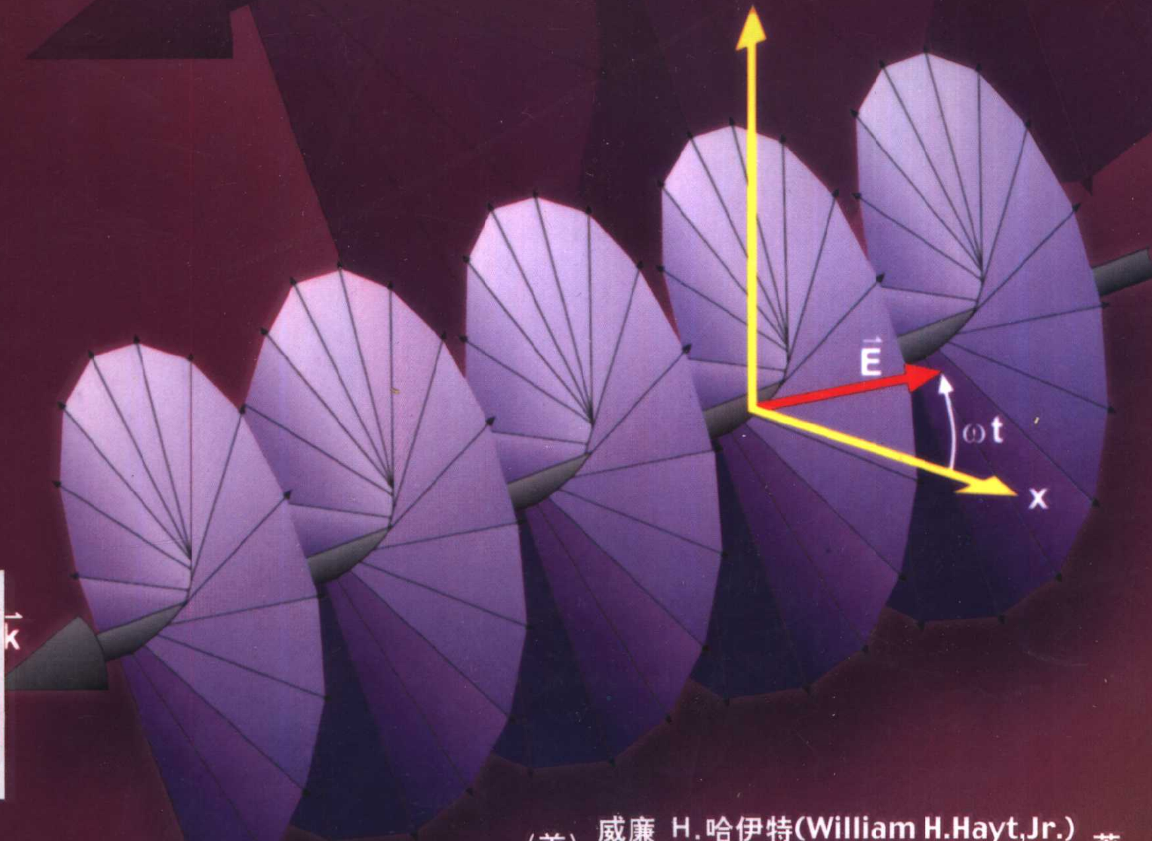
时代教育·国外高校优秀教材精选

工程电磁场（英文版·原书第6版）

Engineering Electromagnetics



（美）威廉 H.哈伊特(William H.Hayt,Jr.)
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机械工业出版社
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序

本书是著名的 McGraw-Hill 教育出版公司于 2001 年在美国纽约出版的《工程电磁场》教科书，适用于工科本科第一学期课程，也很适用于我国高等学校进行“工程电磁场”课程的双语教学。原书（1958 年第 1 版）作者是美国普渡（Purdue）大学的 Hayt 教授，经几次改版，现在第 6 版增加了合作作者 Buck 教授。本书自第 1 版自起一直是美国在电磁场方面的畅销书。全书行文生动流畅，十分精练，叙述概念非常准确。作者在序言中就说明了其写书的原则是注重物理概念的理解和解题能力的培养，所以写得很有特色，主要体现在以下几点：

1. 建立新概念，提出新问题、新内容，做到由浅入深，循序渐进，从正反两方面分析比较。例如讲到用流线描绘点电荷的电场分布时，用了四个图加以比较讨论，使初学者印象深刻；又如讲矢量分析一章时，先说明在研究电磁场的初级课程中，不用矢量分析理论也可以，只是存在缺点和局限性，再提出用矢量分析的必要性和优点，然后进入主题，这样可提高学生的学习兴趣和紧迫感。

2. 讲解新的物理结构模型时，先从广泛意义上日常普遍接触观察到的现象入手。例如讲解电容时，先说明只要两个导体中间隔以介质，有电位差，导体上就会有电荷储存，就产生了电容作用，体现了电容的本性。进而再讲述有特殊结构的电容器和电容的计算方法及储能公式等。这样从感性认识出发，由表及里，达到理论高度，符合认识规律。

3. 强调尊重科学发展历史过程，如说明由安培、法拉第和高斯各人独立工作得到的实验定律，经麦克斯韦提出位移电流后统一成为麦克斯韦方程组，完成了电磁场理论大业。作者在强调经典理论完整性的同时，还引入了传输线上电流电压瞬变过程理论、电磁场计算机辅助分析、光纤通信等新科技，全书贯彻了历史唯物观点。

4. 用一些通俗易懂的方法讲解新内容。例如第 6 章中通过用等高线比拟位场等位线绘出物理模型，用重力场中发生的现象来解释静电场中发生的某些问题。物体的上升和下降是最直观的由重力场作功，与电荷在电场中逆着和顺着电场移动，与电场作功完全相似，但用力学原理，比较更容易理解。

5. 书中某些章节用已知概念来说明分析概念，十分成功。例如书中利用在第 12 章阐述的关于电磁波在不同媒质分界面上反射和透射的理论，来解释偶极子天线发射和接收电磁波的机理，自然使读者比较容易理解其物理意义。

6. 书中很注重内容的前后呼应和上下衔接。每章开始时都先指出与前面内容的关系或异同，立刻就突出了新一章的地位。例如在第 5 章讲解电容时，首先联系前面进过的在静电场中的导体和绝缘体（介质），就使得电容概念的引出比较顺理成章。本书不仅各章内容前后呼应，还考虑了不同课程之间的衔接。例如讲解最后一章（第 14 章）波导与天线时，提出要温习以前各章，把已学知识综合起来形成扎实的专业基础，起到了与后续的“微波技术”等专业课程衔接的桥梁作用。这样可以使学生对这门专业基础课更加重视，而且憧憬着未来课程的学习。

7. 全书配有丰富的例题，并有详细解答，另外，还有 Hayt 教授生前拟就的一套练习题，分布在书中各节（附答案）。每章末尾还有大量习题，其答案可以在www. mhhe. com/engcs/electrical/haytbuck 查阅，并且备有解答手册，这些都十分有利于学生独立思考和学习。

周克定

华中科技大学

2002 年 5 月

出版说明

随着我国加入WTO，国际间的竞争越来越激烈，而国际间的竞争实际上也就是人才的竞争、教育的竞争。为了加快培养具有国际竞争力的高水平技术人才，加快我国教育改革的步伐，教育部近来出台了一系列倡导高校开展双语教学、引进原版教材的政策。以此为契机，机械工业出版社拟于近期推出一系列国外影印版教材，其内容涉及高等学校公共基础课，以及机、电、信息领域的专业基础课和专业课。

引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

为了做好教材的引进工作，机械工业出版社特别成立了由著名专家组成的国外高校优秀教材审定委员会。这些专家对实施双语教学做了深入细致的调查研究，对引进原版教材提出许多建设性意见，并慎重地对每一本将要引进的原版教材一审再审，精选再精选，确认教材本身的质量水平，以及权威性和先进性，以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中，审定委员会还结合我国高校教学课程体系的设置和要求，对原版教材的教学思想和方法的先进性、科学性严格把关，同时尽量考虑原版教材的系统性和经济性。

这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

机械工业出版社

2002年3月

PREFACE

Over the years, I have developed a familiarity with this book in its various editions, having learned from it, referred to it, and taught from it. The second edition was used in my first electromagnetics course as a junior during the early '70's. Its simple and easy-to-read style convinced me that this material could be learned, and it helped to confirm my latent belief at the time that my specialty would lie in this direction. Later, it was not surprising to see my own students coming to me with heavily-marked copies, asking for help on the drill problems, and taking a more active interest in the subject than I usually observed. So, when approached to be the new co-author, and asked what I would do to change the book, my initial feeling was—nothing. Further reflection brought to mind earlier wishes for more material on waves and transmission lines. As a result, Chapters 1 to 10 are original, while 11 to 14 have been revised, and contain new material.

A conversation with Bill Hayt at the project's beginning promised the start of what I thought would be a good working relationship. The rapport was immediate. His declining health prevented his active participation, but we seemed to be in general agreement on the approach to a revision. Although I barely knew him, his death, occurring a short time later, deeply affected me in the sense that someone that I greatly respected was gone, along with the promise of a good friendship. My approach to the revision has been as if he were still here. In the front of my mind was the wish to write and incorporate the new material in a manner that he would have approved, and which would have been consistent with the original objectives and theme of the text. Much more could have been done, but at the risk of losing the book's identity and possibly its appeal.

Before their deaths, Bill Hayt and Jack Kemmerly completed an entirely new set of drill problems and end-of-chapter problems for the existing material at that time, up to and including the transmission lines chapter. These have been incorporated, along with my own problems that pertain to the new topics. The other revisions are summarized as follows: The original chapter on plane waves has now become two. The first (Chapter 11) is concerned with the development of the uniform plane wave and the treatment wave propagation in various media. These include lossy materials, where propagation and loss are now modeled in a general way using the complex permittivity. Conductive media are presented as special cases, as are materials that exhibit electronic or molecular resonances. A new appendix provides background on resonant media. A new section on wave polarization is also included. Chapter 12 deals with wave reflection at single and multiple interfaces, and at oblique incidence angles. An additional section on dispersive media has been added, which introduces the concepts of group velocity and group dispersion. The effect of pulse broadening arising from group dispersion is treated at an elementary level. Chapter 13 is essentially the old transmission lines chapter, but with a new section on transients. Chapter 14 is intended as an introduction to waveguides and antennas, in which the underlying physical concepts are emphasized. The waveguide sections are all new, but the antennas treatment is that of the previous editions.

The approach taken in the new material, as was true in the original work, is to emphasize physical understanding and problem-solving skills. I have also moved the work more in the direction of communications-oriented material, as this seemed a logical way in which the book could evolve, given the material that was already there. The perspective has been broadened by an expanded emphasis toward optics concepts and applications, which are presented along with the more traditional lower-frequency discussions. This again seemed to be a logical step, as the importance of optics and optical communications has increased significantly since the earlier editions were published.

The theme of the text has not changed since the first edition of 1958. An inductive approach is used that is consistent with the historical development. In it, the experimental laws are presented as individual concepts that are later unified in Maxwell's equations. Apart from the first chapter on vector analysis, the mathematical tools are introduced in the text on an as-needed basis. Throughout every edition, as well as this one, the primary goal has been to enable students to learn independently. Numerous examples, drill problems (usually having multiple parts), and end-of-chapter problems are provided to facilitate this. Answers to the drill problems are given below each problem. Answers to selected end-of-chapter problems can be found on the internet at www.mhhe.com/engcs/electrical/haytbuck. A solutions manual is also available.

The book contains more than enough material for a one-semester course. As is evident, statics concepts are emphasized and occur first in the presentation. In a course that places more emphasis on dynamics, the later chapters can be reached earlier by omitting some or all of the material in Chapters 6 and 7, as well as the later sections of Chapter 8. The transmission line treatment (Chapter 13) relies heavily on the plane wave development in Chapters 11 and 12. A more streamlined presentation of plane waves, leading to an earlier arrival at transmission lines, can be accomplished by omitting sections 11.5, 12.5, and 12.6. Chapter 14 is intended as an "advanced topics" chapter, in which the development of waveguide and antenna concepts occurs through the application of the methods learned in earlier chapters, thus helping to solidify that knowledge. It may also serve as a bridge between the basic course and more advanced courses that follow it.

I am deeply indebted to several people who provided much-needed feedback and assistance on the work. Glenn S. Smith, Georgia Tech, reviewed parts of the manuscript and had many suggestions on the content and the philosophy of the revision. Several outside reviewers pointed out errors and had excellent suggestions for improving the presentation, most of which, within time limitations, were taken. These include Madeleine Andrawis, South Dakota State University, M. Yousif El-Ibiary, University of Oklahoma, Joel T. Johnson, Ohio State University, David Kelley, Pennsylvania State University, Sharad R. Laxpati, University of Illinois at Chicago, Masoud Mostafavi, San Jose State University, Vladimir A. Rakov, University of Florida, Hussain Al-Rizzo, Sultan

Qaboos University, Juri Silmberg, Ryerson Polytechnic University and Robert M. Weikle II, University of Virginia. My editors at McGraw-Hill, Catherine Fields, Michelle Flomenhoft, and Betsy Jones, provided excellent expertise and support—particularly Michelle, who was almost in daily contact, and provided immediate and knowledgeable answers to all questions and concerns. My seemingly odd conception of the cover illustration was brought into reality through the graphics talents of Ms Diana Fouts at Georgia Tech. Finally, much is owed to my wife and daughters for putting up with a part-time husband and father for many a weekend.

John A. Buck
Atlanta, 2000

CHAPTER 1

VECTOR ANALYSIS

Vector analysis is a mathematical subject which is much better taught by mathematicians than by engineers. Most junior and senior engineering students, however, have not had the time (or perhaps the inclination) to take a course in vector analysis, although it is likely that many elementary vector concepts and operations were introduced in the calculus sequence. These fundamental concepts and operations are covered in this chapter, and the time devoted to them now should depend on past exposure.

The viewpoint here is also that of the engineer or physicist and not that of the mathematician in that proofs are indicated rather than rigorously expounded and the physical interpretation is stressed. It is easier for engineers to take a more rigorous and complete course in the mathematics department after they have been presented with a few physical pictures and applications.

It is possible to study electricity and magnetism without the use of vector analysis, and some engineering students may have done so in a previous electrical engineering or basic physics course. Carrying this elementary work a bit further, however, soon leads to line-filling equations often composed of terms which all look about the same. A quick glance at one of these long equations discloses little of the physical nature of the equation and may even lead to slighting an old friend.

Vector analysis is a mathematical shorthand. It has some new symbols, some new rules, and a pitfall here and there like most new fields, and it demands concentration, attention, and practice. The drill problems, first met at the end of Sec. 1.4, should be considered an integral part of the text and should all be worked. They should not prove to be difficult if the material in the accompanying section of the text has been thoroughly understood. It take a little longer to "read" the chapter this way, but the investment in time will produce a surprising interest.

1.1 SCALARS AND VECTORS

The term *scalar* refers to a quantity whose value may be represented by a single (positive or negative) real number. The x, y, and z we used in basic algebra are scalars, and the quantities they represent are scalars. If we speak of a body falling a distance L in a time t, or the temperature T at any point in a bowl of soup whose coordinates are x, y, and z, then L, t, T, x, y, and z are all scalars. Other scalar quantities are mass, density, pressure (but not force), volume, and volume resistivity. Voltage is also a scalar quantity, although the complex representation of a sinusoidal voltage, an artificial procedure, produces a *complex scalar*, or *phasor*, which requires two real numbers for its representation, such as amplitude and phase angle, or real part and imaginary part.

A *vector* quantity has both a magnitude[1] and a direction in space. We shall be concerned with two- and three-dimensional spaces only, but vectors may be defined in n-dimensional space in more advanced applications. Force, velocity, acceleration, and a straight line from the positive to the negative terminal of a storage battery are examples of vectors. Each quantity is characterized by both a magnitude and a direction.

We shall be mostly concerned with scalar and vector *fields*. A field (scalar or vector) may be defined mathematically as some function of that vector which connects an arbitrary origin to a general point in space. We usually find it possible to associate some physical effect with a field, such as the force on a compass needle in the earth's magnetic field, or the movement of smoke particles in the field defined by the vector velocity of air in some region of space. Note that the field concept invariably is related to a region. Some quantity is defined at every point in a region. Both *scalar fields* and *vector fields* exist. The temperature throughout the bowl of soup and the density at any point in the earth are examples of scalar fields. The gravitational and magnetic fields of the earth, the voltage gradient in a cable, and the temperature gradient in a soldering-iron tip are examples of vector fields. The value of a field varies in general with both position and time.

In this book, as in most others using vector notation, vectors will be indicated by boldface type, for example, **A**. Scalars are printed in italic type, for example, A. When writing longhand or using a typewriter, it is customary to draw a line or an arrow over a vector quantity to show its vector character. (CAUTION: This is the first pitfall. Sloppy notation, such as the omission of the line or arrow symbol for a vector, is the major cause of errors in vector analysis.)

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[1] We adopt the convention that "magnitude" infers "absolute value"; the magnitude of any quantity is therefore always positive.

1.2 VECTOR ALGEBRA

With the definitions of vectors and vector fields now accomplished, we may proceed to define the rules of vector arithmetic, vector algebra, and (later) of vector calculus. Some of the rules will be similar to those of scalar algebra, some will differ slightly, and some will be entirely new and strange. This is to be expected, for a vector represents more information than does a scalar, and the multiplication of two vectors, for example, will be more involved than the multiplication of two scalars.

The rules are those of a branch of mathematics which is firmly established. Everyone "plays by the same rules," and we, of course, are merely going to look at and interpret these rules. However, it is enlightening to consider ourselves pioneers in the field. We are making our own rules, and we can make any rules we wish. The only requirement is that the rules be self-consistent. Of course, it would be nice if the rules agreed with those of scalar algebra where possible, and it would be even nicer if the rules enabled us to solve a few practical problems.

One should not fall into the trap of "algebra worship" and believe that the rules of college algebra were delivered unto man at the Creation. These rules are merely self-consistent and extremely useful. There are other less familiar algebras, however, with very different rules. In Boolean algebra the product AB can be only unity or zero. Vector algebra has its own set of rules, and we must be constantly on guard against the mental forces exerted by the more familiar rules or scalar algebra.

Vectorial addition follows the parallelogram law, and this is easily, if inaccurately, accomplished graphically. Fig. 1.1 shows the sum of two vectors, $\mathbf{A}$ and $\mathbf{B}$. It is easily seen that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$, or that vector addition obeys the commutative law. Vector addition also obeys the associative law,

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

Note that when a vector is drawn as an arrow of finite length, its location is defined to be at the tail end of the arrow.

Coplanar vectors, or vectors lying in a common plane, such as those shown in Fig. 1.1, which both lie in the plane of the paper, may also be added by expressing each vector in terms of "horizontal" and "vertical" components and adding the corresponding components.

Vectors in three dimensions may likewise be added by expressing the vectors in terms of three components and adding the corresponding components. Examples of this process of addition will be given after vector components are discussed in Sec. 1.4.

The rule for the subtraction of vectors follows easily from that for addition, for we may always express $\mathbf{A} - \mathbf{B}$ as $\mathbf{A} + (-\mathbf{B})$; the sign, or direction, of the second vector is reversed, and this vector is then added to the first by the rule for vector addition.

Vectors may be multiplied by scalars. The magnitude of the vector changes, but its direction does not when the scalar is positive, although it reverses direc-

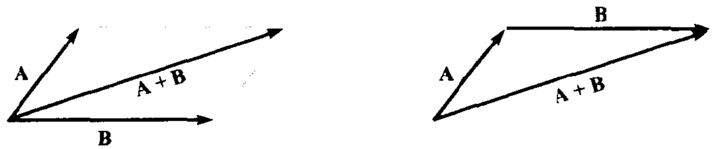

FIGURE 1.1
Two vectors may be added graphically either by drawing both vectors from a common origin and completing the parallelogram or by beginning the second vector from the head of the first and completing the triangle; either method is easily extended to three or more vectors.

tion when multiplied by a negative scalar. Multiplication of a vector by a scalar also obeys the associative and distributive laws of algebra, leading to

$$(r+s)(\mathbf{A}+\mathbf{B}) = r(\mathbf{A}+\mathbf{B}) + s(\mathbf{A}+\mathbf{B}) = r\mathbf{A} + r\mathbf{B} + s\mathbf{A} + s\mathbf{B}$$

Division of a vector by a scalar is merely multiplication by the reciprocal of that scalar.

The multiplication of a vector by a vector is discussed in Secs. 1.6 and 1.7.

Two vectors are said to be equal if their difference is zero, or $\mathbf{A} = \mathbf{B}$ if $\mathbf{A} - \mathbf{B} = 0$.

In our use of vector fields we shall always add and subtract vectors which are defined at the same point. For example, the *total* magnetic field about a small horseshoe magnet will be shown to be the sum of the fields produced by the earth and the permanent magnet; the total field at any point is the sum of the individual fields at that point.

If we are not considering a vector *field*, however, we may add or subtract vectors which are not defined at the same point. For example, the sum of the gravitational force acting on a 150-lb_f (pound-force) man at the North Pole and that acting on a 175-lb_f man at the South Pole may be obtained by shifting each force vector to the South Pole before addition. The resultant is a force of 25 lb_f directed toward the center of the earth at the South Pole; if we wanted to be difficult, we could just as well describe the force as 25 lb_f directed *away* from the center of the earth (or "upward") at the North Pole.[2]

1.3 THE CARTESIAN COORDINATE SYSTEM

In order to describe a vector accurately, some specific lengths, directions, angles, projections, or components must be given. There are three simple methods of doing this, and about eight or ten other methods which are useful in very special cases. We are going to use only the three simple methods, and the simplest of these is the *cartesian*, or *rectangular, coordinate system.*

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[2] A few students have argued that the force might be described at the equator as being in a "northerly" direction. They are right, but enough is enough.

In the cartesian coordinate system we set up three coordinate axes mutually at right angles to each other, and call them the x, y, and z axes. It is customary to choose a *right-handed* coordinate system, in which a rotation (through the smaller angle) of the x axis into the y axis would cause a right-handed screw to progress in the direction of the z axis. If the right hand is used, then the thumb, forefinger, and middle finger may then be identified, respectively, as the x, y, and z axes. Fig. 1.2a shows a right-handed cartesian coordinate system.

A point is located by giving its x, y, and z coordinates. These are, respectively, the distances from the origin to the intersection of a perpendicular dropped from the point to the x, y, and z axes. An alternative method of interpreting coordinate values, and a method corresponding to that which *must* be used in all other coordinate systems, is to consider the point as being at the

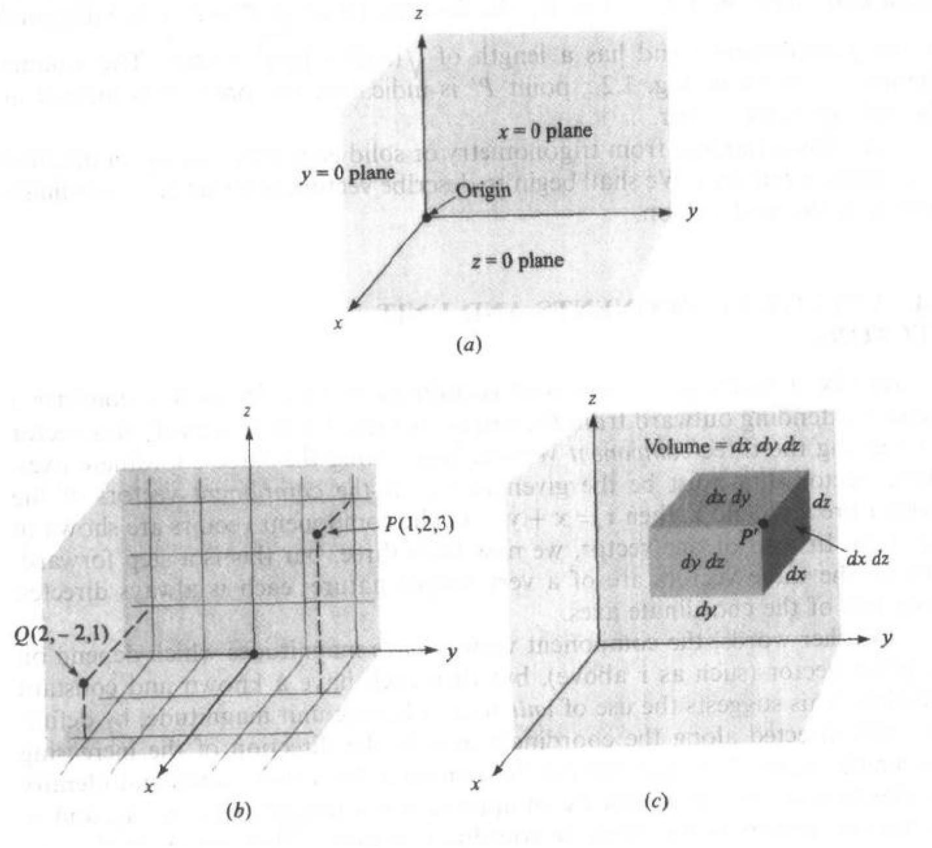

FIGURE 1.2
(a) A right-handed cartesian coordinate system. If the curved fingers of the right hand indicate the direction through which the x axis is turned into coincidence with the y axis, the thumb shows the direction of the z axis. (b) The location of points $P(1, 2, 3)$ and $Q(2, -2, 1)$. (c) The differential volume element in cartesian coordinates; dx, dy, and dz are, in general, independent differentials.

common intersection of three surfaces, the planes $x =$ constant, $y =$ constant, and $z =$ constant, the constants being the coordinate values of the point.

Fig. 1.2*b* shows the points P and Q whose coordinates are $(1, 2, 3)$ and $(2, -2, 1)$, respectively. Point P is therefore located at the common point of intersection of the planes $x = 1$, $y = 2$, and $z = 3$, while point Q is located at the intersection of the planes $x = 2$, $y = -2$, $z = 1$.

As we encounter other coordinate systems in Secs. 1.8 and 1.9, we should expect points to be located at the common intersection of three surfaces, not necessarily planes, but still mutually perpendicular at the point of intersection.

If we visualize three planes intersecting at the general point P, whose coordinates are x, y, and z, we may increase each coordinate value by a differential amount and obtain three slightly displaced planes intersecting at point P', whose coordinates are $x + dx$, $y + dy$, and $z + dz$. The six planes define a rectangular parallelepiped whose volume is $dv = dxdydz$; the surfaces have differential areas dS of $dxdy$, $dydz$, and $dzdx$. Finally, the distance dL from P to P' is the diagonal of the parallelepiped and has a length of $\sqrt{(dx)^2 + (dy)^2 + (dz)^2}$. The volume element is shown in Fig. 1.2*c*; point P' is indicated, but point P is located at the only invisible corner.

All this is familiar from trigonometry or solid geometry and as yet involves only scalar quantities. We shall begin to describe vectors in terms of a coordinate system in the next section.

1.4 VECTOR COMPONENTS AND UNIT VECTORS

To describe a vector in the cartesian coordinate system, let us first consider a vector **r** extending outward from the origin. A logical way to identify this vector is by giving the three *component vectors*, lying along the three coordinate axes, whose vector sum must be the given vector. If the component vectors of the vector **r** are **x**, **y**, and **z**, then $\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$. The component vectors are shown in Fig. 1.3*a*. Instead of one vector, we now have three, but this is a step forward, because the three vectors are of a very simple nature; each is always directed along one of the coordinate axes.

In other words, the component vectors have magnitudes which depend on the given vector (such as **r** above), but they each have a known and constant direction. This suggests the use of *unit vectors* having unit magnitude, by definition, and directed along the coordinate axes in the direction of the increasing coordinate values. We shall reserve the symbol **a** for a unit vector and identify the direction of the unit vector by an appropriate subscript. Thus $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are the unit vectors in the cartesian coordinate system.[3] They are directed along the x, y, and z axes, respectively, as shown in Fig. 1.3*b*.

[3]The symbols **i**, **j**, and **k** are also commonly used for the unit vectors in cartesian coordinates.

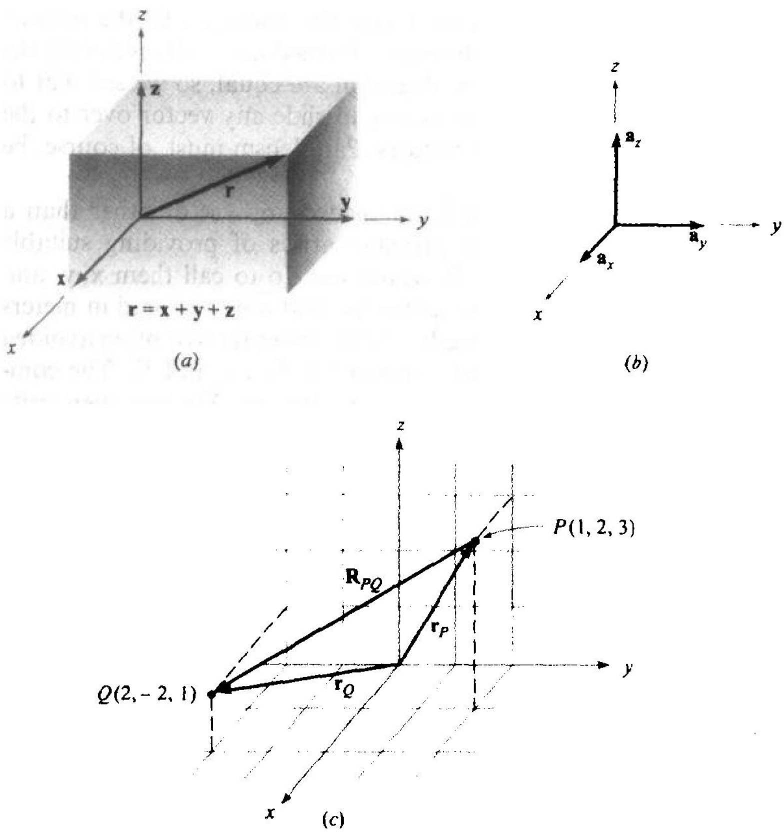

FIGURE 1.3
(*a*) The component vectors **x**, **y**, and **z** of vector **r**. (*b*) The unit vectors of the cartesian coordinate system have unit magnitude and are directed toward increasing values of their respective variables. (*c*) The vector $\mathbf{R}_{PQ}$ is equal to the vector difference $\mathbf{r}_Q - \mathbf{r}_P$.

If the component vector **y** happens to be two units in magnitude and directed toward increasing values of y, we should then write $\mathbf{y} = 2\mathbf{a}_y$. A vector $\mathbf{r}_P$ pointing from the origin to point $P(1, 2, 3)$ is written $\mathbf{r}_P = \mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$. The vector from P to Q may be obtained by applying the rule of vector addition. This rule shows that the vector from the origin to P plus the vector from P to Q is equal to the vector from the origin to Q. The desired vector from $P(1, 2, 3)$ to $Q(2, -2, 1)$ is therefore

$$\begin{aligned}\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P &= (2 - 1)\mathbf{a}_x + (-2 - 2)\mathbf{a}_y + (1 - 3)\mathbf{a}_z \\ &= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z\end{aligned}$$

The vectors $\mathbf{r}_P$, $\mathbf{r}_Q$, and $\mathbf{R}_{PQ}$ are shown in Fig. 1.3*c*.