\sin \alpha}{\tan \alpha} \times \frac{\tan \alpha}{-\cot \alpha} \times \frac{\cos \alpha}{-\sin \alpha} = \frac{\cos \alpha}{\cot \alpha} \\ &= \cos \alpha \times \frac{\sin \alpha}{\cos \alpha} = \sin \alpha \end{aligned}$$

$$(2) \quad \tan 238^\circ = \frac{8}{5} \text{ 時, } \sin 238^\circ \text{ 之值如何? }$$

解： 238° 爲第三象限之角，故其正弦之值爲負，因之
 $\alpha = 238^\circ$ $\sin \alpha$ 以既知數 $\tan \alpha$ 之項表示之時，其
 符號爲負。

$$\sin \alpha = \tan \alpha \cos \alpha = -\frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$$

$$\text{然 } \tan \alpha = \frac{8}{5}$$

$$\text{故 } \sin \alpha = -\frac{\frac{8}{5}}{\sqrt{1 + \left(\frac{8}{5}\right)^2}} = -\frac{8}{\sqrt{89}} = -\frac{8\sqrt{89}}{89}$$

(四) 角之和及差之三角公式 及數值計算問題

(A) 摘要

(1) 二角之和及差之三角公式

$$\left. \begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \end{aligned} \right\} \dots\dots (A)$$

(2) 二倍角或半角之三角公式

$$\left. \begin{aligned} \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2 \sin^2 A = 2 \cos^2 A - 1 \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} \end{aligned} \right\} \dots\dots (B)$$

(3) 半角之三角公式

$$\left. \begin{aligned} 1 + \cos A &= 2 \cos^2 \frac{A}{2} \\ 1 - \cos A &= 2 \sin^2 \frac{A}{2} \\ \frac{1 - \cos A}{1 + \cos A} &= \tan^2 \frac{A}{2} \end{aligned} \right\} \dots\dots (C)$$

(4) 三倍角之三角公式

$$\left. \begin{aligned} \sin 3A &= 3 \sin A - 4 \sin^3 A \\ \cos 3A &= 4 \cos^3 A - 3 \cos A \\ \tan 3A &= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \end{aligned} \right\} \dots\dots (D)$$

(B) 問題及解法

(1) 試變 $\tan A + \tan B$ 為積形

$$\text{解： 原式} = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}$$

$$\begin{aligned}
 &= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \\
 &= \frac{\sin(A+B)}{\cos A \cos B}
 \end{aligned}$$

(2) $\tan(45^\circ + A) - \tan(45^\circ - A)$ 試簡單之

$$\begin{aligned}
 \text{解： 原式} &= \frac{1 + \tan A}{1 - \tan A} - \frac{1 - \tan A}{1 + \tan A} \\
 &= \frac{(1 + \tan A)^2 - (1 - \tan A)^2}{(1 + \tan A)(1 - \tan A)} \\
 &= \frac{4 \tan A}{1 - \tan^2 A} = 2 \tan^2 A
 \end{aligned}$$

(3) $\frac{\sin \theta}{1 + \cos \theta}$ 試簡單之

$$\begin{aligned}
 \text{解： 原式} &= \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \tan \frac{\theta}{2}
 \end{aligned}$$

(4) $\cos 4\theta - 4 \cos 2\theta + 3 = 8 \sin^4 \theta$ 試證之

$$\begin{aligned}
 \text{解： } \cos 4\theta - 4 \cos 2\theta + 3 &= \cos 4\theta - 1 - 4 \cos 2\theta + 4 \\
 &= 4(1 - \cos 2\theta) - (1 - \cos 4\theta)
 \end{aligned}$$

$$= 8\sin^2\theta - 2\sin^2 2\theta$$

$$= 8\sin^2\theta - 2 \times 4\sin^2\theta\cos^2\theta$$

$$= 8\sin^2\theta(1 - \cos^2\theta) = 8\sin^4\theta$$

$$(5) \quad \frac{1 + \sin\theta}{1 - \sin\theta} = \tan^2 \left(45^\circ + \frac{\theta}{2} \right) \text{ 試證明之}$$

$$\text{解: } \frac{1 + \sin\theta}{1 - \sin\theta} = \frac{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} + 2\sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} - 2\sin \frac{\theta}{2} \cos \frac{\theta}{2}} \dots (1)$$

$$= \frac{\left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right)^2}{\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)^2} = \left\{ \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right\}^2$$

$$\text{兩項以 } \cos \frac{\theta}{2} \text{ 除之} = \left\{ \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right\}^2$$

$$= \left\{ \frac{\tan 45^\circ + \tan \frac{\theta}{2}}{1 - \tan 45^\circ \tan \frac{\theta}{2}} \right\}^2$$

$$= \tan^2 \left(45^\circ + \frac{\theta}{2} \right)$$

$$\begin{aligned} \text{(別解)} \quad \frac{1+\sin\theta}{1-\sin\theta} &= \frac{1+\cos(90^\circ-\theta)}{1-\cos(90^\circ-\theta)} = \frac{2\cos^2\left(45^\circ-\frac{\theta}{2}\right)}{2\sin^2\left(45^\circ-\frac{\theta}{2}\right)} \\ &= \cot^2\left(45^\circ-\frac{\theta}{2}\right) = \tan^2\left(45^\circ+\frac{\theta}{2}\right) \end{aligned}$$

(6) 試化 $\sec\theta + \tan\theta$ 爲 $\tan\left(45^\circ + \frac{\theta}{2}\right)$ 之形

$$\text{解: } \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$\begin{aligned} &= \frac{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}} \\ &= \frac{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)} \end{aligned}$$

$$\begin{aligned}
 & \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} = \frac{1 + \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}}{1 - \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}}
 \end{aligned}$$

$$= \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} = \tan \left(45^\circ + \frac{\theta}{2} \right)$$

$$(\text{別解}) \sec \theta + \tan \theta = \frac{1 + \sin \theta}{\cos \theta} = \frac{1 - \cos(90^\circ + \theta)}{\sin(90^\circ + \theta)}$$

$$\begin{aligned}
 & = \frac{2 \sin^2 \left(45^\circ + \frac{\theta}{2} \right)}{2 \sin \left(45^\circ + \frac{\theta}{2} \right) \cos \left(45^\circ + \frac{\theta}{2} \right)} \\
 & = \tan \left(45^\circ + \frac{\theta}{2} \right)
 \end{aligned}$$

數值計算問題