高等学校试用

英语理工科教材选

第三分册　理论力学与材料力学

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机械工业部部属院校选编

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Book Ⅲ

Theoretical Mechanics
and
Mechanics of Materials

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(第三分册 理论力学与材料力学)

姚应时 张幼之 (主编) 苏英 李德立 金善琪 钱冬梅 选编

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编者的话

为了提高机械工业部部属院校学生的外语水平，培养学生阅读英语科技书刊的能力，我们选编了这套“英语理工科教材选”。整套“教材选”共分九个分册，内容包括数学、物理、理论力学、材料力学(与理论力学合为一个分册)、电工学、工业电子学、金属工艺学、机械原理、机械另件(与机械原理合为一个分册)、计算机算法语言、管理工程等十一门业务课程。各业务课都选了三章英语原版教材（个别也有选四章），供机械工业部部属院校试用。

在业务课中使用部分外语原版教材，这是我们的一次尝试，也是业务课教材改革、汲收国外先进科学技术的探索。在选材时，我们考虑了我国现行各课程的体系、内容以及学生的外语程度，尽可能选用适合我国实际的外国材料。

本“教材选”的选编工作，是在机械工业部教育局的直接领导下，由部属院校的有关教研室做了大量调查研究后选定的，并进行注释和词汇整理工作。由马泰来、卢思源、李国瑞、柯秉衡、谢卓杰、戴炜华、戴鸣钟等同志(以姓氏笔划为序)组成的审编小组，对选材的文字、注释、词汇作了审校。戴鸣钟教授担任整套“教材选”的总审。

由于对间仓促，选材、注释和编辑必有不尽完善之处，希广大读者提出宝贵意见，以利改进。

1983年4月

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§ 58. Introduction to Kinematics

Kinematics is the section of mechanics which treats of the geometry of the motion of bodies, without taking into account their inertia (mass) or the forces acting on them.

On the one hand, kinematics is an introduction to dynamics, insofar as the fundamental concepts and relationships of kinematics have to be understood before studying the motion of bodies taking
into account the action of forces. On the other hand, the methods of ①
kinematics are in themselves of practical importance, for example in studying the transmission of motion in mechanisms. That is why the demands of the developing machine-building industry led to the emergence of kinematics as a separate division of mechauics (in the first half of the 19th century).

By motion in mechanics is meant the relative displacement with
time of a body in space with respect to other bodies. ②

In order to locate a moving body (or particle) we assume a coordinate system, which we call the *frame of reference* or reference system, to be fixed relative to the body with respect to which the motion
is being considered. If the coordinates of all the points of a body re- ③
main constant within a given frame of reference, the body is said to be at *rest* relative to that reference system. If, on the other hand, the coordinates of any points of the body change with time, the body is said to be in motion relative to the given frame of reference (and consequently, relative to a body which is fixed with respect to the

frame of reference). When we speak of the motion of a body with respect to a given frame of reference, we shall mean its motion relative
④ to a body fixed with respect to that frame of reference.

Any motion in space takes place with time. In mechanics we deal with three-dimensional Euclidean space in which all dimensions are measured by the methods of Euclidean geometry. The unit of length, by which distance is measured, is the metre. Time in mechanics is considered as universal, i.e., as passing simultaneously in all our frames of reference. The unit of time is one second.*)

Euclidean space and universal time reflect only approximately the actual properties of space and time. Our daily experience shows, however, that for the motions considered in mechanics (at velocities much below the velocity of light) the approximation is sufficiently
⑤ accurate for all practical purposes.

Time is a continuously varying quantity. In problems of kinematics, time t is taken as an independent variable (the argument). All other variables (distance, speed, etc.) are regarded as changing with time, i.e., as functions of time t. Time is measured from some *initial instant* ($t = 0$) which is specified for every problem. Any given *instant of time* t is specified by the number of seconds that has passed between the initial and the given time. The difference between successive instants of time is called the *time interval*.

The principles of kinematics, evolved from and confirmed by prac-
⑥ tical experience, are based on the axioms of geometry. No other laws or axioms are necessary for the kinematic study of motion.

For the solution of problems of kinematics, the specific motion under consideration has to be described.

To describe the motion, or the law of motion, of a given body (particle) kinematically means to specify the position of that body (particle) relative to a given frame of reference for any moment of time. One of the main problems of kinematics is that of describing the motion of particles or bodies in terms of mathematical expressions. Hence, we shall commence the investigation of the motion of any object with determining the ways of describing that motion.

The *principal problem of kinematics* is that of determining all the kinematic characteristics of the motion of a body as a whole or of any of its particles (path, velocity, acceleration, etc.) when the law of motion for the given body is known. For the solution of this problem we must know either the equations of motion for the given body or for another body kinematically associated with it.

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*) The International System of Units (see § 101) defines the *metre* as the length equal to 1 650 763,73 wave lengths in vacuo of the radiation corresponding to the transition between the levels of $2p_{10}$ and $5d_5$ of the krypton-86 atom; the *second* is defined as 1/31 556 925.9747 of the tropical year for 1900 January 0 at 12 hous ephemeris time.

We shall start the study of kinematics with an investigation of the motion of the simplest body—a particle (kinematics of a particle), proceeding later to the examination of the kinematics of rigid bodies.

§ 59. Methods of Describing Motion of a Particle. Path

We shall begin the study of kinematics with examining the methods of describing motion. To describe the motion of a particle, it is necessary to specify its position in a chosen frame of reference at any given time. There are three methods of describing motion: (1) the natural method, (2) the coordinate method, (3) the vector method.

(1) **Natural Method of Describing Motion.** The continuous curve described by a particle moving with respect to a given frame of reference is called the *path* of that particle. If the path is a straight line, the motion is said to be *rectilinear*, if the path is a curve, the motion is *curvilinear*.

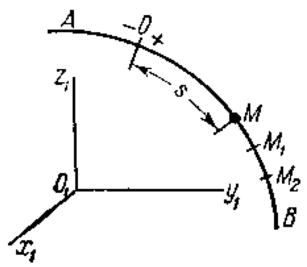

Fig. 136

The natural method of describing motion is convenient when the particle's path is known at once. Let the curve AB in Fig. 136 be the path of a particle M moving with respect to a frame of reference $O_1x_1y_1z_1$. Take any fixed point O on the path as the origin of another frame of reference; now, taking the path as an arc-coordinate axis, assume the positive and negative directions, as is done with rectangular axes. The position of the particle M on the path is now speci- ⑦
fied by a single coordinate s, equal to the distance from O to M measured along the arc of the path and taken with the appropriate sign. The displacement of particle M carries it through positions M_1, M_2, . . ., i.e., the distance s changes with time. In order to know the position of M on the path at any instant, we must know the relation

$$s = f(t). \tag{1}$$

Eq. (1) expresses the *law of motion of particle M* along its path.

Thus, in order to describe the motion of a particle by the natural method, a problem must state: (1) *the path of the particle*; (2) *the origin on the path*, showing the positive and negative *directions*; (3) *the equation of the particle's motion along the path in the form* $s = f(t)$.

For example, if a particle is moving from an origin O along a curve so that its distance from O increases in proportion to the square of the time, the equation of motion will be

$$s = at^2,$$

where a is the displacement of the particle in the first second. At time $t_2 = 2$ s the particle will be at distance $4a$ from O, etc. Consequently, knowing Eq. (1) we can determine the position of a moving particle at any instant.

Note that s in Eq. (1) denotes the moving particle's *position*, not the distance travelled by it. For example, if the particle in

Fig. 137

Fig. 136 travels from O to M_1 and then reverses its motion to point M, its coordinate at that moment is $s = \widehat{OM}$, but the distance it travelled is $\widehat{OM_1} + \widehat{M_1M}$, i.e., not s.

In the case of *rectilinear motion*, if axis Ox is directed along the particle's path (Fig. 137), we have $s = x$, yielding the *law of rectilinear motion* of a particle as

$$x = f(t). \tag{2}$$

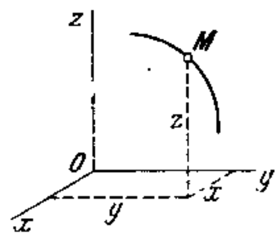

Fig. 138

(2) Coordinate Method of Describing Motion. The natural method of describing motion offers a very clear picture, but a particle's path may not be known, which is why the coordinate method is employed more frequently.

The position of a particle with respect to a given frame of reference $Oxyz$ can be specified by its Cartesian coordinates x, y, z (Fig. 138). When motion takes place, the three coordinates will change with time. If we want to know the equation of motion of a particle, i.e., its location in space at any instant, we must know its coordinates for any moment of time, i.e., the relations

$$x = f_1(t), \quad y = f_2(t), \quad z = f_3(t) \tag{3}$$

should be known.

Eqs. (3) are the *equations of motion of a particle in terms of the Cartesian rectangular coordinates*. They describe the curvilinear motion of a particle by the coordinate method *).

If a particle moves in one plane, then, taking the plane for the xy plane, we obtain two equations of motion:

$$x = f_1(t), \quad y = f_2(t) \tag{4}$$

*) The motion of a particle can be described in other coordinate systems: polar (see § 71), spherical, etc.

Finally, in the case of rectilinear motion, if axis x is directed along the path, the motion is described by the single equation (2) obtained before (in this case the coordinate and natural methods of describing motion coincide).

Eqs. (3) or (4) are, at the same time, the equations of the particle's *path* in parametric form, where the time t is the parameter. By eliminating time t from the equations of motion we can obtain the equation of the path in the usual form, i.e., in the form of a relation between the particle's coordinates.

Examples. (1) Let a particle's motion in a plane Oxy be given by the equations

$$x = 2t, \quad y = 12t^2. \tag{a}$$

From these equations at time $t = 0$ the particle is at M_0 (0, 0), i.e., at the origin of the coordinate system; at time $t_1 = 1$ s it is at M_1 (2, 12), etc. Thus, equations (a) actually define the particle's (9)
position at any instant. By assigning t different values and drawing a graph of the particle's displacement we can construct its path.

Another way of determining the path is by eliminating t from the equations (a). From the first equation $t = \frac{x}{2}$; substituting it for t in the second equation, we obtain $y = 3x^2$. Hence the path is a parabola with the apex at the origin of the coordinate axes and the axis parallel to axis Oy. (10)

(2) Consider the case when a particle's motion is described by the equations:

$$x = a \sin \pi t, \quad y = a \cos \pi t, \quad z = a \cos \pi t. \tag{b}$$

Squaring the first two equations and adding them, we obtain: $x^2 + y^2 = a^2$. Also, from the second and third equations $y = z$. Thus, the path lies on the line of intersection of a circular cylinder of radius a, whose axis coincides with axis Oz, with a plane $y = z$ bisecting the spatial angle between planes Oxy and Oxz, i.e., an ellipse with semiaxes a and $a\sqrt{2}$ lying in the plane $y = z$. (11)

For other examples of determining path see Problems 53, 54, 56 (§ 65).

(3) **Vector Method of Describing Motion.** Let a particle M be moving relative to any frame of reference $Oxyz$. The position of the particle at any instant can be specified by a vector $\boldsymbol{r}$ drawn from the origin O to the particle M (Fig. 139). Vector $\boldsymbol{r}$ is called the *radius vector* of the particle M.

When the particle moves, the vector $\boldsymbol{r}$ changes with time both in magnitude and direction. Thus, $\boldsymbol{r}$ is a variable vector (a vector function) depending on the argument t:

$$\boldsymbol{r} = \boldsymbol{r}\,(t). \tag{5}$$

Eq. (5) describes the curvilinear motion of a particle in vector form and can be used to construct a vector $\boldsymbol{r}$ for any particular mo-
12 ment of time and to determine the position of the moving particle at that moment.

The locus of the tips of vector $\boldsymbol{r}$ defines the path of the moving particle.

The vector method is convenient for establishing general dependencies, as it describes a particle's motion in terms of one vector
13 equation (5) instead of the three scalar equations (3).

The relationship between the coordinate and vector methods of describing motion can easily be established by introducing unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ directed along the x, y,
14 z axes, respectively (see Fig. 139). As the projections of vector $\boldsymbol{r}$ on the coordinate axes are equal to the coordinates of the particle M, i.e. $r_x = x$, $r_y = y$, $r_z = z$, we obtain

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}. \qquad (6)$$

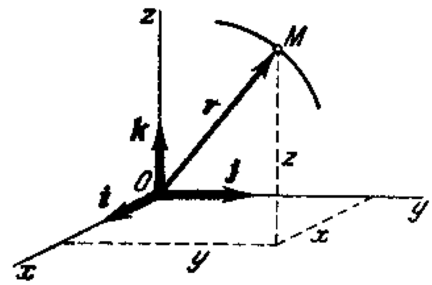

Fig. 139

Hence if, for example, the motion of a particle in plane Oxy is given in coordinate form by the equations $x = 2t$, $y = 12t^2$, the vector equation (5) of that motion will be

$$\boldsymbol{r} = 2t\boldsymbol{i} + 12t^2\boldsymbol{j}.$$

Using this equation we can construct vector $\boldsymbol{r}$ and determine the particle's position at any instant t. For example, at $t_1 = 1$ s, $\boldsymbol{r}_1 = 2\boldsymbol{i} + 12\boldsymbol{j}$ and can be constructed as the diagonal of the corresponding parallelogram, etc.

Conversely, if a particle's motion is described in vector form by, for example, the equation $\boldsymbol{r} = (1 - t)\,\boldsymbol{i} + 2t^2\boldsymbol{j} - 3t\boldsymbol{k}$, the equation of motion in coordinate form will be $x = (1 - t)$, $y = 2t^2$, $z = -3t$.

§ 60*. Conversion From Coordinate to Natural Method of Describing Motion

It has been shown that if motion is described by the equations (3) or (4), the path of the particle can be determined. It is, furthermore, known that $ds^2 = dx^2 + dy^2 + dz^2$, or $ds = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\,dt$, where $\dot{x} = \frac{dx}{dt}$, etc. Hence, assuming that at $t = 0$ the displacement $s = 0$, we obtain*)

$$s=\int_0^t \sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\,dt. \tag{7}$$

After integrating, Eq. (7) yields the equation of motion along the path in the form (1). If the motion is described by Eqs. (4), the equation (7) will lack the member with the derivative of z. ⑮

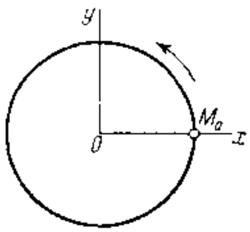

Fig. 140

Problem 52. The motion of a point in plane Oxy is described by the equations

$$x = a\cos\omega t,\quad y = a\sin\omega t, \tag{a}$$

where a and ω are constants. Determine the path and the equation of motion along it.

Solution. Squaring the equations (a) and adding them, we obtain: $x^2 + y^2 = a^2$.

Hence, the path is a circle of radius a with the centre at the origin of the coordinate system (Fig. 140). Computing the derivatives of x and y with respect to t, we obtain

$$\dot{x} = -a\omega\sin\omega t,\quad \dot{y} = a\omega\cos\omega t.$$

Substituting into Eq. (7), we have

$$s=\int_0^t a\omega\,dt \quad\text{or}\quad s=a\omega t. \tag{b}$$

Equation (b) describes the particle's motion along the path in the form (1). From equations (a), when $t = 0$, we have $x = a$, and $y = 0$, i.e., the particle is at M_0; as t increases x decreases and y increases, assuming positive values. Consequently, the counting off of s starts at point M_0 and the displacement along the circle is in the direction indicated by the arrow in Fig. 140. It will be observed from equation (b) that as the particle moves the displacement s increases in proportion to the time, the increment being $a\omega$ each second, such motion is called uniform.

In this case the conversion from the natural method made for a more graphic visualisation of the motion than could be presented by equations (a) directly. ⑯

*) By taking the square root with the plus sign we thereby determine the positive direction of the displacement s (in the direction the point starts moving at time $t = 0$).

§ 61. Velocity Vector of a Particle

One of the basic kinematic characteristics of motion of a particle is a vector quantity called velocity. First let us introduce the concept of average velocity of a particle in a given time interval. Let a moving particle occupy at time t a position M defined by the radius vector $\boldsymbol{r}$, and at time t_1 a position M_1 defined by the radius vector $\boldsymbol{r}_1$ (Fig. 141). The displacement during the time interval $\Delta t = t_1 - t$ is defined by a vector $\overline{MM_1}$ which we shall call the *displacement vector of the particle*. The vector is directed along a chord if the particle is

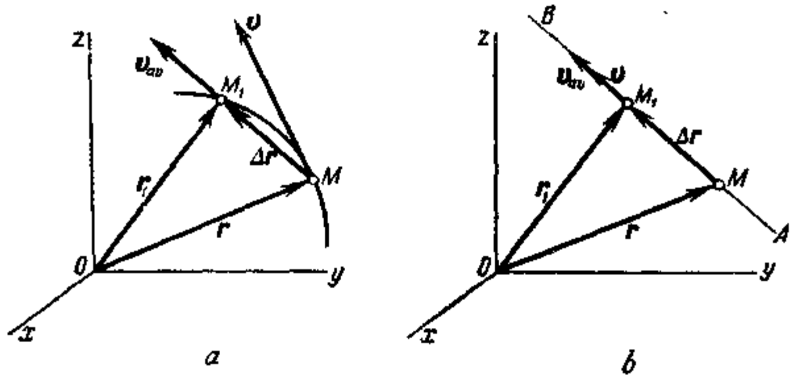

Fig. 141

in curvilinear motion (Fig. 141*a*), and along the path AB in rectilinear motion (Fig. 141*b*).

From triangle OMM_1 we have $\boldsymbol{r} + \overline{MM_1} = \boldsymbol{r}_1$, whence

$$\overline{MM_1} = \boldsymbol{r}_1 - \boldsymbol{r} = \Delta \boldsymbol{r}.$$

The ratio of the displacement vector of a particle to the corresponding time interval defines a vector quantity called the *average* (both magnitude and direction) *velocity* of the particle during the given time interval Δt: ⑰

$$\boldsymbol{v}_{av} = \frac{\overline{MM_1}}{\Delta t} = \frac{\Delta \boldsymbol{r}}{\Delta t}. \qquad (8)$$

The magnitude of the average velocity given by Eq. (8) is

$$v_{av} = \frac{MM_1}{\Delta t}. \qquad (8')$$

Vector $\boldsymbol{v}_{av}$ has the same direction as vector $\overline{MM_1}$, i.e., along the chord MM_1 in the direction of the motion of the particle in the case of curvilinear motion, and along the path itself in the case of rectilinear motion (the direction of the vector is not altered by being divided by Δt).

Obviously, the smaller the time interval $\Delta t = t_1 - t$ for which the average velocity has been calculated, the more precisely will (18) $\boldsymbol{v}_{av}$ characterise the particle's motion. To obtain a characteristic of motion independent of the choice of the time interval Δt, the concept of *instantaneous velocity of a particle* is introduced.

The instantaneous velocity of a particle at any time t is defined as the vector quantity $\boldsymbol{v}$ towards which the average velocity $\boldsymbol{v}_{av}$ tends when the time interval Δt tends to zero:

$$\boldsymbol{v} = \lim_{\Delta t \to 0} (\boldsymbol{v}_{av}) = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t}.$$

The limit of the ratio $\Delta \boldsymbol{r}/\Delta t$ as $\Delta t \to 0$ is the first derivative of the vector $\boldsymbol{r}$ with respect to t and is denoted, like the derivative of a scalar function, by the symbol $d\boldsymbol{r}/dt$*).

Finally we obtain:

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}. \qquad (9)$$

Thus, *the vector of instantaneous velocity of a particle is equal to the first derivative of the radius vector of the particle with respect to time.*

As the limiting direction of the secant MM_1 is a tangent, the vector of instantaneous velocity is tangent to the path of the particle in the direction of motion.

Eq. (9) also shows that the velocity vector $\boldsymbol{v}$ is equal to the ratio of the infinitesimal displacement $d\boldsymbol{r}$ of the particle tangent to its path to the corresponding time interval dt.

In rectilinear motion the velocity vector $\boldsymbol{v}$ is always directed along the straight line in which the particle is moving and can change only in magnitude; in curvilinear motion the direction of the velocity vector changes continuously. The dimension of velocity is displacement/time, and the customary units are m/s or km/h.

§ 62. Acceleration Vector of a Particle

Acceleration characterises the time rate of change of velocity in magnitude and direction.

Let a moving particle occupy a position M and have a velocity $\boldsymbol{v}$ at a given time t, and let it at time t_1 occupy a position M_1 and have a velocity $\boldsymbol{v}_1$ (Fig. 142). The increase in velocity in the time interval $\Delta t = t_1 - t$ is $\Delta \boldsymbol{v} = \boldsymbol{v}_1 - \boldsymbol{v}$. To construct vector $\Delta \boldsymbol{v}$, lay off vector

*) In general, for any variable vector $\boldsymbol{u}$ depending on an argument t

$$\lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{u}}{\Delta t} = \frac{d\boldsymbol{u}}{dt}.$$