国外电子与通信教材系列

线性电路分析基础

（第二版）

Elementary Linear Circuit Analysis

Second Edition

英文版

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线性电路分析基础

（第二版） 英文版

Elementary Linear Circuit Analysis, Second Edition

《线性电路分析基础》（第二版）是美国大学电类专业“线性电路分析”课程的教科书。全书分 14 章，从基本元件和定律开始，依次讲述电路分析方法、重要的电路概念、储能元件、一阶电路、二阶电路、计算机辅助电路分析、正弦分析、功率、重要的交流概念、拉普拉斯变换、双端口网络、傅里叶级数、傅里叶变换等线性电路分析中最基础的内容。

本书内容安排适应电子信息科学技术的发展，文字叙述简洁，公式推导准确，并有大量的例题，每章后的小结、习题以及习题答案便于读者理解、掌握基本概念并能将其灵活运用到工程实践中。此外，采纳本书作为教材的教师可免费获得与本书配套的教师指导手册。

本书属于基础课教材，内容经典，组织编排独特，适于教学，可作为电子类理工科高等学校各类专业的“线性电路分析”课程的教科书及教学参考书，也可作为有关工程技术人员进修教材。

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CHAPTER 1

Basic Elements and Laws

INTRODUCTION

The study of electric circuits is fundamental in electrical engineering education, and can be quite valuable in other disciplines as well. The skills acquired not only are useful in such electrical engineering areas as electronics, communications, microwaves, control, and power systems but also can be employed in other seemingly different fields.

By an **electric circuit** or **network** we mean a collection of electrical devices (for example, voltage and current sources, resistors, inductors, capacitors, transformers, amplifiers, and transistors) that are interconnected in some manner. The various uses of such circuits, though important, is not the major concern of this text. Instead, our prime interest will be with the process of determining the behavior of a given circuit—which is referred to as **analysis**.

We begin our study by discussing some basic electric elements and the laws that describe them. It is assumed that the reader has been introduced to the concepts of electric charge, potential, and current in various science and physics courses in high school and college.

1.1 IDEAL SOURCES

Electric charge† is measured in **coulombs** (abbreviated C) in honor of the French scientist Charles de Coulomb (1736–1806); the unit of work or energy—the **joule**

† An electron has a negative charge of 1.6×10^{-19} C.

(J)—is named for the British physicist James P. Joule (1818–1889). Although the unit for energy expended on electric charge is J/C, we give it the special name **volt** (V) in honor of the Italian physicist Alessandro Volta (1745–1827), and we way that it is a measure of **electric potential difference** or **voltage**. These units are part of the **Système International d'Unités** (International System of Units). Units of this system are referred to as **SI** units. Unless indicated to the contrary, SI units are the units used in this book.

An **ideal voltage source**, which is represented in Fig. 1.1, is a device that produces a voltage or potential difference of v volts across its **terminals** *regardless of what is connected to it.*

For the device shown in Fig. 1.1, terminal 1 is marked plus (+) and terminal 2 is marked minus (−). This denotes that terminal 1 is at an electric potential that is v volts higher than that of terminal 2. (Alternatively, the electric potential of terminal 2 is v volts lower than that of terminal 1.)

The quantity v can have either a positive or a negative value. For the latter case, it is possible to obtain an equivalent source with a positive value. For suppose that $v = -5$ V for the voltage source shown in Fig. 1.1. Then the potential at terminal 1 is −5 V higher than that of terminal 2. However, this is equivalent to saying that terminal 1 is at a potential of +5 V lower than terminal 2. Consequently, the two ideal voltage sources shown in Fig. 1.2 are equivalent.

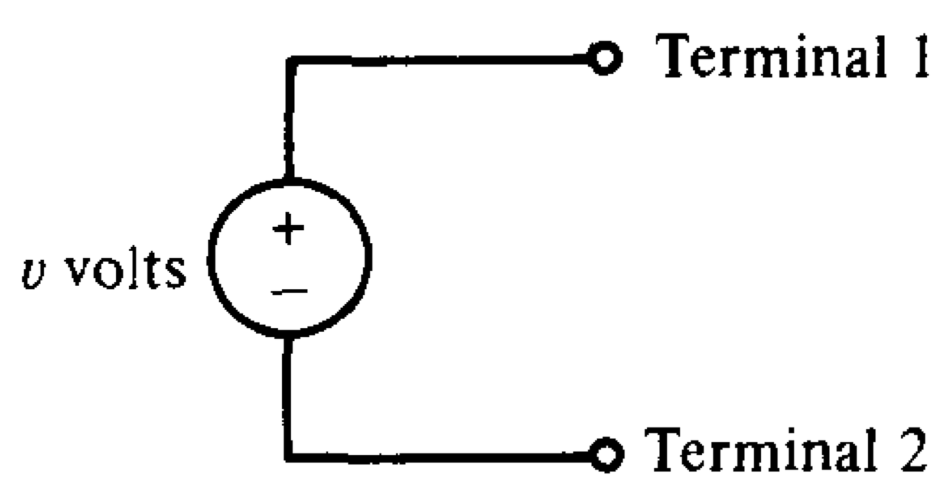

Fig. 1.1 Ideal voltage source.

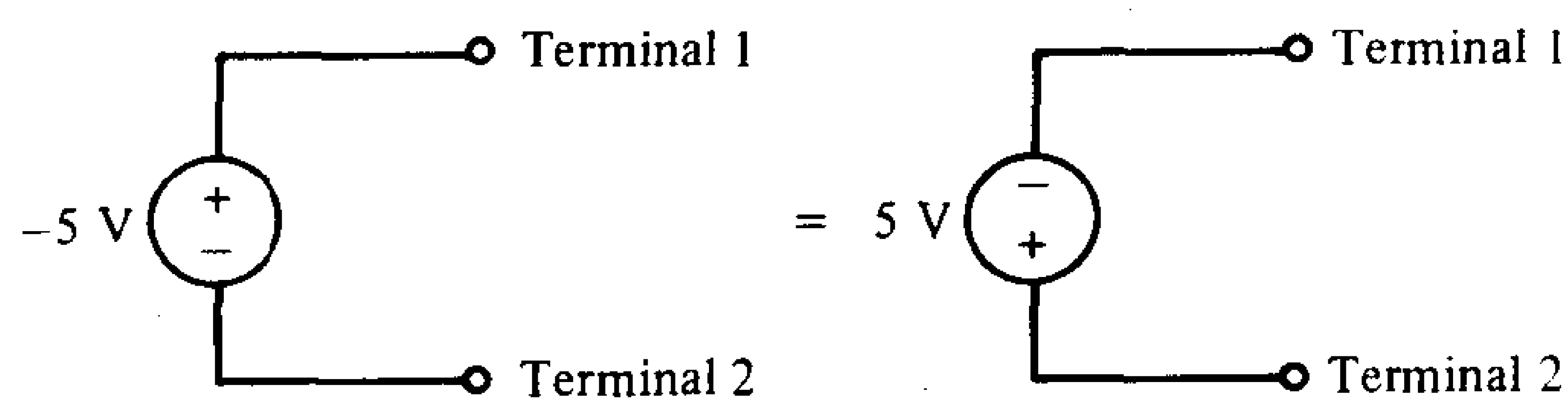

Fig. 1.2 Equivalent ideal voltage sources.

In the discussion above, we may have implied that the value of an ideal voltage source is constant, that is, it does not change with time. Such a situation is plotted in Fig. 1.3 for the case that $v = 3$ V. For occasions such as this, an ideal voltage source is commonly represented by the equivalent notation shown in Fig. 1.4. We refer to such a device as an ideal **battery**. Although an actual battery is not ideal, there are many circumstances under which an ideal battery is a very good approximation. One

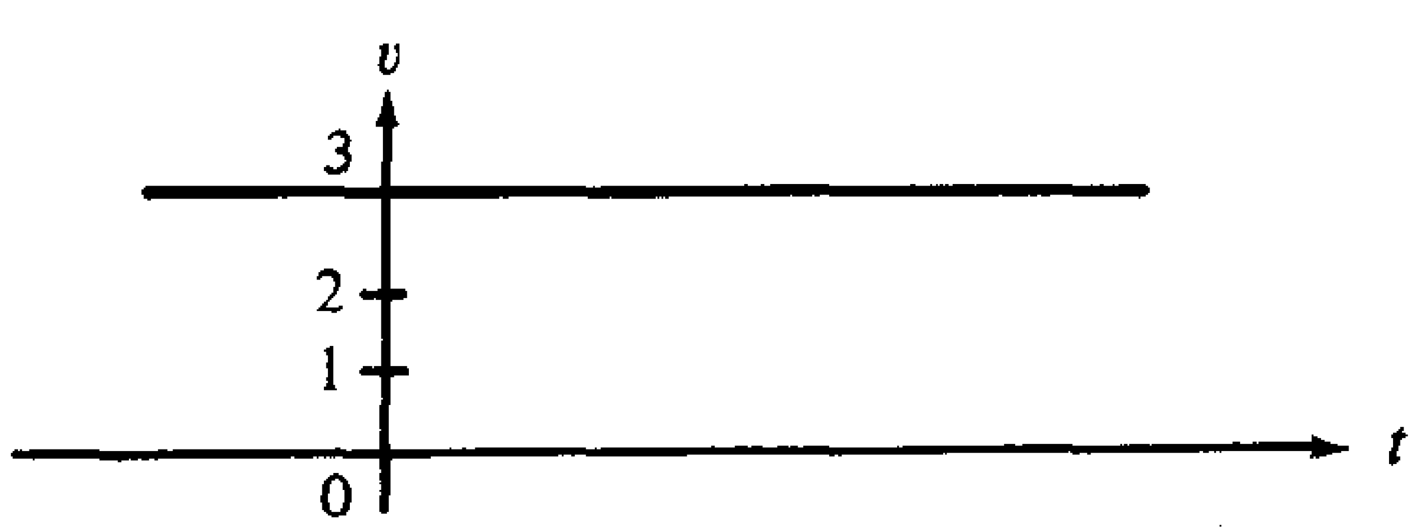

Fig. 1.3 Constant voltage.

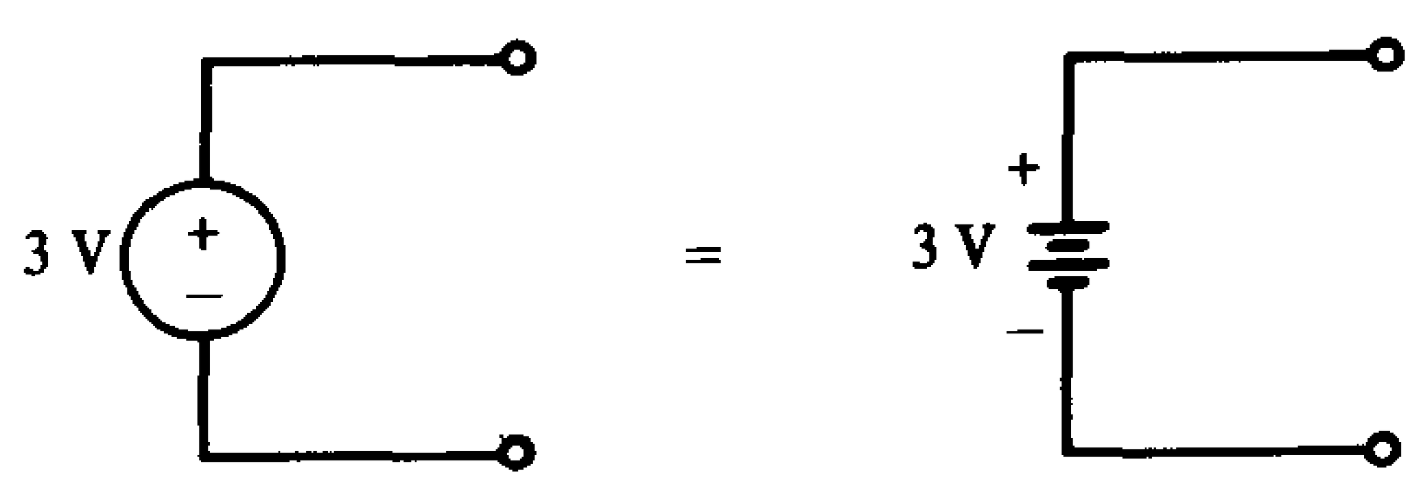

Fig. 1.4 Battery symbols.

such example is the 9-V battery that you use for your portable transistor radio—or you may have the type that uses four or six C or D $1\frac{1}{2}$-V batteries. A 12-V automobile storage battery is another case in point. In general, however, the voltage produced by an ideal voltage source will be a function of time. A few of the multitude of possible voltage waveforms are shown in Fig. 1.5.

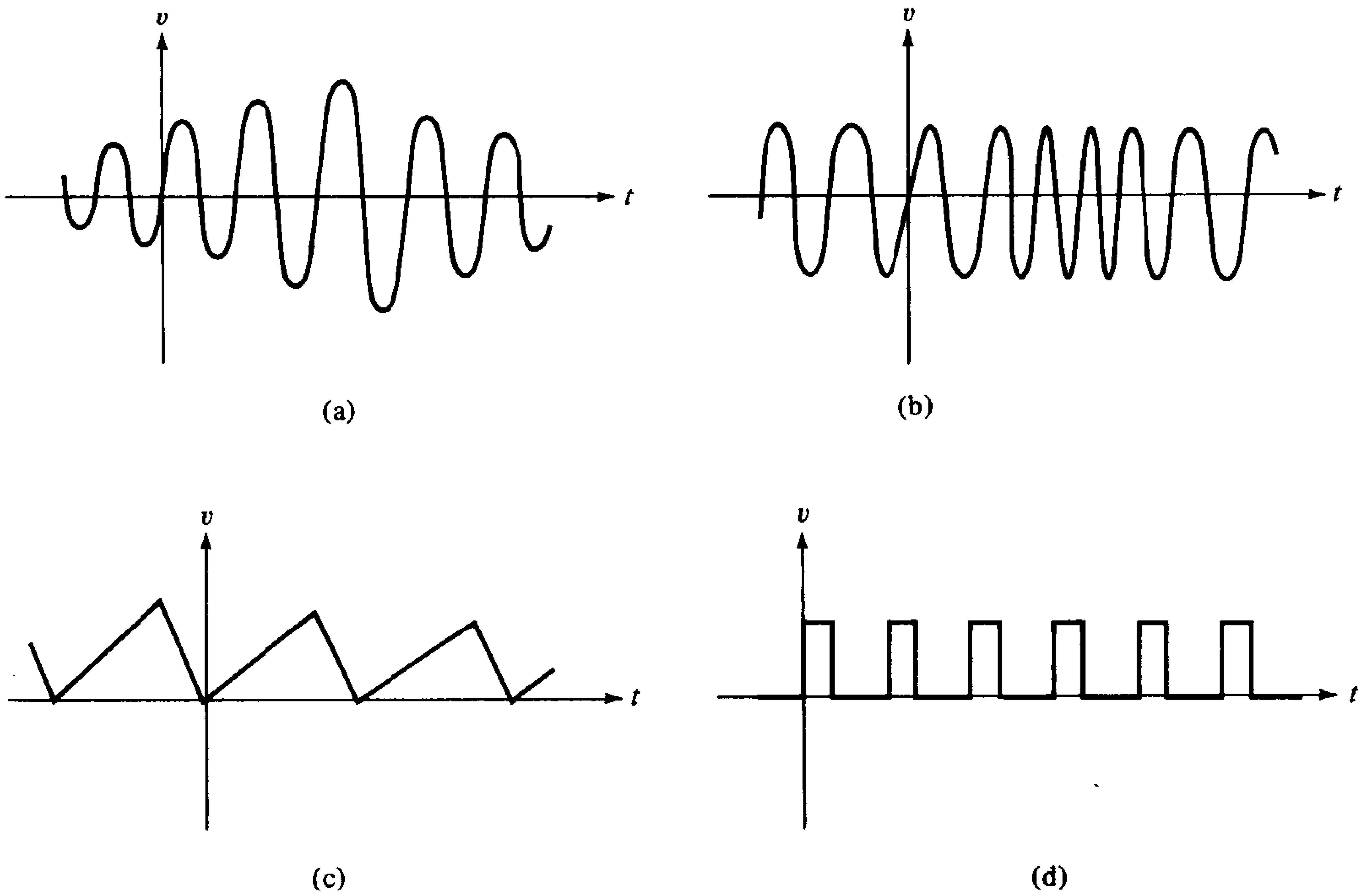

Fig. 1.5 Typical voltage waveforms.

Since the voltage produced by a source is, in general, a function of time, say $v(t)$, then the most general representation of an ideal voltage source is that shown in Fig. 1.6. There should be no confusion if the units "volts" are not included in the representation of the source. Thus, the ideal voltage source in Fig. 1.7 is identical to the one in Fig. 1.6 with "volts" being understood.

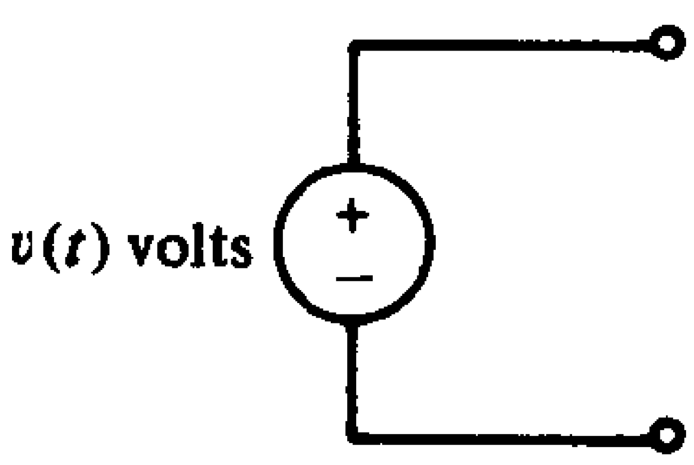

Fig. 1.6 Generalized ideal voltage source.

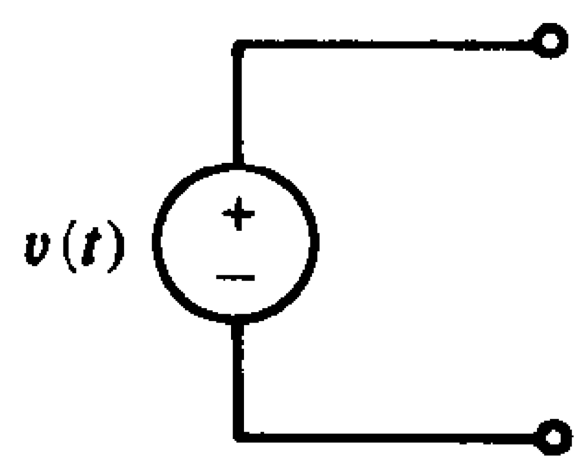

Fig. 1.7 Equivalent generalized ideal voltage source.

DRILL EXERCISE 1.1

An ideal voltage source has a value of $v(t) = 10e^{-t}$ V. What is the voltage produced by this source when (a) $t = 0$ seconds, (b) $t = 1$ second, (c) $t = 2$ seconds, (d) $t = 3$ seconds, and (e) $t = 4$ seconds?

Answer: (a) 10 V; (b) 3.68 V; (c) 1.35 V; (d) 0.498 V; (e) 0.183 V

Placing an electric potential difference (voltage) across some material generally results in a flow of electric charge. Negative charge (in the form of electrons) flows from a given electric potential to a higher potential. Conversely, positive charge tends to flow from a given potential to a lower potential. Charge is usually denoted by q, and since this quantity is generally time dependent, the total amount of charge that is present in a given region is designated by $q(t)$.

We define **current**, denoted $i(t)$, to be the flow rate of the charge; that is,

$$i(t) = \frac{dq(t)}{dt}$$

is the current in the region containing $q(t)$. Following the convention of Benjamin Franklin (a positive thinker), the direction of current has been chosen to be opposite to the direction of electron flow. The units of current (coulombs per second, or C/s) are referred to as **amperes** (A) or **amps** for short, in honor of the French physicist André Ampère (1775–1836).

An **ideal current source**, represented as shown in Fig. 1.8, is a device which, *when connected to anything*, will always move I amperes in the direction indicated by the arrow.

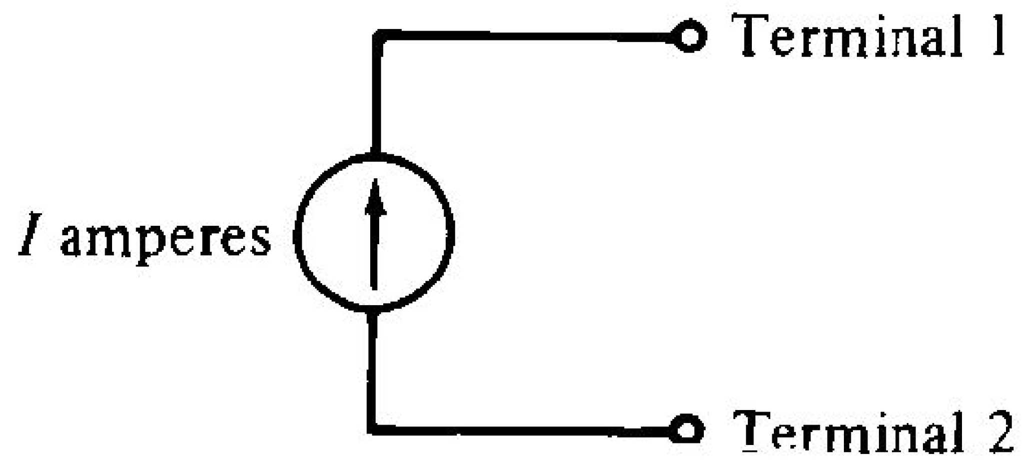

Fig. 1.8 Ideal current source.

As a consequence of the definition, it should be quite clear that the ideal current sources in Fig. 1.9 are equivalent.

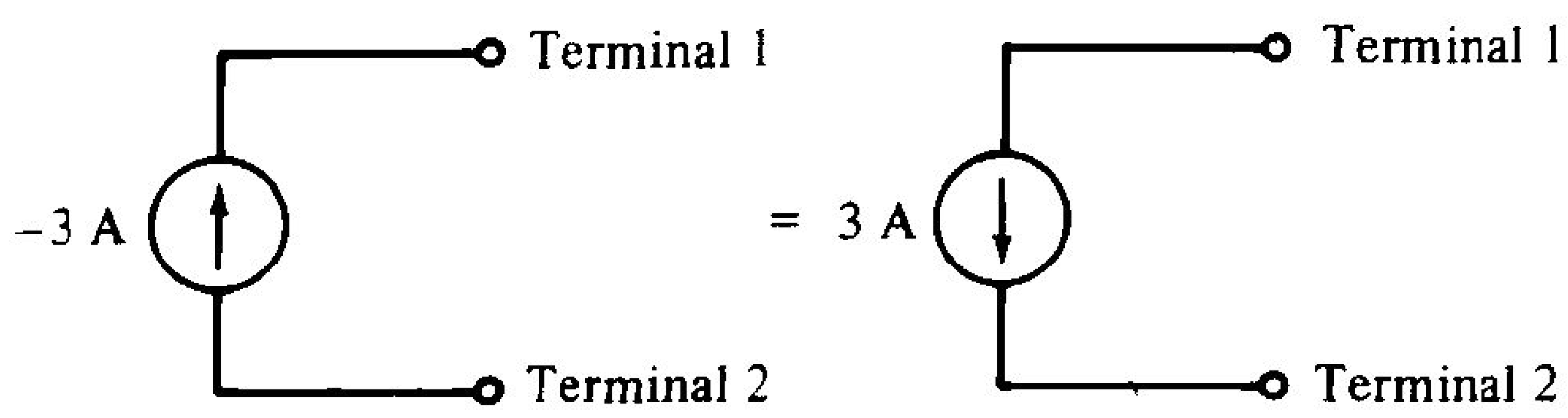

Fig. 1.9 Equivalent ideal current sources.

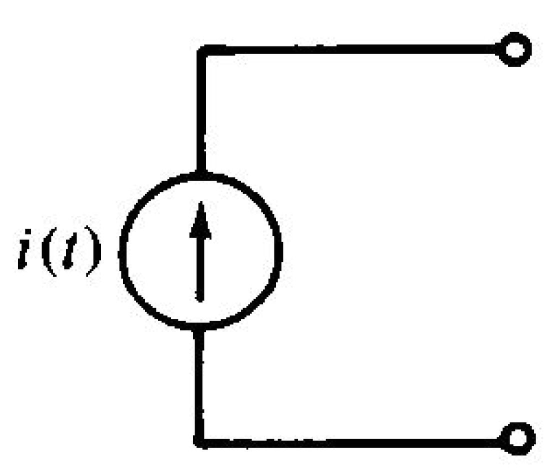

Fig. 1.10 Generalized ideal current source.

Again, in general, the amount of current produced by an ideal source will be a function of time. Thus, the general representation of an ideal current source is shown in Fig. 1.10, where the units "amperes" are understood.

DRILL EXERCISE 1.2

The total charge in some region is described by the function $q(t) = 3e^{-4t} + 0.02 \sin 120\pi t$ C. Find the magnitude of the current in this region.
Answer: $-12e^{-4t} + 7.54 \cos 120\pi t$ A

1.2 RESISTORS AND OHM'S LAW

Suppose that some material is connected to the terminals of an ideal voltage source $v(t)$ as shown in Fig. 1.11. Suppose that $v(t) = 1$ V. Then the electric potential at the top of the material is 1 V above the potential at the bottom. Since an electron has a negative charge, electrons in the material will tend to flow from bottom to top. Therefore, we say that current tends to go from top to bottom through the material. Hence, for the given polarity, when $v(t)$ is a positive number, $i(t)$ will be a positive number with the direction indicated. If $v(t) = 2$ V, again the potential at the top is greater than at the bottom, so $i(t)$ will again be positive. However, because the potential is now twice as large as before, the current will be greater. (If the material is a "linear" element, the current will be twice as great.) Suppose now that $v(t) = 0$ V. Then the potentials at the top and the bottom of the material are the same. The result is no flow of electrons and, hence, no current. In this case, $i(t) = 0$ A. But suppose that $v(t) = -2$ V. Then the top of the material will be at a potential lower than at the bottom of the material. A current from bottom to top will result, and $i(t)$ will be a negative number. Due to the physical law of the conservation of electric charge, $i(t)$ goes through the voltage source as indicated.

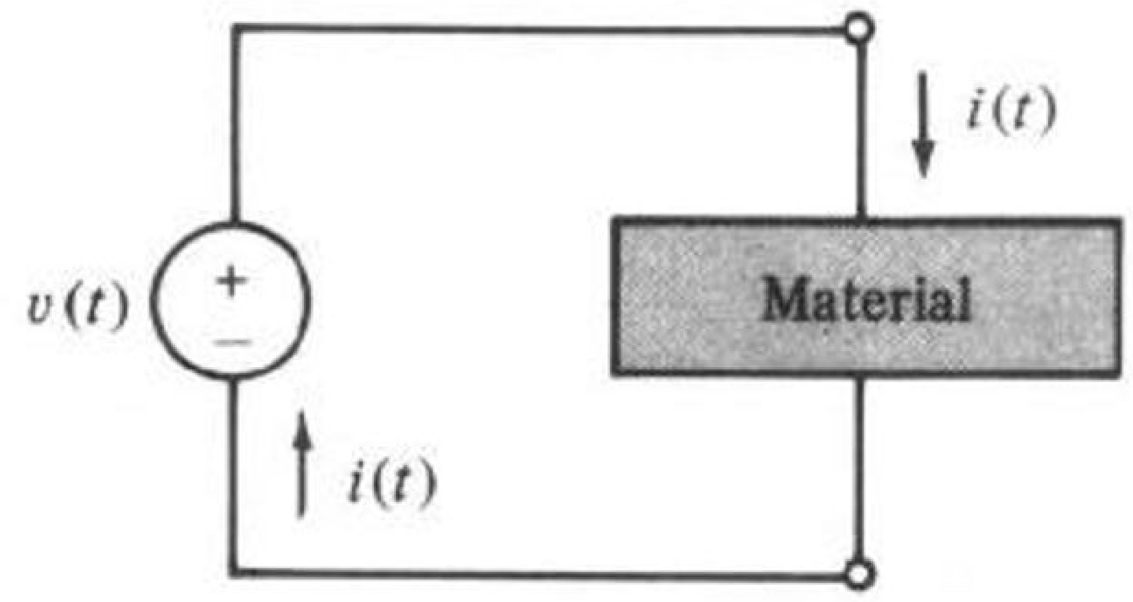

Fig. 1.11 Material with an applied voltage.

If in Fig. 1.11 the resulting current $i(t)$ is always directly proportional to the voltage for any function $v(t)$, then the material is called a **linear resistor**, or **resistor** for short.

Since voltage and current are directly proportional for a resistor, there exists a proportionality constant R, called **resistance**, such that

$$v(t) = Ri(t)$$

In dividing both sides of this equation by $i(t)$, we obtain

$$R = \frac{v(t)}{i(t)}$$

The units of resistance (volts per ampere) are referred to as **ohms**† and are denoted by the capital Greek letter omega, Ω. The accepted circuit symbol for a resistor whose resistance is R ohms is shown in Fig. 1.12. A plot of voltage versus current for a (linear) resistor is given in Fig. 1.13.

Fig. 1.12 Circuit symbol for a resistor.

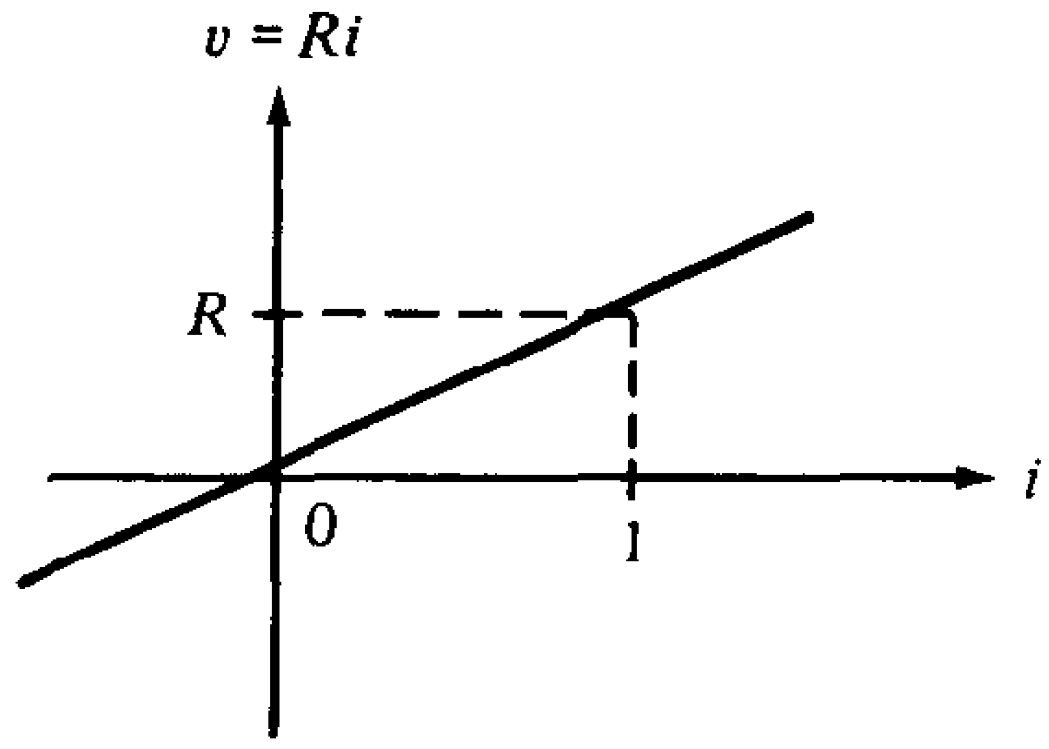

Fig. 1.13 Plot of voltage versus current for a resistor.

It was Ohm who discovered that if a resistor R has a voltage $v(t)$ *across* it and a current $i(t)$ *through* it, then if one is the cause, the other is the effect. Furthermore, if the polarity of the voltage and the direction of the current are as shown in Fig. 1.14, then it is true that

$$\boxed{v(t) = Ri(t)}$$

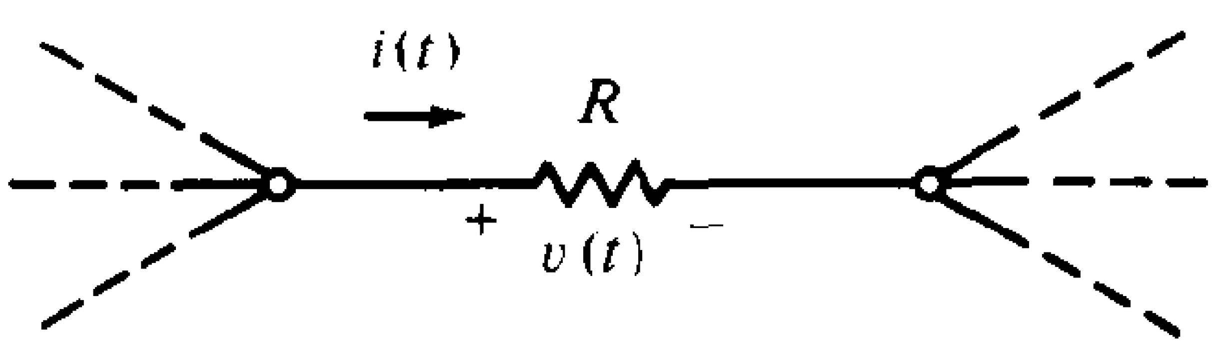

Fig. 1.14 Current and voltage convention for Ohm's law.

† Named for the German physicist Georg Ohm (1787–1854).

This equation is often called **Ohm's law**. From it, we may immediately deduce that

$$\boxed{R = \frac{v(t)}{i(t)}} \quad \text{and} \quad \boxed{i(t) = \frac{v(t)}{R}}$$

These last two equations are also referred to as Ohm's law.

EXAMPLE 1.1
For the resistor given in Fig. 1.14, suppose that $R = 10\ \Omega$. When $i(t) = 2$ A, then $v(t) = Ri(t) = 10(2) = 20$ V.

But, now consider the case that $i(t) = -3$ A. Under this circumstance, $v(t) = Ri(t) = 10(-3) = -30$ V. This can be represented either as shown in Fig. 1.15(a), or alternatively, as shown in Fig. 1.15(b), where $i_1(t) = -i(t)$.

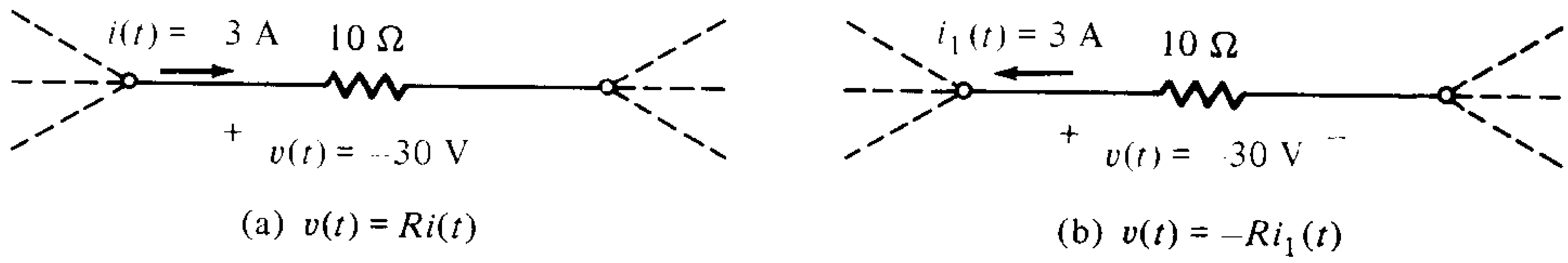

Fig. 1.15 Equivalent forms of Ohm's law.

From Example 1.1, we see that because of the use of negative numbers, we can have situations where currents are indicated going through resistors from + to − as depicted in Fig. 1.14, or going from − to + as in Fig. 1.16. For the former case, Ohm's law is $v(t) = Ri(t)$. However, for the latter case, since Fig. 1.17 is equivalent to Fig. 1.16, and since Fig. 1.17 is in the form of Fig. 1.14, we use Ohm's law to write its alternative version

$$v_1(t) = R[-i_1(t)]$$

or

$$v_1(t) = -Ri_1(t)$$

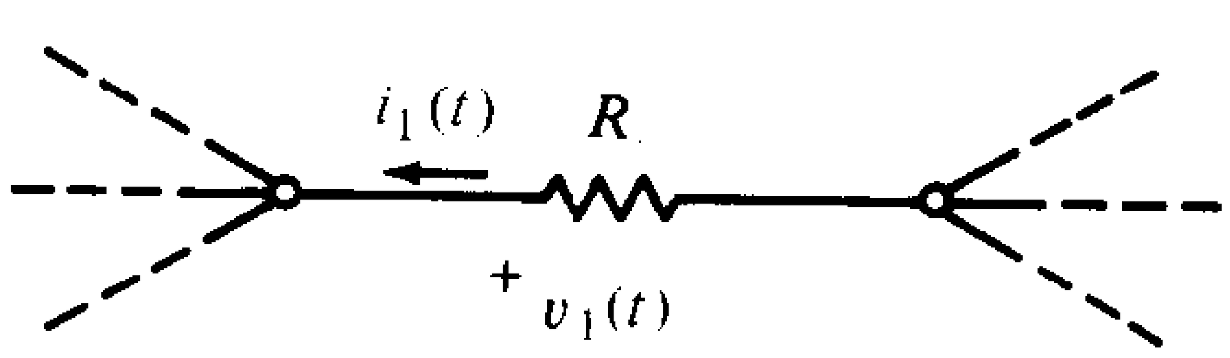

Fig. 1.16 Situation requiring negative sign for Ohm's law.

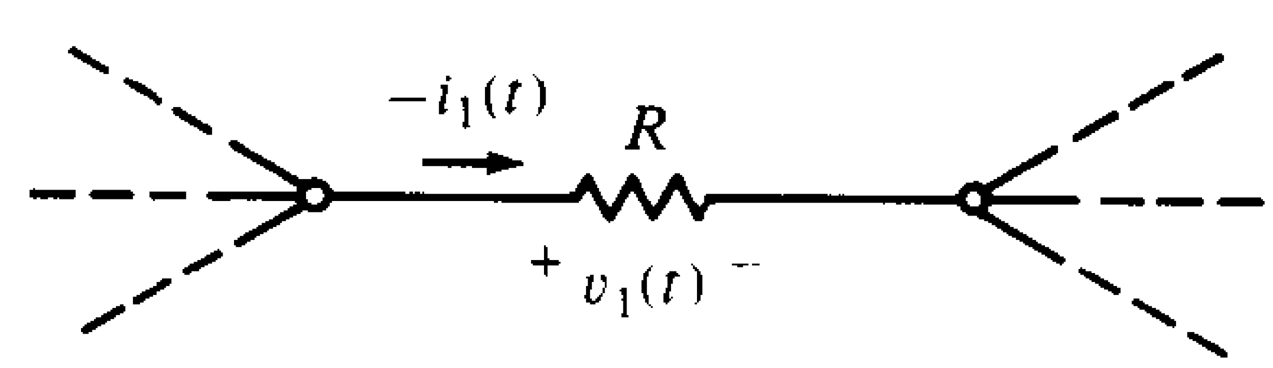

Fig. 1.17 Equivalent form of Fig. 1.16.

EXAMPLE 1.2
Consider the circuit shown in Fig. 1.18. We use the letter "k" to represent the prefix "kilo," which indicates a value of 10^3. Following is a table of the more common symbols.

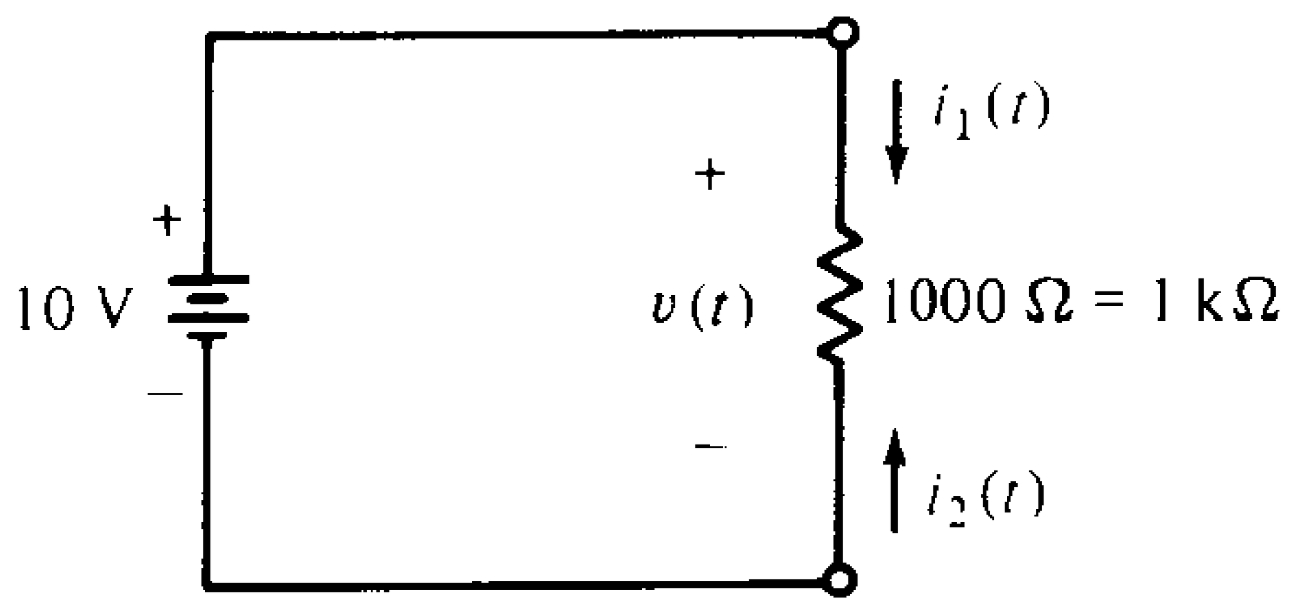

Fig. 1.18 Circuit for Example 1.2.

Value	Prefix	Symbol
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G

For the circuit in Fig. 1.18, the voltage across the 1-kΩ resistor is, by the definition of an ideal voltage source, $v(t) = 10$ V. Thus, by Ohm's law, we get

$$i_1(t) = \frac{v(t)}{R} = \frac{10}{1000} = \frac{1}{100} = 0.01 \text{ A} = 10 \text{ mA}$$

and

$$i_2(t) = -\frac{v(t)}{R} = -\frac{10}{1000} = -\frac{1}{100} = -0.01 \text{ A} = -10 \text{ mA}$$

Note that $i_2(t) = -i_1(t)$ as expected.

For the circuit shown in Fig. 1.19, by the definition of an ideal current source, $i(t) = 25\ \mu\text{A} = 25 \times 10^{-6}$ A. By Ohm's law, we have that

$$v(t) = -Ri(t) = -(2 \times 10^6)(25 \times 10^{-6}) = -50 \text{ V}$$

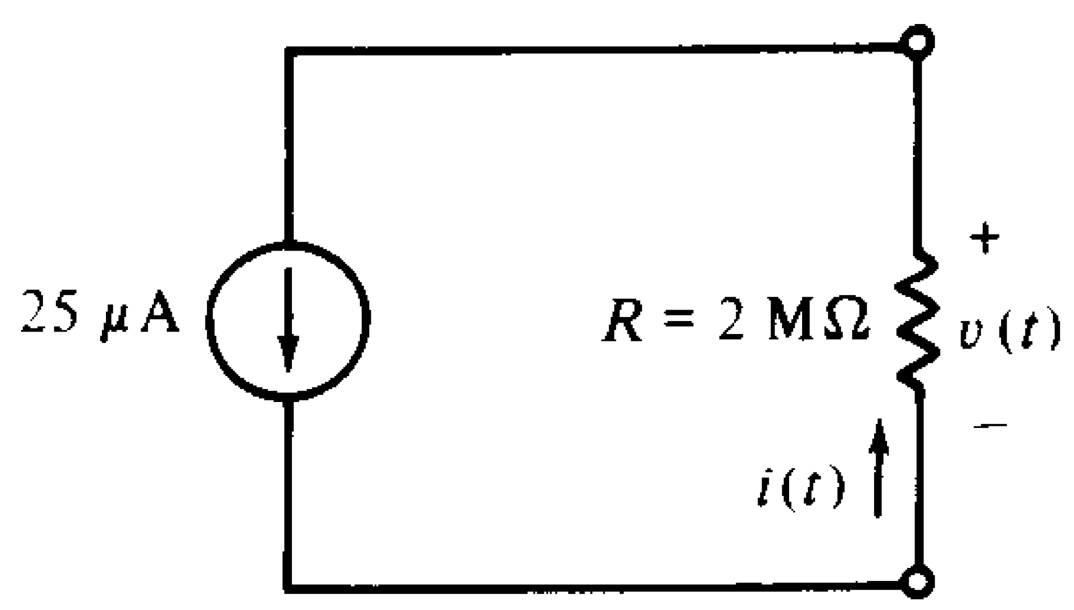

Fig. 1.19. Another circuit for Example 1.2.

DRILL EXERCISE 1.3

For the circuit shown in Fig. DE1.3, find (a) v_1, (b) v_2, (c) i_3, (d) i_4, and (e) R.
Answer: (a) -1 V; (b) 6 V; (c) -2 mA; (d) 2 mA; (e) 3 kΩ

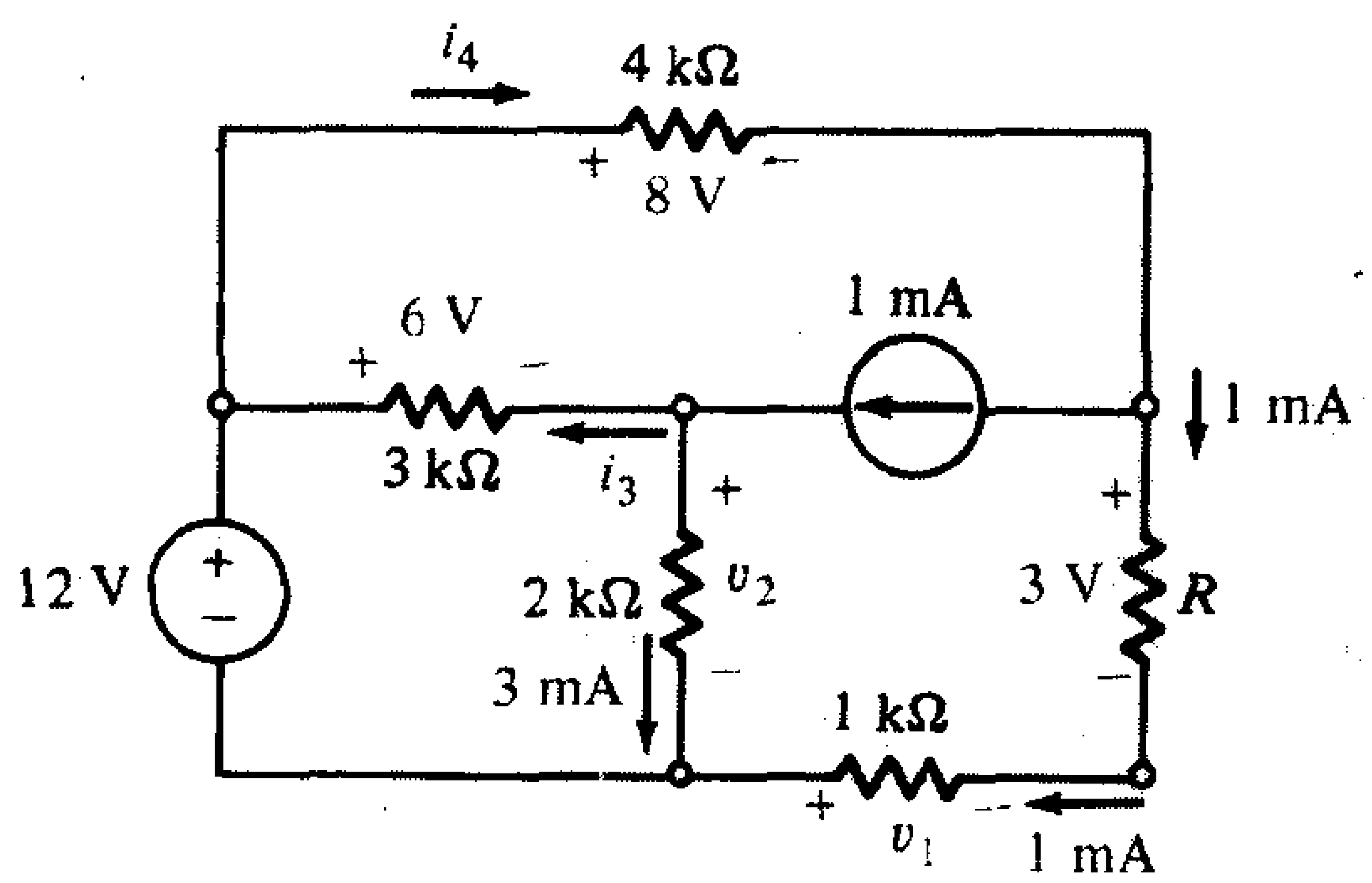

Fig. DE1.3

Given a resistor R connected to an ideal voltage source $v(t)$ as shown in Fig. 1.20, we conclude the following. Since $i(t) = v(t)/R$ for any particular ideal source $v(t)$, the amount of current $i(t)$ that results can be made to be any finite value by choosing the appropriate value for R (e.g., to make $i(t)$ large, make R small). Thus, we see that an ideal voltage source is capable of supplying *any* amount of current, and that amount depends on what is connected to the source—only the voltage is constrained to be $v(t)$ volts at the terminals.

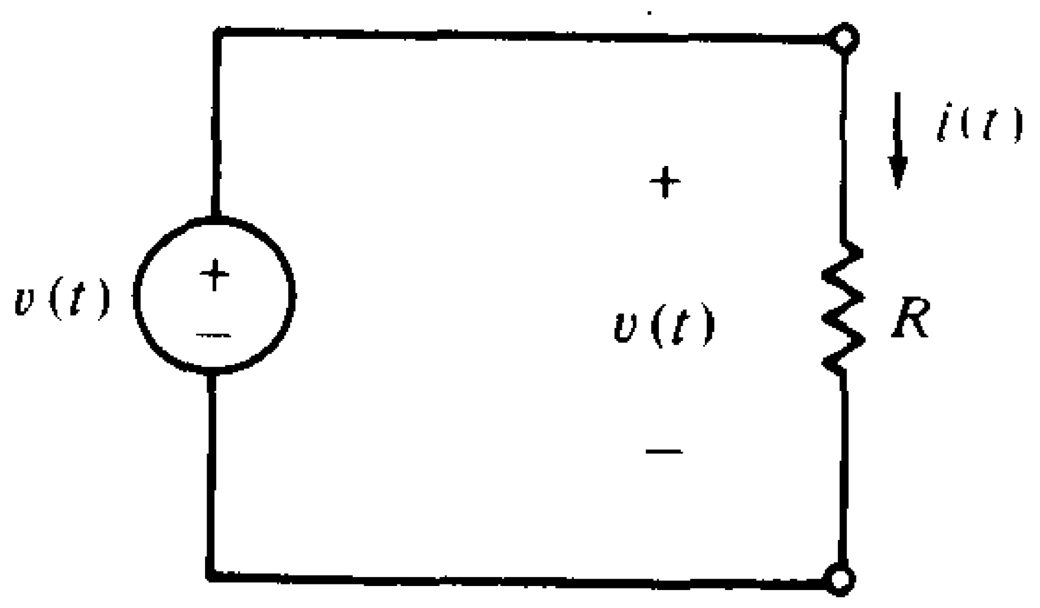

Fig. 1.20 Current through a voltage source.

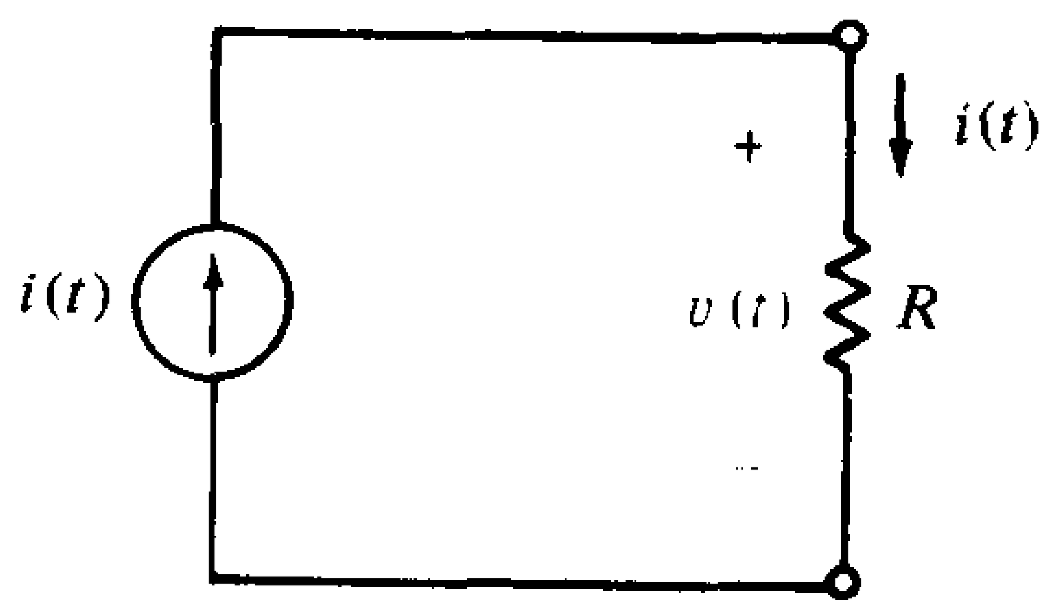

Fig. 1.21 Voltage across a current source.

If $v(t)$ is positive, then for the circuit shown in Fig. 1.21, the current comes out of the + side of the voltage source (assuming a positive resistance, or course). However, as we shall see later, when other types of elements are connected to a voltage source, the current through the voltage source can be in either direction—the direction depends on what exactly is connected to the source.

When a resistor R is connected to an ideal current source as in Fig. 1.21, we know that $v(t) = Ri(t)$. Therefore, for a given current source $i(t)$, the voltage $v(t)$ that results can be made to be any finite value by appropriately choosing R[e.g., to make $v(t)$ large, make R large]. Hence, we conclude that an ideal current source is capable of producing *any* amount of voltage across its terminals, and that amount depends on what is connected to the source—only the current is constrained to be $i(t)$ amperes through the source.

For the circuit in Fig. 1.21, if $i(t)$ is positive, then the polarity of the (positive-valued) voltage $v(t)$ is as indicated. In general, however, the polarity (as well as the magnitude) of the voltage across a current source depends on what exactly is connected to the source.

Physical (nonideal) sources do not have the ability to produce unlimited currents and voltages. As a matter of fact, an actual source may approximate an ideal source only for a limited range of values.

Now consider the two ideal voltage sources whose terminals are connected as shown in Fig. 1.22. By definition of an ideal 3-V source, $v(t)$ must be 3 V. However, by the definition of a 5-V ideal voltage source, $v(t) = 5$ V. Clearly, both conditions cannot be satisfied simultaneously. (Note that connecting a resistor, or anything else, for that matter, to the terminals will not alleviate this problem.) Therefore, to avoid this paradoxical situation, we will insist that two ideal voltage sources never have their terminals connected together as in Fig. 1.22; the only exception is two sources with the same value and the same polarity.

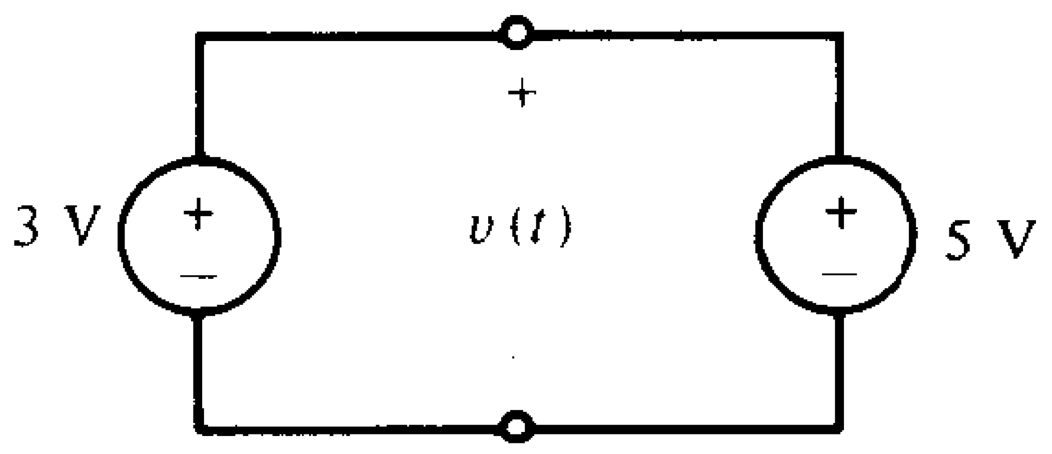

Fig. 1.22 Example of nonallowable connection of ideal voltage sources.

Now consider a resistor whose value is zero ohms. An equivalent representation, called a **short circuit**, of such a resistance is given in Fig. 1.23. By Ohm's law, we have that

$$v(t) = Ri(t) = 0i(t) = 0 \text{ V}$$

Thus, no matter what finite value $i(t)$ has, $v(t)$ will be zero. Hence, we see that *a zero-ohm resistor is equivalent to an ideal voltage source whose value is zero volts,* provided that the current through it is finite. Therefore, for a zero resistance to be synonymous with a constraint of zero volts (and to avoid the unpleasantness of infinite currents), we will insist that we never be allowed to place a short circuit directly

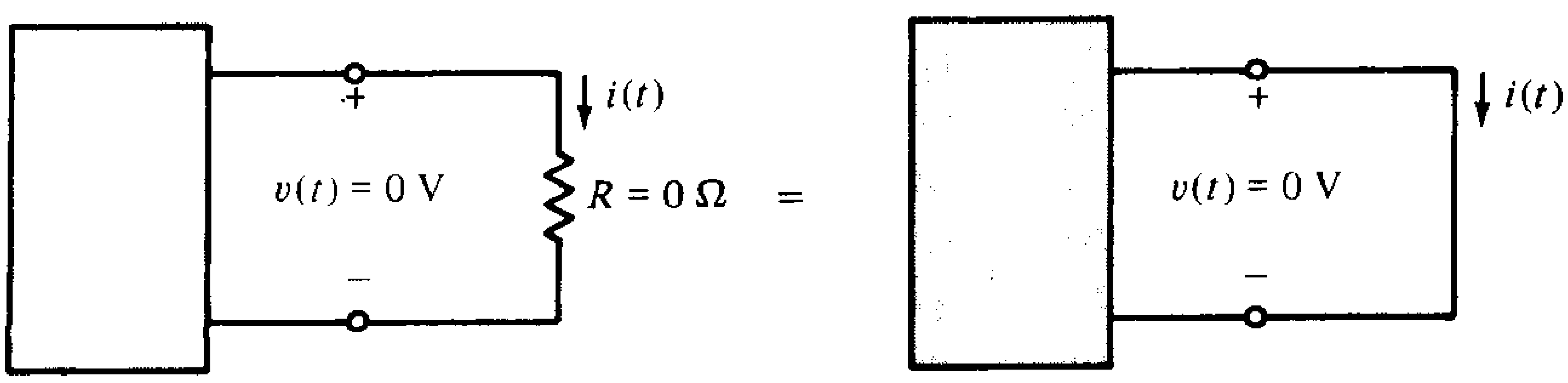

Fig. 1.23 Short-circuit equivalents.

across a voltage source. In actuality, the reader will be spared a lot of grief by never attempting this in a laboratory or field situation.

Next consider a resistor having infinite resistance. An equivalent representation, called an **open circuit**, of such a situation is depicted in Fig. 1.24. By Ohm's law,

$$i(t) = \frac{v(t)}{R} = 0 \text{ A}$$

as long as $v(t)$ has a finite value. Thus, we may conclude that *an infinite resistance is equivalent to an ideal current source whose value is zero amperes.* Furthermore, we will always assume that an ideal current source has something connected to its terminals.

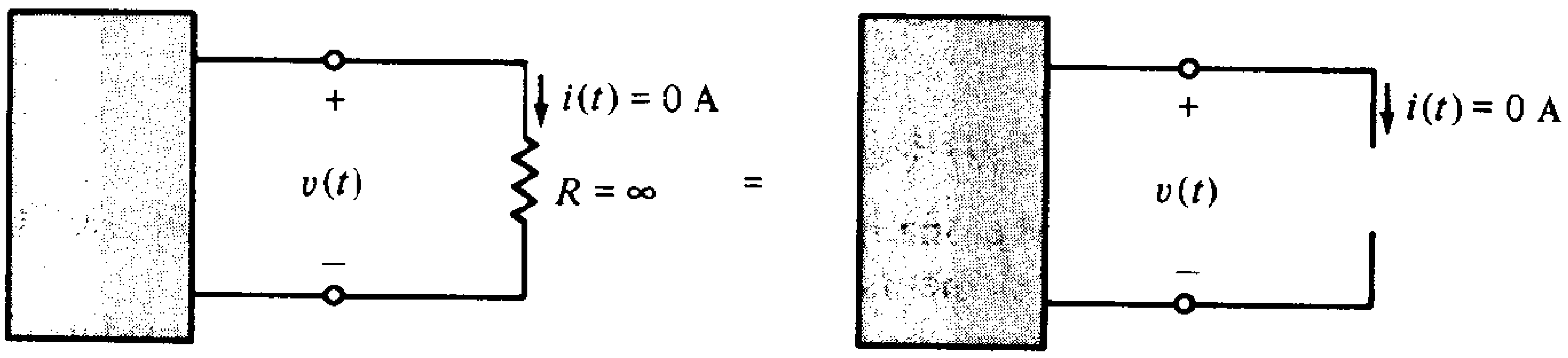

Fig. 1.24 Open-circuit equivalents.

1.3 KIRCHHOFF'S CURRENT LAW (KCL)

It is a consequence of the work of the German physicist Gustav Kirchhoff (1824–1887) that enables us to analyze an interconnection of any number of elements (voltage sources, current sources, and resistors, as well as elements not yet discussed). We will refer to any such interconnection as a **circuit** or a **network**.

For a given circuit, a connection of two or more elements[†] shall be called a **node**. An example of a node is depicted in the partial circuit shown in Fig. 1.25. In

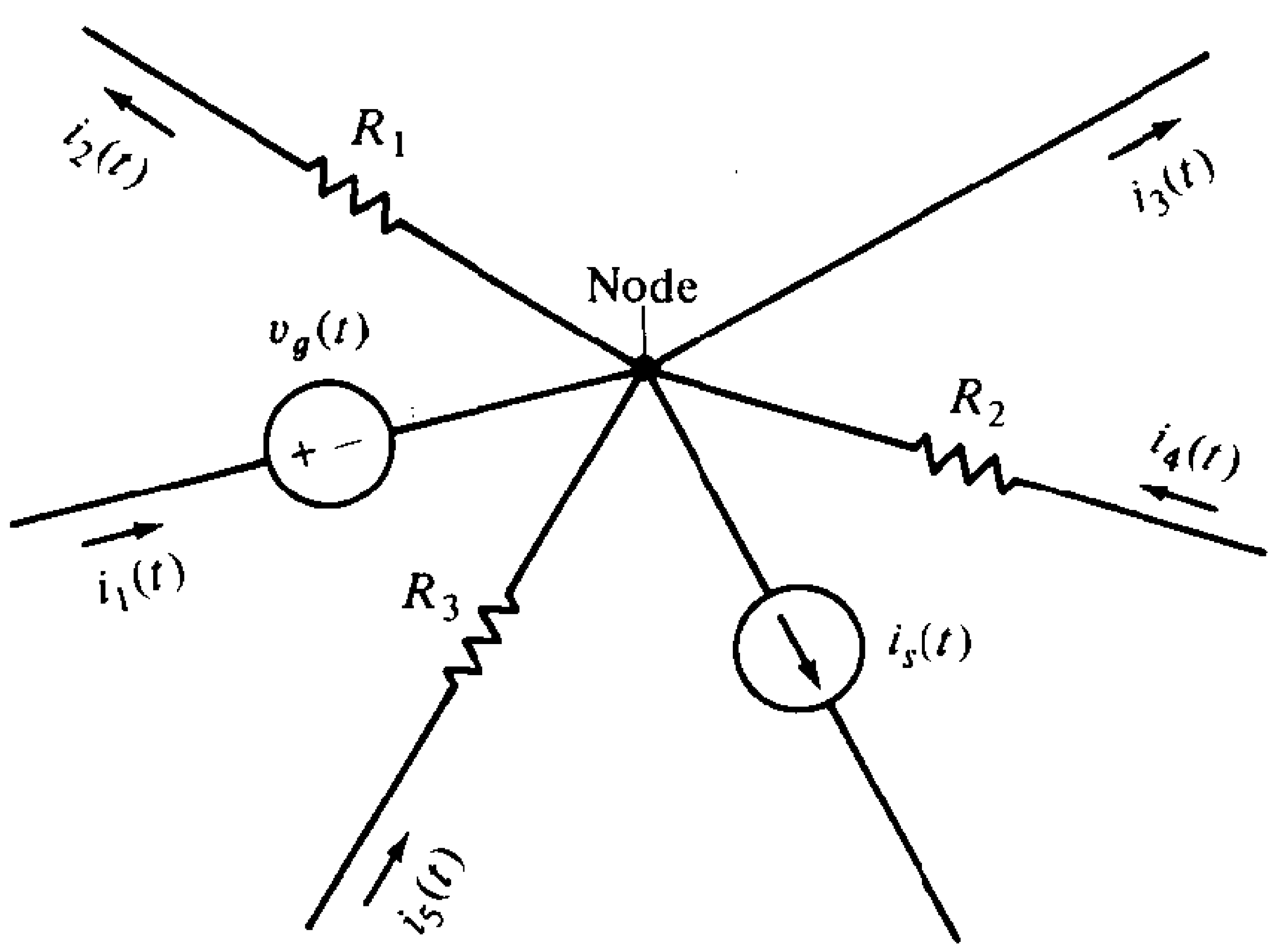

Fig. 1.25 Portion of a circuit.

[†] One element can be an open circuit.

addition to using a solid dot, we may also indicate a node by a hollow dot, as was done for a terminal. Conversely, we may use a solid dot for the terminal of a device.

We now present the first of Kirchhoff's two laws, his current law (KCL), which is essentially the law of conservation of electric charge.

KCL: At any node of a circuit, at every instant of time, the sum of the currents into the node is equal to the sum of the currents out of the node.

Specifically for the portion of the network shown in Fig. 1.25, by applying KCL we obtain the equation

$$i_1(t) + i_4(t) + i_5(t) = i_2(t) + i_3(t) + i_s(t)$$

Note that one of the elements [the one in which $i_3(t)$ flows] is a short circuit—KCL holds regardless of the kinds of elements in the circuit.

An alternative, but equivalent, form of KCL can be obtained by considering currents directed into a node to be positive in sense and currents directed out of a node to be negative in sense (or vice versa). Under this circumstance, the alternative form of KCL can be stated as follows:

KCL: At any node of a circuit, the currents algebraically sum to zero.

Applying this form of KCL to the node in Fig. 1.25 and considering currents directed in to be positive in sense, we get

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_s(t) + i_5(t) = 0$$

A close inspection of the last two equations, however, reveals that they are the same!

From this point on, we will simplify our notation somewhat by often abbreviating functions of time t such as $v(t)$ and $i(t)$ as v and i, respectively. For instance, we may rewrite the last two equations, respectively, as

$$i_1 + i_4 + i_5 = i_2 + i_3 + i_s$$

and

$$i_1 - i_2 - i_3 + i_4 - i_s + i_5 = 0$$

It should always be understood, however, that lowercase letters such as v and i, in general, represent time-varying quantities.†

† A constant is a special case of a function of time.

EXAMPLE 1.3

Let us find the voltage v in the two-node circuit given in Fig. 1.26 in which the directions of i_1, i_2, and i_3 and the polarity of v were chosen arbitrarily. (The directions of the 2-A and 13-A sources are given.)

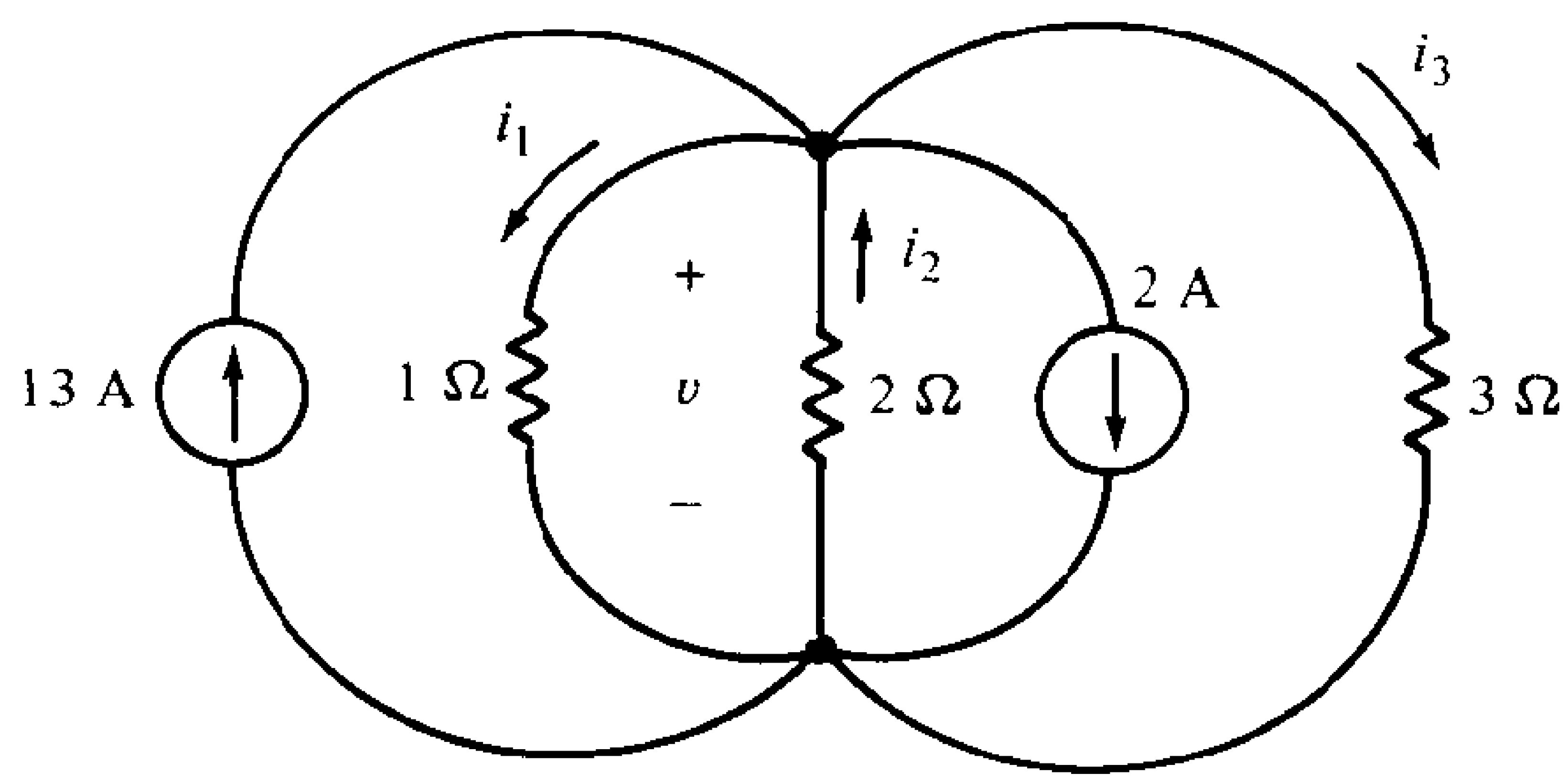

Fig. 1.26 Circuit for Example 1.3.

By KCL (at either of the two nodes), we have

$$13 - i_1 + i_2 - 2 - i_3 = 0 \qquad (1.1)$$

From this we can write

$$i_1 - i_2 + i_3 = 11$$

By Ohm's law,

$$i_1 = \frac{v}{1} \qquad i_2 = -\frac{v}{2} \qquad i_3 = \frac{v}{3}$$

Substituting these into the preceding equation yields

$$\frac{v}{1} - \left(-\frac{v}{2}\right) + \frac{v}{3} = 11$$

from which

$$v = 6 \text{ V}$$

Having solved for v, we now find that

$$i_1 = \frac{v}{1} = \frac{6}{1} = 6 \text{ A} \qquad i_2 = -\frac{v}{2} = -\frac{6}{2} = -3 \text{ A} \qquad i_3 = \frac{v}{3} = \frac{6}{3} = 2 \text{ A}$$

Note that a reordering of the circuit elements, as shown in Fig. 1.27, will result in the same equation (1.1) when KCL is applied. Since Ohm's law remains unchanged, the same answers are obtained.

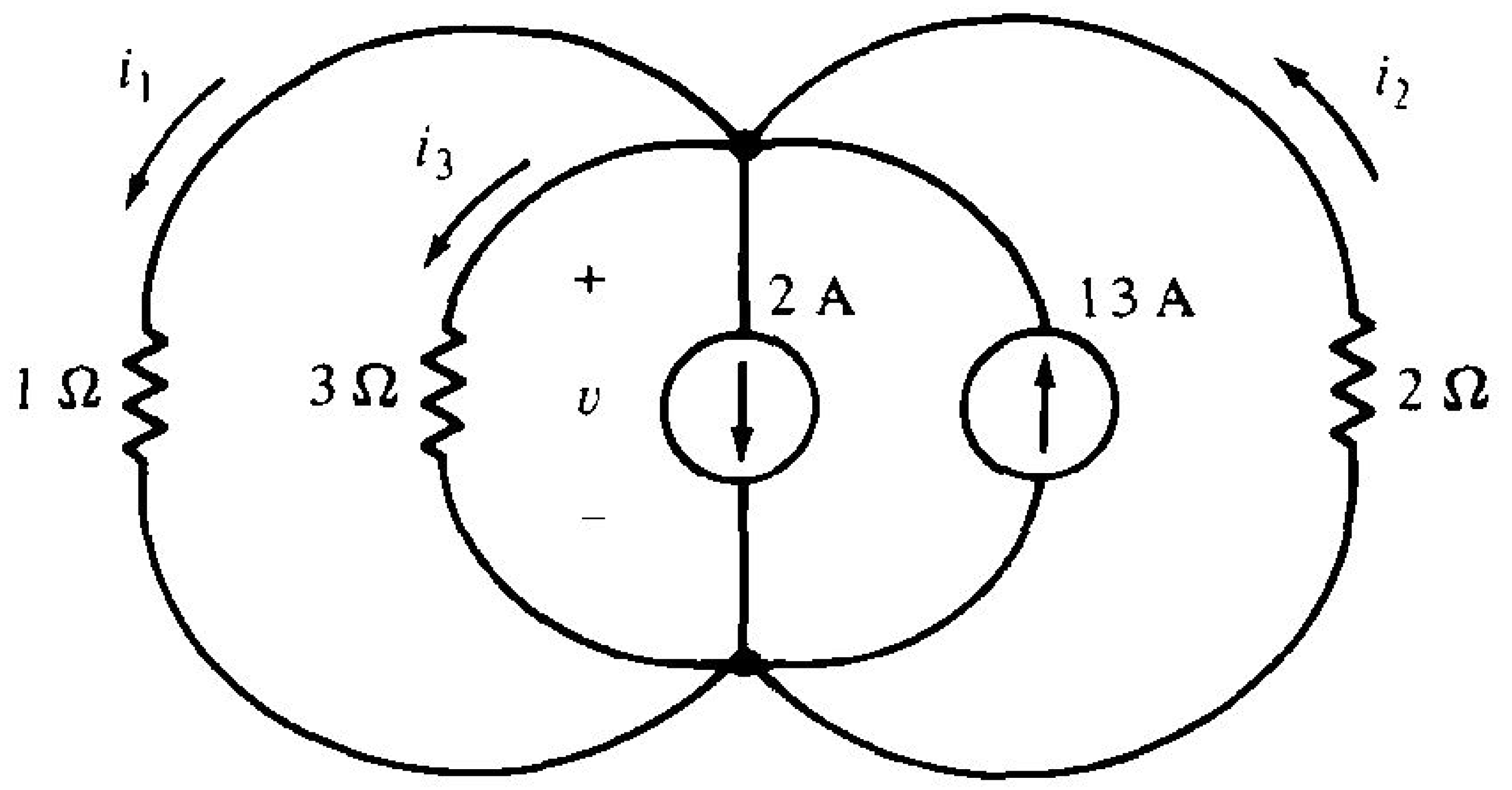

Fig. 1.27 Circuit equivalent to Fig. 1.26.

DRILL EXERCISE 1.4

For the circuit shown in Fig. DE1.4, suppose that $i_s = 10$ mA. Find (a) v, (b) i_1, (c) i_2, (d) i_3, and (e) i_4.
Answer: (a) 4.8 V; (b) −4.8 mA; (c) 2.4 mA; (d) −1.6 mA; (e) 1.2 mA

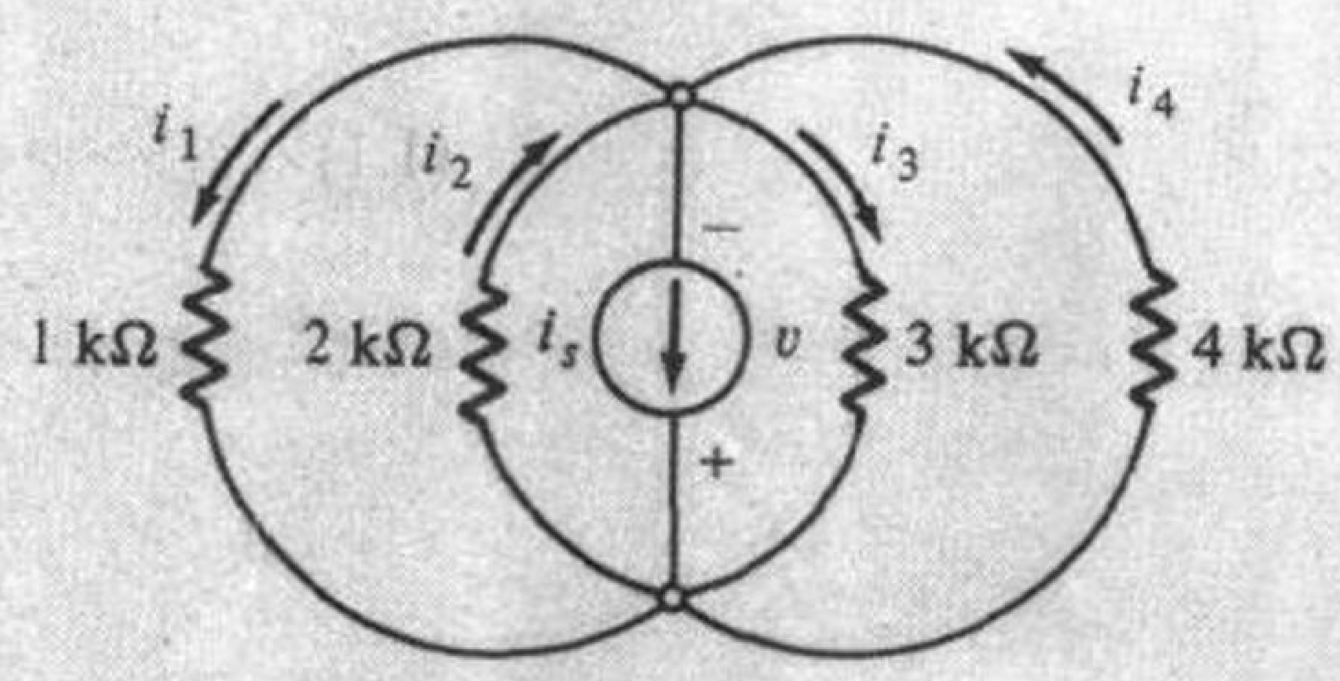

Fig. DE1.4

Just as KCL applies to any node of a circuit (i.e., to satisfy the physical law of conservation of charge, the current going in must equal the current coming out), so must KCL hold for any closed region.

For the circuit shown in Fig. 1.28, three regions have been arbitrarily identified. Applying KCL to Region 1, we get

$$i_1 + i_4 + i_5 = i_2$$

Applying KCL to Region 2, we obtain

$$i_5 + i_6 + i_7 = 0$$

and by applying KCL to Region 3, we get

$$i_2 + i_7 = i_4 + i_g$$

Note that Region 3 apparently contains two nodes. However, since they are connected by a short circuit, there is no difference in voltage between these two points.

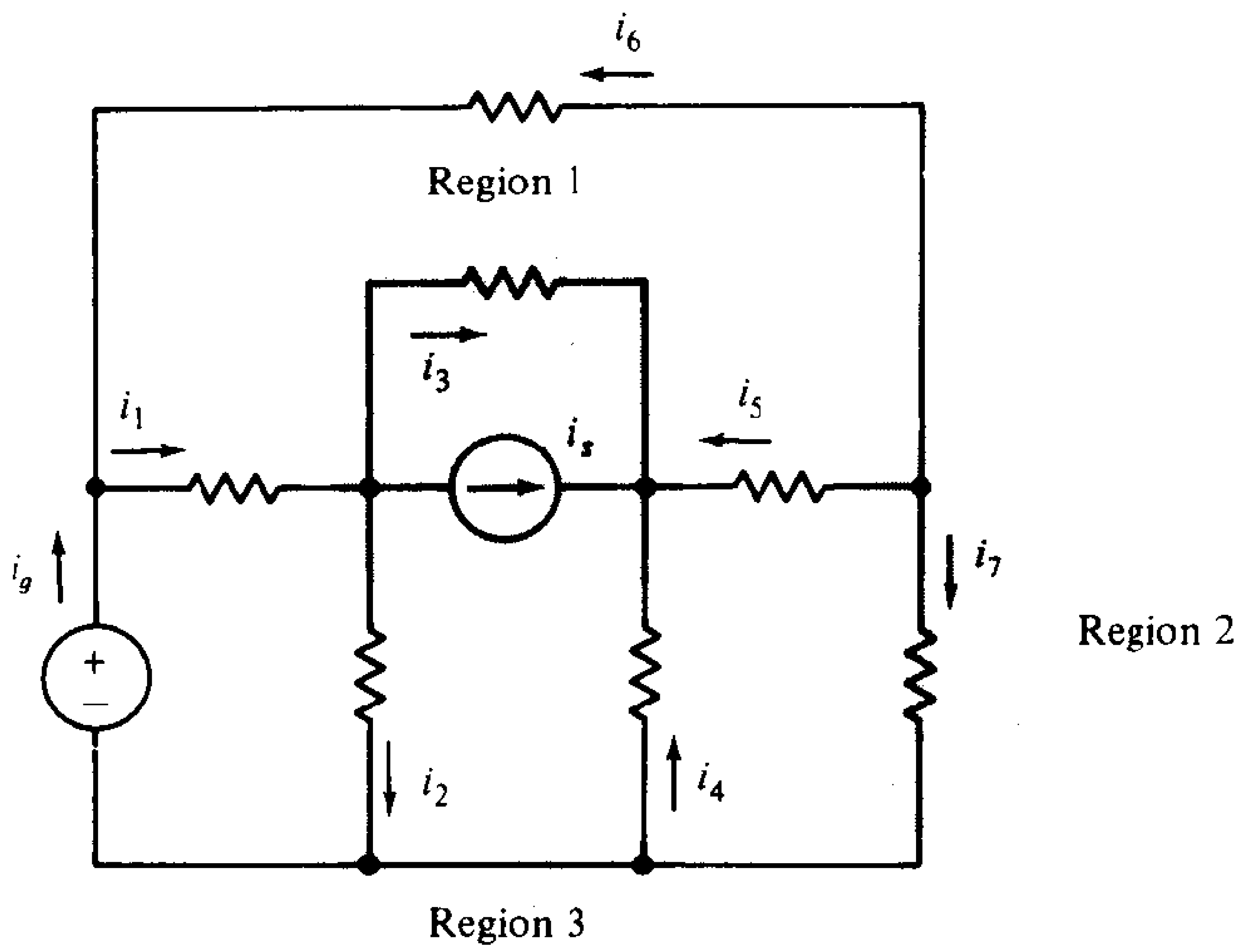

Fig. 1.28 Circuit with three arbitrarily selected regions.

Therefore, we can shrink the short circuit so as to coalesce the two points into a single node without affecting the operation of the circuit. Applying KCL at the resulting node again yields $i_2 + i_7 = i_4 + i_g$.

The converse process of expanding a node into apparently different nodes interconnected by short circuits also does not affect the operation of a circuit. For example, the portion of a circuit shown in Fig. 1.25 has the equivalent form shown in Fig. 1.29. Applying KCL to the region indicated results in the same equation as is obtained when KCL is applied to the node shown in Fig. 1.25. Thus, although it may appear that there are four distinct nodes in the region depicted in Fig. 1.29, they actually constitute a single node.

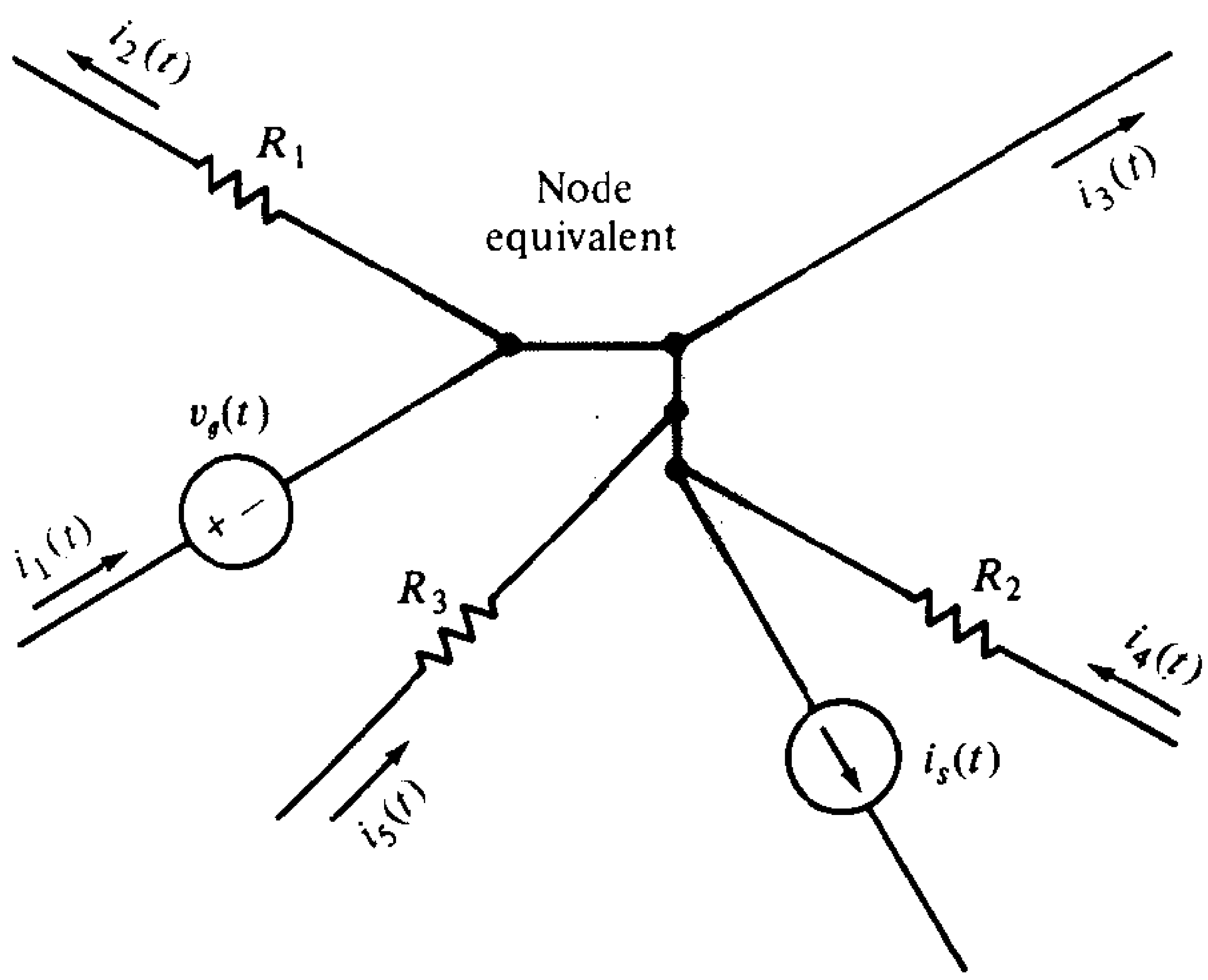

Fig. 1.29 Equivalent of circuit portion in Fig. 1.25.