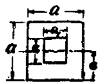
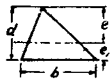
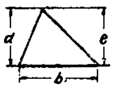
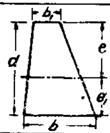
第4編　應用力學

4—1 各種斷面之力學性質表

斷面	斷面積 F	中軸至外緣距離 e, e_1	慣性矩 J	斷面係数 $W=\frac{J}{e}$	廻轉半徑 $r=\sqrt{\frac{J}{F}}$
	$a^2-a_1^2$	$e=\frac{a}{2}$	$\frac{a^4-a_1^4}{12}$	$\frac{a^4-a_1^4}{6a}$	$\sqrt{\frac{a^2+a_1^2}{12}}$
	$\frac{bd}{2}$	$e=\frac{2d}{3},\ e_1=\frac{d}{3}$	$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d}{\sqrt{18}}=0.236d$
	$\frac{bd}{2}$	$e=d$	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{d}{\sqrt{6}}=0.408d$
	$\frac{(b+b_1)}{2}d$	$e=\frac{2b+b_1}{b+b_1}\cdot\frac{d}{3}$ $e_1=\frac{b+2b_1}{b+b_1}\cdot\frac{d}{3}$	$\frac{b^2+4bb_1+b_1^2}{36(b+b_1)}d^3$	$\frac{b+4bb_1+b_1^2}{12(2b+b_1)}d^2$	$\frac{d\sqrt{2(b^2+4bb_1+b_1^2)}}{6(b+b_1)}$
	$\frac{\pi d^2}{4}=0.785d^2$	$e=\frac{d}{2}$	$\frac{\pi d^4}{64}=0.049d^4$	$\frac{\pi d^3}{32}=0.098d^3$	$\frac{d}{4}$

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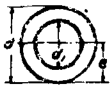
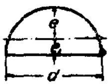
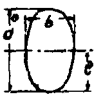
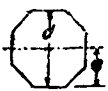
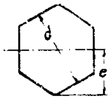
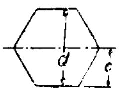
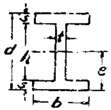
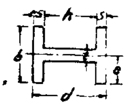
斷　面	斷面積 F	中軸至外緣距離 $e,\ e_1$	慣性矩 J	斷面係数 $W=\frac{J}{e}$	廻轉半徑 $r=\sqrt{\frac{J}{F}}$
	$\frac{\pi(d^2-d_1^2)}{4}=0.785(d^2-d_1^2)$	$e=\frac{d}{2}$	$\frac{\pi(d^4-d_1^4)}{64}=0.049(d^4-d_1^4)$	$\frac{\pi(d^4-d_1^4)}{32d}=0.098(d^4-d_1^4)\div d$	$\frac{\sqrt{d^2+d_1^2}}{4}$
	$\frac{\pi d^2}{8}=0.392d^2$	$e=\frac{(3\pi-4)d}{6\pi}=0.288d$ $e_1=\frac{2d}{3\pi}=0.212d$	$\frac{9\pi^2-64}{1152\pi}d^4=0.007d^4$	$\frac{9\pi^2-64}{192(3\pi-4)}d^3=0.024d^3$	$\frac{\sqrt{9\pi^2-64}}{12\pi}d=0.132d$
	$\frac{\pi bd}{4}=0.785bd$	$e=\frac{d}{2}$	$\frac{\pi bd^3}{64}=0.049bd^3$	$\frac{\pi bd^2}{32}=0.098bd^2$	$\frac{d}{4}$
	$2d^2\tan 22\frac{1}{2}°=0.828d^2$	$e=\frac{d}{2}$	$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 22\frac{1}{2}°)}{4\cos^2 22\frac{1}{2}°}\right]=0.055d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 22\frac{1}{2}°)}{4\cos^2 22\frac{1}{2}°}\right]=0.109d^3$	$\frac{d}{4}\sqrt{\frac{1+2\cos^2 22\frac{1}{2}°}{3\cos^2 22\frac{1}{2}°}}=0.257d$

4—1 各種斷面之力學性質表

斷面	斷面積 F	中軸至外緣距離 e, e_1	慣性矩 J	斷面係數 $W=\frac{J}{e}$	迴轉半徑 $r=\sqrt{\frac{J}{F}}$
	$\frac{3}{2}d^2\tan 30^\circ$ $=0.866d$	$e=\frac{d}{2\cos 30^\circ}$ $=0.577d$	$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $=0.060d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $=0.104d^3$	$\frac{d}{4}\sqrt{\frac{1+2\cos^2 30^\circ}{3\cos^2 30^\circ}}$ $=0.264d$
	$\frac{3}{2}d^2\tan 30^\circ$ $=0.866d^2$	$e=\frac{d}{2}$	$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $=0.060d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $=0.120d^3$	$\frac{d}{4}\sqrt{\frac{1+2\cos^2 30^\circ}{3\cos^2 30^\circ}}$ $=0.264d$
	$bd-h(b-t)$	$e=\frac{d}{2}$	$\frac{bd^3-h^3(b-t)}{12}$	$\frac{bd^3-h^3(b-t)}{6d}$	$\sqrt{\frac{bd^3-h^3(b-t)}{12[bd-h(b-t)]}}$
	$bd-h(b-t)$	$e=\frac{b}{2}$	$\frac{2sb^3+ht^3}{12}$	$\frac{2sb^3+ht^3}{6b}$	$\sqrt{\frac{2sb^3+ht^3}{12[bd-h(b-t)]}}$

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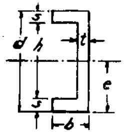
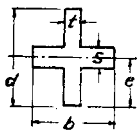
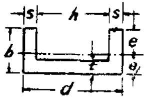
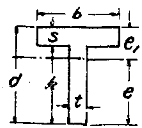
断面	断面積 F	中軸至外縁距離 $e,\ e_1$	慣性矩 J	断面係数 $W=\frac{J}{e}$	廻轉半徑 $r=\sqrt{\frac{J}{F}}$
	$bd-h(b-t)$	$e=\frac{d}{2}$	$\frac{bd^3-h^3(b-t)}{12}$	$\frac{bd^3-h^3(b-t)}{6d}$	$\sqrt{\frac{bd^3-h^3(b-t)}{12[bd-h(b-t)]}}$
	$td+s(b-t)$	$e=\frac{d}{2}$	$\frac{td^3+s^3(b-t)}{12}$	$\frac{td^3+s^3(b-t)}{6d}$	$\sqrt{\frac{td^3-s^3(b-t)}{12[td+s(b-t)]}}$
	$bd-h(b-t)$	$e=\frac{\frac{1}{2}b^2d+h(b-t)^2}{bd-h(b-t)}$ $e_1=b-e$	$\frac{2b^3s+ht^3}{3}-Ae_1^2$	$\frac{J}{b-e_1}$	$\sqrt{\frac{J}{F}}$
	$bs+ht$	$e_1=\frac{d^2t+s^2(b-t)}{2A}$ $e=d-e_1$	$\frac{te^3+be_1^3}{3}-\frac{(b-t)(e_1-s)^3}{3}$	$\frac{J}{d-e_1}$	$\sqrt{\frac{J}{F}}$

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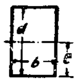
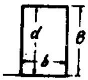
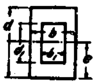
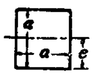
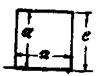
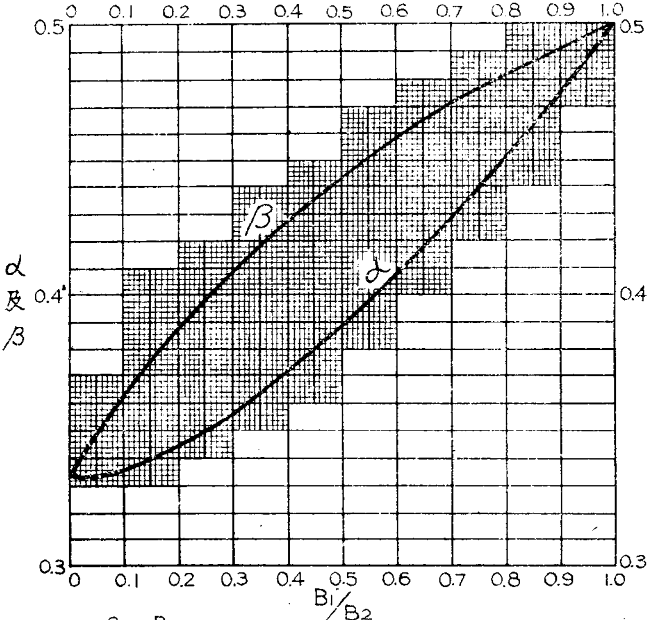
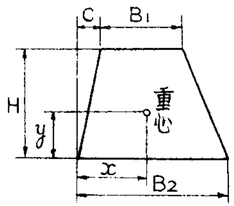
斷　面	斷面積 F	中軸至外緣距離 $e,\ e_1$	慣性矩 J	斷面係数 $W=\frac{J}{e}$	廻轉半徑 $r=\sqrt{\frac{J}{F}}$
	bd	$e=\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{d}{\sqrt{12}}=0.289d$
	bd	$e=d$	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	$\frac{d}{\sqrt{3}}=0.577d$
	$bd-b_1d_1$	$e=\frac{d}{2}$	$\frac{bd^3-b_1d_1^3}{12}$	$\frac{bd^3-b_1d_1^3}{6d}$	$\sqrt{\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}}$
	a^2	$e=\frac{a}{2}$	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}=0.289a$
	a^2	$e=a$	$\frac{a^4}{3}$	$\frac{a^3}{3}$	$\frac{a}{\sqrt{3}}=0.577a$

4—2 求梯形重心位置圖表





按下式求梯形重心位置：

$$x = \alpha \cdot B_2 + \beta \cdot C$$

$$y = \beta \cdot H$$

係数 α, β 從上面圖表求得.

一般式：

$$x = \frac{(B_1^2 + B_1 B_2 + B_2^2) + (2B_1 + B_2)C}{3(B_1 + B_2)}$$

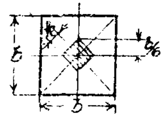
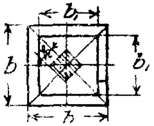
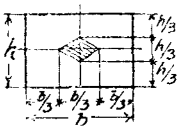
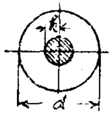
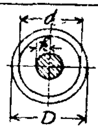
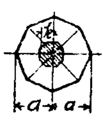
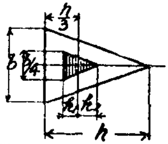
$$y = \frac{H}{3} \cdot \frac{2B_1 + B_2}{B_1 + B_2}$$

以上式中之 $B_1/B_2 = \gamma$ 則：

$$x = \frac{(\gamma^2 + \gamma + 1)}{3(\gamma + 1)} B_2 + \frac{(2\gamma + 1)}{3(\gamma + 1)} C$$

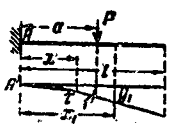
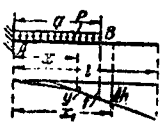
$$y = \frac{(2\gamma + 1)}{3(\gamma + 1)} \cdot H$$

4—3 核心斷面式樣及核心半徑表

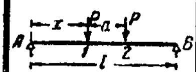
方形		$k=\frac{b}{6\sqrt{2}}=0.1179b$ 核心斜長$=\frac{b}{3}$
中空方形		$k=\frac{b}{6\sqrt{2}}\left[1+\left(\frac{b_1}{b}\right)^2\right]$ $=0.1179b\left[1+\left(\frac{b_1}{b}\right)^2\right]$
長方形		$k=\frac{b\cdot h}{6\sqrt{b^2+h^2}}$ 核心斜長$=\frac{b}{3}$ 或 $\frac{h}{3}$
圓形		$k=\frac{d}{8}$
中空圓形		$k=\frac{D}{d}\left[1+\left(\frac{d}{D}\right)^2\right]$
八角形		$k=0.2256a$
三角形		$k_1=\frac{h}{6}$ $k_2=\frac{h}{12}$

註： k=核心半徑(*Radius of the kern*)

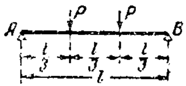
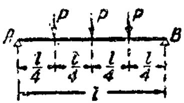
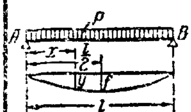
4—4 梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	剪力及支點反力	彎曲力矩	撓度(變位)	傾斜角度
1		$A=P$ $Q_x=P$	$M_x=-P(a-x)$ $\max M=-p\cdot a$	$0<x<a: y=\frac{Px^2}{2EJ}\left(a-\frac{x}{3}\right)$ $a<x_1<l: y=\frac{Pa^2}{2EJ}\left(x_1-\frac{a}{3}\right)$ $x=a: \quad f=\frac{Pa^3}{3EJ}$	$\tau=\frac{Pa^2}{2EJ}$
2		$A=p\cdot a$ $Q_x=p(a-x)$ $\max Q=p\cdot a$	$M_x=-\frac{p}{2}(a-x)^2$ $\max M=-\frac{pa^2}{2}$	$0<x<a: y=\frac{px^2a^2}{2EJ}\left[\frac{1}{2}-\right.$ $\left.-\frac{1}{3}\left(\frac{x}{a}\right)+\frac{1}{12}\left(\frac{x}{a}\right)^2\right]$ $a<x_1<l: y_1=\frac{pa^3}{6EJ}\left(x_1-\frac{a}{4}\right)$ $x=a: \quad f=\frac{pa^4}{8EJ}$	$\tau=\frac{pa^3}{6EJ}$

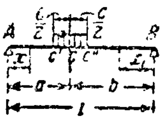
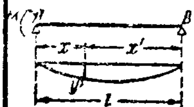
4—4 梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度（變位）	傾斜角度
3		$A-C:\ A = Q_x = \frac{Pb}{l}$ $C-B:\ B = Q_{x_1} = \frac{Pa}{l}$	$A-C:\ M_x = \frac{Pb}{l}x$ $C-B:\ M_{x_1} = \frac{Pa}{l}x_1$ C 點：$\max M = \frac{Pab}{l}$	$A-C:\ y = \frac{Pbx}{6EJl}(l^2-b^2-x^2)$ $C-B:\ y_1 = \frac{Pax_1}{6EJl}(l^2-a^2-x^2)$ $B-C:\ f = \frac{Pa^2b^2}{3EJl}$；倘 $a=b:\ f = \frac{Pl^3}{48EJ}$	$\tau_A = \frac{Pab}{6EJ}\times\left(1+\frac{b}{l}\right)$ $\tau_B = \frac{Pab}{6EJ}\times\left(1+\frac{a}{l}\right)$
4		$A = \frac{2P}{l}\left(l-x-\frac{a}{2}\right)$ $B = \frac{2P}{l}\left(x+\frac{a}{2}\right)$	$M_1 = \frac{2P}{l}\left(l-x-\frac{a}{2}\right)x$ $M_2 = \frac{2P}{l}(l-x-a)\times\left(x+\frac{a}{2}\right)$ 在 $x = \frac{l}{2}-\frac{a}{4}$；$M_1 = \max M = \frac{Pl}{2}\left(1-\frac{a}{2l}\right)^2$；倘 $\frac{a}{l} \geq 0{,}586$，$\max M = \frac{1}{4}Pl$ 則僅按一個力求之		

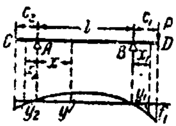
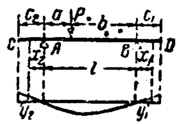
4—4 梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度(變位)	傾斜角度
5		$A=B=P$	$\max M=\frac{Pl}{3}$	中部之撓度 $f=\frac{23}{648}\frac{Pl^3}{EJ}$	
6		$A=B=\frac{3}{2}P$	$\max M=\frac{Pl}{2}$	中部之撓度 $f=\frac{19}{384}\frac{Pl^3}{EJ}$	
7		$A=B=\frac{1}{2}pl$ $Q_x=p\left(\frac{l}{2}-x\right)$	$M_x=\frac{1}{2}px(l-x)$ 當 $x=\frac{l}{2}$ 時 $\max M=\frac{1}{8}pl^2$	$y=\frac{pl^4}{24EJ}\left[\frac{x}{l}-2\left(\frac{x}{l}\right)^3+\left(\frac{x}{l}\right)^4\right]$ 當 $x=\frac{l}{2}$ 時 $f=\frac{5pl^4}{384EJ}$	$\tau=\frac{pl^3}{24EJ}$

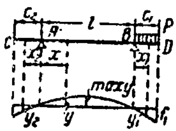
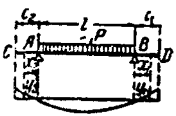
4—4　梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度(變位)	傾斜角度
8		$A=p\dfrac{bc}{l}$ $B=p\dfrac{ac}{l}$	$A-C'$:　$M_x=p\dfrac{bc}{l}x$ $C''-B$:　$M_{x_1}=p\dfrac{ac}{l}x_1$ 在 C 點：$M_c=pc\left(\dfrac{ab}{l}-\dfrac{c}{8}\right)$ 當 $x=a-\dfrac{c}{2}+\dfrac{cb}{l}$： $\max M=p\dfrac{abc}{l^2}\left(l-\dfrac{c}{2}\right)$	C 點之撓度 $f=\dfrac{pc}{6EJ}\left[\dfrac{ab}{l}\left(2al-2a^2-\dfrac{c^2}{4}\right)+\dfrac{c^3}{64}\right]$	$\tau_A=\dfrac{pb}{6EJ}\times\dfrac{c}{l}\left(l^2-b^2-\dfrac{c^2}{4}\right)$, $\tau_B=\dfrac{pa}{6EJ}\times\dfrac{c}{l}\left(l^2-a^2-\dfrac{c^2}{4}\right)$
9		$A=-\dfrac{M}{l}$ $B=+\dfrac{M}{l}$	$M_x=M\dfrac{x_1}{l}$	$y=\dfrac{Mxx'}{6EJ}\left(1+\dfrac{x'}{l}\right)$	$\tau=\dfrac{Ml}{6EJ}\left(1-\dfrac{3x'^2}{l^2}\right)$

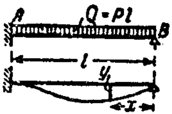
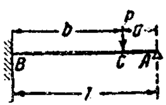
4—4 梁之支點反力、剪力,彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度(變位)	傾斜角度
10		$A=-P\frac{c_1}{l}$ $B=P\frac{l+c_1}{l}$ $A-B$: $Q_x=-P\frac{c_1}{l}$ $B-D$: $Q_{x_1}=P$	$A-B$: $M_x=-P\frac{c_1 x}{l}$ $B-D$: $M_{x_1}=-P(c_1-x_1)$	$A-B$: $y=\frac{Pc_1 l^2}{6EJ}\left(\frac{x}{l}\right)\left[1-\left(\frac{x}{l}\right)^2\right]$ $x=0{,}577\,l$. $\max y=-\frac{Pc_1 l^2}{9\sqrt{3}EJ}$ $B-D$: $y_1=\frac{Pc_1^2 x_1}{6EJ}\left[2\frac{l}{c_1}+3\frac{x_1}{c_1}\left(\frac{x_1}{c_1}\right)^2\right]$ $B-D$: $f_1=\frac{Pc_1^2}{3EJ}(l+c_1)$ $A-C$: $y_2=\frac{Pc_1 l}{6EJ}x_2$	$\tau_1=\frac{Pc_1^2}{6EJ}\times$ $\times\left(3+2\frac{l}{c_1}\right)$ $\tau_2=-\frac{Pc_1 l}{6EJ}$
11		與第3號簡支梁同		$A-B$: 與簡支梁同 $B-D$: $y_1=-\frac{Pabx_1}{6EJl}(2a+b)$ $A-C$: $y_2=-\frac{Pabx_2}{6EJl}(a+2b)$	

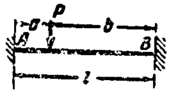
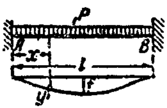
4—4 梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度(變位)	傾斜角度
12		$A=-\frac{p}{2}\frac{c_1^2}{l}$ $B=pc_1\left(1+\frac{c_1}{2l}\right)$ $A-B$: $Q_x=-\frac{p}{2}\frac{c_1^2}{l}$ $B-D$: $Q_{x1}=p(c_1-x_1)$	$A-B$: $M_x=-\frac{p}{2}\frac{c_1^2x}{l}$ $B-D$: $M_{x1}=-\frac{p}{2}(c_1-x_1)^2$	$A-B$: $y=-\frac{pc_1^2l^2}{12EJ}\left(\frac{x}{l}\right)\left[1-\left(\frac{x}{l}\right)^2\right]$ $x=0{,}577\,l.$ $\max y=-\frac{pc_1^2l^2}{18\sqrt{3}\,EJ}$ $B-D$ $y_1=\frac{pc_1^3x_1}{6EJ}\left[\frac{l}{c_1}+\frac{3}{2}\frac{x_1}{c_1}-\left(\frac{x_1}{c_1}\right)^2+\frac{1}{4}\left(\frac{x_1}{c_1}\right)^3\right]$ $B-D$ $f_1=\frac{pc_1^3}{24EJ}(4l+3c_1)$ $A-C$: $y_2=\frac{pc_1^2l}{12EJ}x_2$	$\tau_1=\frac{pc_1^2}{6EJ}\times\left(1+\frac{l}{c}\right)$ $\tau_2=-\frac{pc_1^2l}{12EJ}$
13		與第7號簡支梁同		$A-B$: 與簡支梁同 $B-D$: $y_1=-\frac{pl^3}{24EJ}x_1$ $A-C$: $y_2=-\frac{pl^3}{24EJ}x_2$	

4—4 梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度(變位)	傾斜角度
14		$A=\frac{5}{8}Q$ $B=\frac{3}{8}Q$	$M_x=\frac{Qx}{2}\left(\frac{3}{4}-\frac{x}{l}\right)$ 在 $x=\frac{l}{2}$: $M_x=\frac{Ql}{16}$ $M_A=-\frac{Ql}{8}$ $+\max M=\frac{9}{128}Ql$ 在 $x=\frac{3}{8}l$	$y=\frac{Ql^3}{48EJ}\left(\frac{x}{l}-3\frac{x^3}{l^3}+2\frac{x^4}{l^4}\right)$ $\max f=\frac{Ql^3}{185EJ}$	
15		$A=\frac{P}{2}\frac{b^2}{l^3}(a+2l)$ $B=\frac{P}{2}\left(\frac{3a}{l}-\frac{a^3}{l^3}\right)=$ $=P-A$	$M_B=\frac{-Pa(l^2-a^2)}{2l^2}$ $M_C=\frac{Pa}{2}\left(2-\frac{3a}{l}+\frac{a^3}{l^3}\right)$	$f_C=\frac{Pb^3a^2}{12EJl^3}(4l-b)$	

4—4 梁之支點反力、剪力，彎矩及撓度計算公式

編號	梁的類型及荷重	支點反力及剪力	彎曲力矩	撓度（變位）	傾斜角度
16		$A=P\frac{b^2}{l^3}(2a+l)$ $B=P\frac{a^2}{l^3}(2b+l)$	$M_A=-P\frac{ab^2}{l^2}$ $M_B=-P\frac{a^2b}{l^2}$ 荷重作用處： $M_x=\frac{2Pa^2b^2}{l^3}$	荷重作用處之撓度： $y=\frac{Pa^3b^3}{3EJl^3}$	
17		$A=B=\frac{pl}{2}$	$M_A=M_B=-\frac{pl^2}{12}$ $M_x=-\frac{pl^2}{2}\left[\frac{1}{6}-\frac{x}{l}+\left(\frac{x}{l}\right)^2\right]$； 當 $x=\frac{l}{2}$ 時： $M=\frac{pl^2}{24}$	$y=\frac{pl^4}{24EJ}\left(\frac{x}{l}\right)^2\left(1-\frac{x}{l}\right)^2$ 當 $x=\frac{l}{2}$ 時： $f=\frac{pl^4}{384EJ}$	